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AN INVESTIGATION OF STUDENTS' ALGEBRAIC PROFICIENCY FROM A STRUCTURE SENSE PERSPECTIVE

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Abstract

Structure sense can be interpreted as an intuitive ability towards symbolic expressions, including skills to perceive, to interpret, and to manipulate symbols in different roles. This ability shows student algebraic proficiency in dealing with various symbolic expressions and is considered important to be mastered by secondary school students for advanced study or professional work. This study, therefore, aims to investigate students' algebraic proficiency in terms of structure sense. To reach this aim, we set up a qualitative case study with the following three steps. First, after conducting a literature study, we designed structure sense tasks according to structure sense characteristics for the topic of equations. Second, we administered an individual written test involving 28 grade XI students (16-17 year-old). Third, we analyzed students' written work using a structure sense perspective. The results showed that about two-thirds of the participated students lack of structure sense in which they tend to use more procedural strategies than structure sense strategies in solving equations. We conclude that the perspective of structure sense provides a fruitful lens for assessing students' algebraic proficiency.

Keywords: Algebraic Proficiency, Structure Sense, Equations, Procedural and Structure Sense Strategies

Abstrak

Kepekaan struktur(structure sense) dapat dimaknai sebagai sebuah kemampuan intuitif terhadap bentuk-bentuk simbolik yang meliputi keterampilan untuk memahami, menginterpretasi, dan memanipulasi simbol dalam peran yang berbeda-beda. Kemampuan ini menunjukkan kecakapan aljabar siswa dalam menangani berbagai bentuk-bentuk simbolik dan dipandang sebagai kemampuan penting untuk dikuasai oleh siswa sekolah menengah baik untuk studi lanjut maupun kebutuhan profesional dunia kerja. Oleh karena itu, penelitian ini bertujuan untuk menginvestigasi kecakapan aljabar siswa dalam perspektif kepekaan struktur. Untuk mencapai tujuan ini, kami melakukan sebuah penelitian kualitatif dengan desain studi kasus melalui tiga langkah berikut. Pertama, setelah melakukan kajian literatur mengenai kepekaan struktur, kami mendesain soal-soal mengenai kepekaan struktur berdasarkan karakteristik dari kepekaan struktur pada topik persamaan. Kedua, kami melakukan tes individu tertulis terhadap 28 siswa kelas XI (usia 16-17 tahun). Ketiga, hasil tes tertulis dari siswa tersebut dianalisis dengan menggunakan perspektif kepekaan struktur. Hasil analisis menunjukkan bahwa sekitar dua pertiga siswa yang mengikuti tes memiliki kemampuan kepekaan struktur yang rendah, di mana mereka lebih cenderung menggunakan strategi prosedural ketimbang strategi yang menekankan penggunaan kemampuan kepekaan struktur dalam menyelesaikan persamaan. Kami simpulkan bahwa perspektif kepekaan struktur menyediakan alat analisis yang bermanfaat dalam mengevaluasi kecakapan aljabar para siswa.

Kata Kunci: Kecakapan Aljabar, Kepekaan Struktur, Persamaan, Strategi Prosedural dan Strategi Kepekaan Struktur

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Algebra as one of the branches of mathematics is considered to be an important domain for secondary school students over the world for either advanced study or professional work (Bednarz et al., 1996; Carraher et al., 2006; Katz, 2007; Kop et al., 2020). Proficiency in algebra domain is therefore one of the preconditions for school students to pursue their future careers. Algebraic proficiency, which can be described as a matter of proficiency with symbolic representations, in general includes procedural fluency and conceptual understanding (MacGregor & Price, 1999; McCallum, 2007; Stiphout et al., 2013). In previous studies, these two aspects of algebraic proficiency are often assessed through the lens of symbol sense and structure sense (Bokhove & Drijvers, 2010; 2012; Novotna & Hoch, 2008; Stiphout et al., 2013). The results of these studies showed that the lens of structure sense is fruitful to explain lack of student conceptual understanding and procedural skills in algebra.

The lack of student algebraic proficiency occurred over the world, including in Indonesia. Results of international survey studies, such as Trends in International Mathematics and Science Study (TIMSS) in 2011, showed that Indonesian students have a low score in algebra, i.e., the students were in 38th position out of 42 countries (Mullis et al., 2012). Regarding this low score in algebra, previous studies have investigated student difficulties in school algebra, in which the results revealed that Indonesian students lack of both algebraic procedural skills and conceptual understanding (Apsari, Putri et al., 2020; Apsari, Sariyasa et al., 2020; Jupri et al., 2014a; 2015; Sugiarti & Retnawati, 2019; Wahyuni et al., 2020). These aspects of algebraic proficiency for Indonesian students therefore need to be further explored. In Indonesia, however, research for investigating students' algebraic proficiency in terms of structure sense is still limited and is focused on initial algebra learning (e.g., Jupri et al., 2014b), and on abstract algebra for university students (e.g., Junarti et al., 2019; Widodo et al., 2018). In other words, an investigation of student algebraic proficiency for students who have learned a more advanced study of school algebra in Indonesia, to certain extent, is still unexplored. Therefore, this current study aims to investigate secondary school students' algebraic proficiency from a structure sense perspective.

The lens of structure sense was initially used to explain students' difficulties with using knowledge of arithmetic structures in the context of learning initial algebra (Linchevski & Livneh, 1999). Next, this term was refined and developed by Hoch and Dreyfus (2006) as a collection of abilities towards symbolic expressions, including skills to interpret, to manipulate and to perceive symbols.

For the purpose of the present study, we distinguish between the use of procedural and structure sense strategies when dealing with algebra problems. One exhibits the use of procedural strategy if she or he can, for instance, solve equations using certain algebraic procedures without considering the efficiency of the procedures (Hoch & Dreyfus, 2006; Jupri & Sispiyati, 2020). For example, to solve the equation $(2x-5)^2-5(2x-5)-6=0$, rather than using a substitution $(2x-5)$ to obtain a simpler equation that can be solved using a more efficient strategy, someone inefficiently transforms the equation into $4x^2-30x+44=0$ and solve it using a quadratic formula. Someone displays structure sense strategies when dealing with algebra problems if she or he can: (1) recognize a familiar structure in its simplest form; (2) deal with a compound term as a single entity; and (3) choose appropriate manipulations to make best use of a structure (Hoch & Dreyfus, 2006; 2010; Novotna & Hoch, 2008). For example, to solve the equation $(x-3)^4-(x+3)^4=0$ by using structure sense strategies, someone can deal with $(x-3)^2$ and

$(x+3)^2$ as single entities, can use these as substitutions to obtain a simpler equation, and can manipulate the equation into $((x-3)^2+(x+3)^2)\,((x-3)^2-(x+3)^2)=0$.

METHOD

To investigate students' algebraic proficiency from a structure sense perspective, we carried out a qualitative case study (Yin, 2015)—which is part of larger study on investigating secondary school students' algebraic proficiency—with the following three steps. First, we carried out literature study on the theory of and previous research on structure sense for algebra education either for school or university students (Hoch & Dreyfus, 2010; Junarti et al., 2019; Novotna & Hoch, 2008). The theory of structure sense, particularly the three characteristics of structure sense ability (Hoch & Dreyfus, 2006; 2010; Novotna & Hoch, 2008), was used for designing three types of tasks. The tasks and corresponding characteristics of the structure sense used in this study are presented in Table 1. These algebra tasks were theoretically validated by four experts in mathematics education to ensure its appropriateness to secondary school students' level and to the structure sense characteristics. The designed tasks are on the topic of equations, including quadratic or related to quadratic equations. A general structure of equations designed for this study is of the form $A^2-B^2=0$, i.e., equations of the form of difference of squares. We predicted that Task Type 1 is the easiest one, Task type 2 is more difficult than the Task Type 1, and Task Type 3 is the most difficult one for students.

Table 1. Structure sense tasks on solving equations

Task Type	Structure Sense Characteristics	Tasks
1.	Recognize a familiar structure in its simplest form, i.e., recognize difference of squares and factor accordingly	$64-x^2=0$
2.	Deal with a compound term as a single entity and through an appropriate substitution recognize a familiar structure in a more complex form, i.e., see compound terms $(2x-1)$ and $(x+2)$ as singles entities, recognize difference of squares, and factor accordingly	$(2x-1)^2-(x+2)^2=0$
3.	Choose appropriate manipulations to make best use of a structure, i.e., see the possibility of difference of squares, extract common factor, deal with (x^2-2x) and $(x-2)$ as single entities, and factor accordingly	$(x^2-2x)^2-x^2+4x-4=0$

Second, we administered an individual written test involving 28 grade XI students (16-17-year-old) after they had been taught the topic of quadratic equations. The students came from the same one class from one of secondary schools in Bandung, Indonesia. The written test, using the three tasks shown in Table 1, was lasted for about forty minutes. In this test, as written in the direction, students were requested to write down their solutions on answer sheets and were not allowed to use calculators or smartphones during the test. For solving each task, we requested students to use two different strategies. For this purpose, two blank spaces below each task are provided for students to put different solution strategies. In this way, we expected students to use both procedural and structure sense strategies in the solution processes. In addition to students' written work on answer sheets, as part of data triangulation, we also collected students' scratch paper for helping us in interpreting students' solution processes.

Third, in the data analysis, we analyzed students' written work and their corresponding scratch papers by classifying student solution strategies into procedural and structure sense strategies. Through this classification, we decided whether a student applies characteristics of structure sense ability or not for each task. For instance, if the student uses an appropriate substitution for solving the Task Type 2—which concerns a structure sense characteristic, then this is classified as a structure sense strategy. Otherwise it would be classified a procedural strategy. Next, as the results of this classification, we concluded whether students lacked of conceptual understanding or not: A student is considered to be lacked of conceptual understanding if she or he tends to use procedural rather than structure sense strategies; and a student is perceived to have good conceptual understanding if she or he tends to use more efficient structure sense strategies. Students' success in dealing with the tasks, either using structure sense or procedural strategy, is qualitatively perceived to acquire good algebraic proficiency. If a student does not provide any answer (blank answer sheet), then she or he is considered to provide an incorrect solution. In addition to analyze student solution strategies, we also analyzed student difficulties when students use either procedural or structure sense strategies. Failure in use of a certain strategy in solving a task is considered an indication of student difficulty in dealing with the algebra task and is perceived lack of algebraic proficiency.

RESULTS AND DISCUSSION

General Findings

Table 2 presents findings of students' written work on solving quadratic and related quadratic equations. In general, as predicted in the design process, the Task Type 3 is the most difficult for most of the students, and the Task Type 2 is more difficult than the Task type 1. Even if the Task Type 1 is the easiest one, about 32% of the participated students made mistakes and as a consequence provided incorrect solutions.

Concerning solution strategies used by the students, procedural and structure sense strategies were observed in students' written work, in which procedural strategies emerged more frequent than structure sense strategies for each type of tasks. For the Task Type 1, 75% of the participated students are able to use structure sense strategies; surprisingly enough there is no students who used the structure sense strategies for the Task Type 2; and only about 11% of participated students used the structure sense strategies for solving the Task Type 3. These results may suggest that procedural fluency is better mastered by students than conceptual understanding. In other words, most participated students lacked of algebraic proficiency from the perspective of structure sense. In the following sections, we address findings and discussion for each type of tasks.

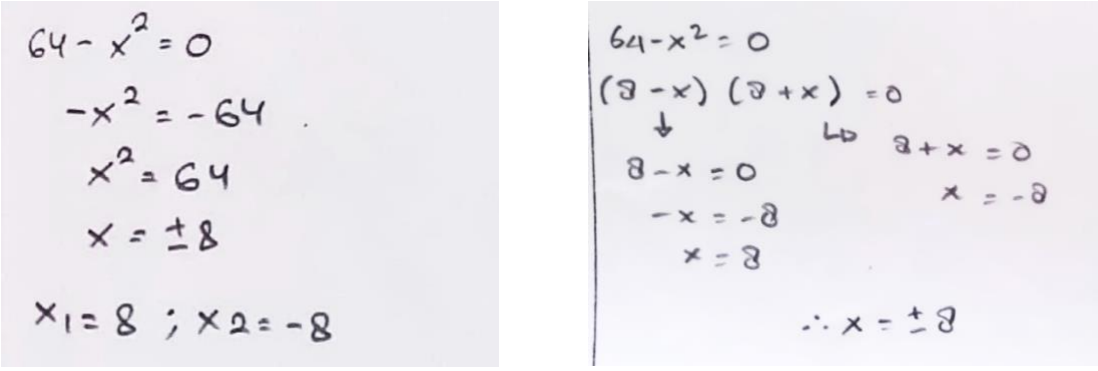
Table 2. Results from data analysis of the written test (N = 28)

Task Type	Tasks	#Correct Solution (%)	Solution Strategies	
			#Procedural Strategy (%)	#Structure Sense Strategy (%)
1	$64 - x^2 = 0$	19 (67.8)	28 (100.0)	21 (75.0)
2	$(2x - 1)^2 - (x + 2)^2 = 0.$	11 (39.3)	23 (82.1)	0 (0.0)
3	$(x^2 - 2x)^2 - x^2 + 4x - 4 = 0$	2 (7.1)	25 (89.3)	3 (10.7)

Results and Discussion for Task Type 1

From the findings shown in Table 2, we found that Task Type 1, i.e., solving $64 - x^2 = 0$, seems to be the easiest task because about 68% of participated students answered it correctly. Even if all students in the first chance used procedural strategies, 75% of the students used structure sense strategies in the second chance. A typical procedural strategy for solving the equation $64 - x^2 = 0$ is as follows. A student rewrites the equation, for instance, into $x^2 = 64$. Next, the student concludes $x = \pm\sqrt{64} = \pm 8$. So, the solution of the equation includes $x = 8$ or $x = -8$. Incorrect solutions occurred when students conclude $x = \sqrt{64} = 8$ as the only solution for the equation.

A typical observed structure sense strategy from students' written work is as follows. A student sees the equation $64 - x^2 = 0$ as $8^2 - x^2 = 0$. Next, by applying the property of $A^2 - B^2 = (A + B)(A - B)$, the student writes $(8 + x)(8 - x) = 0$ and concludes $x = -8$ or $x = 8$ as the solutions of the equation. Recognizing a familiar structure in its simplest form concerns a structure sense characteristic, which shows an algebraic proficiency and is part of symbol sense behavior (Bokhove & Drijvers, 2010; 2012; Kop et al., 2020; Stiphout et al., 2013). Figure 1 shows representative examples of students' written work for solving the equation of Task Type 1. Figure 1(a) shows the use of procedural strategy and Figure 1(b) shows the use of structure sense strategy. In this structure sense strategy, a student can recognize a familiar structure of difference of squares and use factorization method in the solution process accordingly.

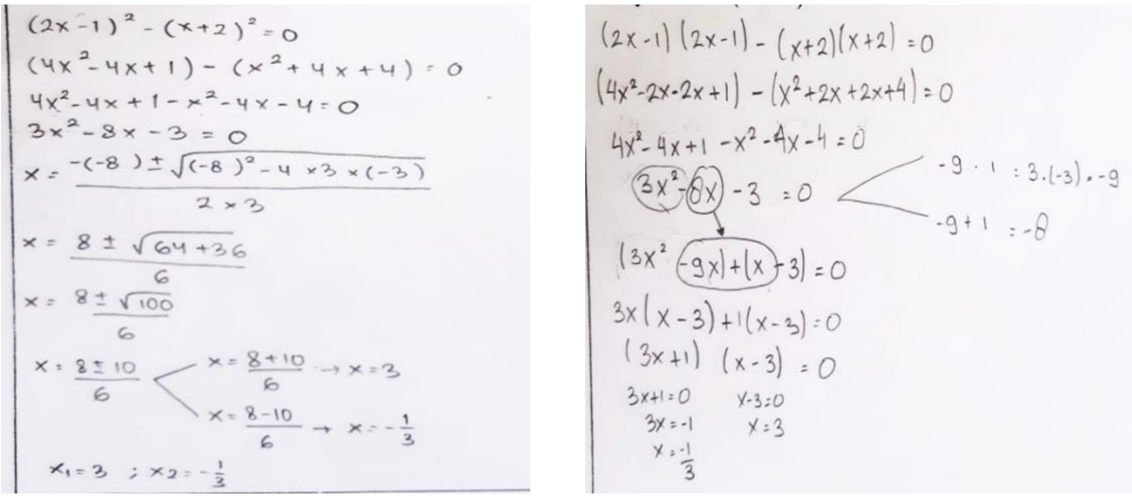


(a) (b)

Figure 1. Representative examples of students' written work on the Task Type 1: Part (a) shows the use of procedural strategy and part (b) shows the use of structure sense strategy

Results and Discussion for Task Type 2

We found that 11 out of 28 students (39.3%) are able to solve the Task Type 2 correctly, i.e., solving the equation $(2x-1)^2-(x+2)^2=0$. Surprisingly enough, as shown in Table 2, all 23 students who tried to answer this task used procedural strategies and no one provides a structure sense strategy. A typical procedural strategy for solving the equation is as follows: a student expands $(2x-1)^2$ and $(x+2)^2$ and does subtraction to obtain $3x^2-8x-3=0$. Next, the student solves this task using either factorization method or quadratic formula to obtain $x=-1/3$ or $x=3$ as the solution of the equation. Incorrect solutions occurred when students made mistakes while expanding, factoring, or using the quadratic formula. Figure 2 presents representative examples of students' written work for this task using the procedural strategy. Figure 2(a) shows the use of the quadratic formula and Figure 2(b) shows the use of the factorization method.



(a) (b)

Figure 2. Representative examples of students' written work for the Task Type 2: Part (a) shows the use of the quadratic formula and Part (b) shows the use of the factorization method

The absence of structure sense strategies for the Task Type 2 seems to be caused of students' inability to see the compound terms $(2x-1)$ and $(x+2)$ as single entities. As a consequence, students did not see that the equation has a familiar form $A^2-B^2=0$. So, rather than factoring the equation into $[(2x-1)+(x+2)][(2x-1)-(x+2)]=0$, students directly expand $(2x-1)^2$ and $(x+2)^2$ and do the factorization method or use the quadratic formula. Another possible cause of the absence of the structure sense strategies might be because the equation itself is relatively easy to solve using expanding and factorization method or the use of the quadratic formula. This result may suggest that students lacked of structure sense ability particularly in dealing with compound terms as single entities. The inability of seeing compound terms as single entities suggests students' failure to read through and to gain meaning before manipulating algebraic expressions (Arcavi, 1994; 2005; Bokhove & Drijvers, 2010; Hoch & Dreyfus, 2006), and may indicate lacked of conceptual or relational understanding (Skemp, 1976) and algebraic thinking ability (Apsari, Putri et al., 2020; Kusumaningsih et al., 2018).

Results and Discussion for Task Type 3

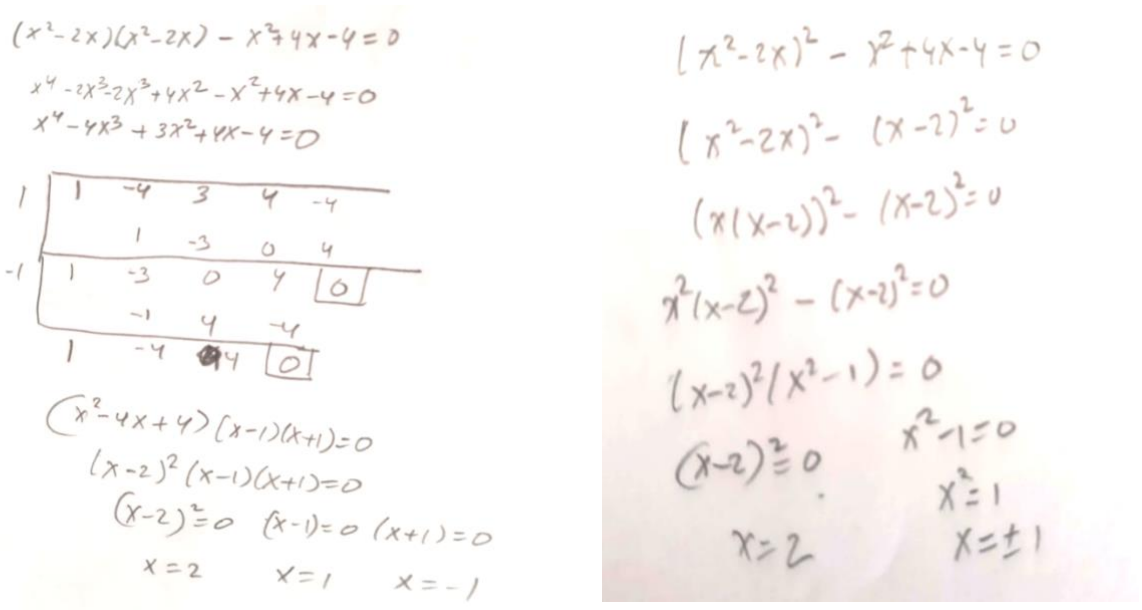
As shown in Table 2, for the case of solving the equation $(x^2-2x)^2-x^2+4x-4=0$, a number of 25 students used procedural strategies in the first chance and three students used structure sense strategies in the second chance. Of the 28 students, two students solved the task correctly in which these students used both procedural and structure sense strategies.

A typical procedural strategy used by students observed in students' written work is as follows. After expanding the term $(x^2-2x)^2$ into $x^4-4x^3+4x^2$, a student simplifies the equation into $x^4-4x^3+3x^2+4x-4=0$. Next, the student factorizes this fourth-degree equation using the Horner method to obtain $(x-2)^2(x-1)(x+1)=0$ and concludes that $x=1, x=-1$, and $x=2$ as solutions for the equation. Students' difficulties in applying this procedural strategy includes difficulties in applying the Horner method and in seeing the structure of the equation having the form of $A^2-B^2=0$, where $A=x^2-2x$ and $B=x-2$.

A typical structure sense strategy used by students is as follows. A student rewrites the equation $(x^2-2x)^2-x^2+4x-4=0$ into $(x^2-2x)^2-(x-2)^2=0$. Next, the student sees the compound terms (x^2-2x) and $(x-2)$ as single entities. As a consequence, the student sees the equation has the form $A^2-B^2=0$. By factoring this form into $[(x^2-2x)+(x-2)][(x^2-2x)-(x-2)]=0$ or $(x-2)(x+1)(x-2)(x-1)=0$, the student concludes that $x=-1, x=1$, and $x=2$ are solutions for the equation. Figure 3 presents examples of students' written work for the case of the Task Type 3. Figure 3(a) shows the use of procedural strategy and Figure 3(b) shows the use of structure sense strategy in solving the equation. For the case of Figure 3(b), the structure sense strategy used by the student is slightly different. After rewriting the original equation into $(x^2-2x)^2-(x-2)^2=0$, the student factorized the term (x^2-2x) into $(x(x-2))$. As a consequence, the student factorized the whole equation into $x^2(x-2)^2-(x-2)^2=(x-2)^2(x^2-1)=0$, and finally concluded $x=1$,

$x = -1$, and $x = 2$ as the solutions of the equation. This difference still produced an efficient structure sense strategy.

The lack of use of structure sense strategies for the case of the Task Type 3 not only shows lack of student conceptual understanding on algebraic expressions (Stiphout et al., 2013), but also indicates lack of, in terms of realistic mathematics education theory, ability to do vertical mathematization (De Lange, 2006; Freudenthal, 1991; Van den Heuvel-Panhuizen & Drijvers, 2014). This concerns difficulties to see a familiar structure from a symbolic expression for doing symbolic manipulation in the world of mathematics.



(a) (b)

Figure 3. Examples of students' written work for the Task Type 3: Part (a) shows the use of procedural strategy and part (b) shows the use of structure sense strategy

CONCLUSION

The fact that in general grade XI students tend to use procedural rather than symbol sense strategies for solving equations shows that these students may lack of algebraic proficiency, particularly the aspect of conceptual understanding. The use of procedural strategies relates to the aspect of procedural skills and the use of structure sense strategies indicates more on the aspect of conceptual understanding.

The frequent use of procedural rather than structure sense strategies for solving equations with compound and complex terms may indicate that students encountered difficulties in recognizing a familiar structure that they should have already known. This finding can lead to further investigation, for instance, whether the occurrence of the procedural strategy is caused by the emphasize use of this strategy in the learning and teaching process, or whether students have no experience in using a more efficient structure sense strategy in equation solving.

Even if the procedural strategy seems to be mastered by participated students, still incorrect solutions frequently appeared with this type of strategy. This suggests that the procedural strategy could not be regarded as has been acquired more by students from the learning and teaching processes than the structure sense strategy. The use of procedural strategies by the students are not perfect yet as they still encountered difficulties and mistakes in equation solving. Therefore, for further research, we suggest to do a more comprehensive study in investigating student algebraic proficiency using the perspective of structure sense, not only limited to the use of students' written work on answer sheets and its corresponding data of scratch papers, but also using for instance interview data.

Concerning unpredicted findings, such as the use of different strategies of structure sense or even the absence of this type of strategy, for future research we recommend to do data analysis using structure sense characteristics in a more specific manner. This can be done for instance by investigating students' ability more deeply for each characteristics of structure sense, i.e., an ability in recognizing a familiar structure in its simplest form, in dealing with a compound term as a single entity and through an appropriate substitution recognize a familiar structure in a more complex form, and in choosing appropriate manipulations to make best use of a structure. In this way, the lens of structure sense is used more sharply in determining students' algebraic proficiency, particularly in determining students' acquisition of conceptual understanding and procedural skills.

In spite of the conclusions above, we acknowledge that this study has several limitations. As we only have students' written work data, including students' answer sheets and scratch papers, triangulation of the data for this explorative study is limited. In addition, as this study included a small number of research participants (i.e., 28 grade XI students), we could not make generalization. As a consequence, a larger number of research participants in future research might provide better information about students' algebraic proficiency in Indonesia.

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REFERENCES

Apsari, R. A., Putri, R. I. I., Sariyasa, Abels, M., & Prayitno, S. (2020). Geometry representation to develop algebraic thinking: A recommendation for a pattern investigation in pre-algebra class. *Journal on Mathematics Education*, *11*(1), 45-58. http://doi.org/10.22342/jme.11.1.9535.45-58

Apsari, R. A., Sariyasa, S., Putri, R. I. I., Gunawan, G., & Prayitno, S. (2020). Understanding students' transition from arithmetic to algebraic thinking in the pre-algebraic lesson. *Journal of Physics: Conference Series*, *1471*(1), 012056. http://doi.org/10.1088/1742-6596/1471/1/012056

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, *14*(3), 24–35. http://flm-journal.org/Articles/BFBFB3A8A2A03CF606513A05A22B

Arcavi, A. (2005). Developing and using symbol sense in mathematics. *For the Learning of Mathematics*, *25*(2), 42–47. http://www.jstor.org/stable/40248497

Bednarz, N., Kieran, C., & Lee, L. (1996) Approaches to Algebra: Perspectives for research and teaching. In: Bernarz N., Kieran C., Lee L. (eds) *Approaches to algebra*. Mathematics education Library, vol 18. Springer, Dordrecht. http://doi.org/10.1007/978-94-009-1732-3_1

Bokhove, C., & Drijvers, P. (2010). Digital activities for capturing symbol sense. *For the Learning of Mathematics*, *30*(3), 43-49. http://flm-journal.org/Articles/45D5ED606856931839C164A32C1092

Bokhove, C., & Drijvers, P. (2012). Effects of feedback in an online algebra intervention. *Technology, Knowledge and Learning*, *17*(1), 43-59. https://doi.org/10.1007/s10758-012-9191-8

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*(2), 87-115. http://www.jstor.org/stable/30034843

De Lange, J. (2006). Mathematical literacy for living from OECD-PISA perspective. *Tsukuba Journal of Educational Study in Mathematics*, *25*(1), 13–35. http://www.human.tsukuba.ac.jp/~mathedu/2503

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer Academic Publishers. http://doi.org/10.1007/0-306-47202-3

Hoch, M., & Dreyfus, T. (2006). Structure sense versus manipulation skills: An unexpected result. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 305-312). Prague, Czech Republic: PME.

Hoch, M., & Dreyfus, T. (2010). Developing Katy's algebraic structure sense. In *Proceedings of the 6th Congress of the European Society for Research in Mathematics* (pp. 529-538). Lyon, France: INRP.

Junarti, Sukestiyarno, Y. L., Mulyono, & Dwidayati, N. K. (2019). The profile of structure sense in abstract algebra instruction in an Indonesian mathematics education. *European Journal of Educational Research*, *8*(4), 1081-1091. http://doi.org/10.12973/eu-jer.8.4.1081

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014a). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683-710. http://doi.org/10.1007/s13394-013-0097-0

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014b). Student difficulties in solving equations from an operational and a structural perspective. *International Electronic Journal of Mathematics Education*, *9*(1), 39-55. https://www.iejme.com/article/student-difficulties-in-solving-equations-from-an-operational-and-a-structural-perspective

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2015). Improving grade 7 students' achievement in initial algebra through a technology-based intervention. *Digital Experiences in Mathematics Education*, *1*(1), 28-58. http://doi.org/10.1007/s40751-015-0004-2

Jupri, A., & Sispiyati, R. (2020). Students' algebraic proficiency from the perspective of symbol sense. *Indonesian Journal of Science and Technology*, *5*(1), 86-94. http://doi.org/10.17509/ijost.v5i1.23102

Katz, V. J. (Ed.) (2007). *Algebra: Gateway to a technological future*. The Mathematical Association of America. http://www.maa.org/news/new-report-algebra-gateway-technological-future

Kop, P. M., Janssen, F. J., Drijvers, P. H., & van Driel, J. H. (2020). The relation between graphing formulas by hand and students' symbol sense. *Educational Studies in Mathematics*, *105*(2), 137-161. http://doi.org/10.1007/s10649-020-09970-3

Kusumaningsih, W., Darhim, Herman, T., & Turmudi. (2018). Improvement algebraic thinking ability using multiple representation strategy on realistic mathematics education. *Journal on Mathematics Education*, *9*(2), 281-290. https://doi.org/10.22342/jme.9.2.5404.281-290

Linchevski, L., & Livneh, D. (1999). Structure sense: The relationship between algebraic and numerical contexts. *Educational Studies in Mathematics*, *40*(2), 173-196. http://doi.org/10.1023/A:1003606308064

MacGregor, M., & Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. *Journal for Research in Mathematics Education*, *30*(4), 449-467. http://doi.org/10.2307/749709

McCallum, W. G. (2007). Assessing the strands of student proficiency in elementary algebra. In A. H. Schoenfeld (Ed.), *Assessing mathematical proficiency* (pp. 157-162). Cambridge, New York: Cambridge University Press. http://doi.org/10.1017/CBO9780511755378

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Boston: TIMSS & PIRLS International Study Center. http://timss.bc.edu/timss2011/international-results-mathematics.html

Novotna, J., & Hoch, M. (2008). How structure sense for algebraic expressions or equations is related to structure sense for abstract algebra. *Mathematics Education Research Journal*, *20*(2), 93-104. http://doi.org/10.1007/BF03217479

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*(1), 20-26. https://www.atm.org.uk/write/MediaUploads/Resources/Richard_Skemp.pdf

Stiphout, I. V., Drijvers, P., & Gravemeijer, K. (2013). The development of students' algebraic proficiency. *International Electronic Journal of Mathematics Education*, *8*(2-3), 62-80. http://doi.org/10.29333/iejme

Sugiarti, L., & Retnawati, H. (2019). Analysis of student difficulties on algebra problem solving in junior high school. *Journal of Physics: Conference Series*, *1320*(1), 012103. http://doi.org/10.1088/1742-6596/1320/1/012103

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*, 713-717. http://doi.org/10.1007/978-94-007-4978-8

Wahyuni, R., Prabawanto, S., & Herman, T. (2020). Students' difficulties in solving algebra task in middle school. *Journal of Physics: Conference Series*, *1521*(3), 032071. http://doi.org/10.1088/1742-6596/1521/3/032071

Widodo, S. A., Prahmana, R. C. I., Purnami, A. S., & Turmudi. (2018). Teaching materials of algebraic equation. *Journal of Physics: Conference Series*, *943*(1), 012017. http://doi.org/10.1088/1742-6596/943/1/012017

Yin, R. K. (2015). *Qualitative research from start to finish*. New York, London: Guilford publications.