LEARNING ONE-DIGIT DECIMAL NUMBERS BY MEASUREMENT AND GAME PREDICTING LENGTH

Puji Astuti

Undergraduate Program on Mathematics Education, Sriwijaya University, Jalan Padang Selasa No. 524, Palembang-30139, Indonesia
email: p.astutipuji@gmail.com

Abstract

This paper aims to describe how students develop understanding of one-digit decimals. To achieve the aim, Local Instruction Theory (LIT) about the process of learning decimals and the means designed to support that learning are developed. Along with this idea, the framework of Realistic Mathematics Education (RME) is proposed. Based on the aim, design research methodology is used. This paper discusses learning activities of three meetings from teaching experiment of the focus group students of the fourth grade elementary school in Surabaya: SDIT Al Ghilmani. The data indicated that the learning activities promoted the students' understanding of one-digit decimal numbers.

Keyword: measurement, decimal numbers, number line

Abstrak

Artikel ini bertujuan untuk menggambarkan bagaimana siswa mengembangkan pemahaman desimal satu digit. Untuk mencapai tujuan tersebut, *Local Instruction Theory (LIT)* tentang proses belajar desimal dan sarana yang dirancang untuk mendukung pembelajaran tersebut dikembangkan. Seiring dengan ide ini, teori Pendidikan Matematika Realistik (RME) diusulkan. Berdasarkan tujuan, metodologi penelitian adalah *design research*. Makalah ini membahas tiga pertemuan kegiatan belajar mengajar kelas eksperimen untuk grup fokus kelas 4 SDIT Al Ghilmani, Surabaya. Data menunjukkan bahwa kegiatan pembelajaran mendukung pemahaman siswa mengenai desimal satu digit.

Kata Kunci: pengukuran, bilangan desimal, garis bilangan

The concept of decimals is included in the mathematics curriculum, *Kurikulum Tingkat Satuan Pendidikan* (KTSP) of the fifth grade of elementary school in Indonesia, and is considered to be of great significance especially due to its application and use in everyday life (Michaelidou, Gagatsis, & Pantazi, 2004). Also, it is important for students to learn decimals because the decimal concepts support each other learning mathematics topics especially measurements, fractions, proportions, and percentages (van Galen et al., 2008). In brief, decimals are essential in both mathematics topics bonds and its applications.

Nevertheless, studies (Moloney & Stacey, 1997; Asnawati, 1999; Irwin, 2001; Moskal & Magone, 2001; Steinle, 2004; Lai & Tsang, 2009; Pramudiani, 2011; Sengul & Guldbagci, 2012) and field data reported students' weaknesses in decimals. Most common areas of problems in learning decimals are weak understanding of place value and weak notions of the magnitude of decimal numbers.

With these reasons, this study intends to support students building an understanding the concepts of decimals. In the same vein with van Galen et al. (2008), this study tries to link decimals with fractions to support students being able to provide meaning for the notation. Markovits and Sowder stated that the development of decimal knowledge can be supported by relationships that exist between fractions and decimals (as cited in Moskal & Magone, 2000).

Activities to measure lengths and a game are proposed. Length measurement is chosen to foster ideas of fractions. Decimals could be introduced by relating tenths as one-digit decimals. Furthermore, the measurement scale could be easier to relate to number line as a model of decimal numbers. The game then designed in this study is to emerge decimals on number line. As Thomson and Walker (as cited in Michaelidou et al., 2008) suggested that the number line contributes to the development of concepts not only related to the identification and comparison of decimals but also to the ability to perform operations.

This research is conducted with aim to describe how students develop the understanding of one-digit decimals. The research question addressed is: "How can students develop the understanding of one-digit decimals?"

METHOD

Based on the research question and the aim of the research, then the type of the research is categorized as design research. This research contributes to develop a Local Instruction Theory (LIT) to support students develop the understanding of decimals. The LIT consists of both provisional instructional activities and a conjectured learning process that anticipates students' thinking and understanding might evolve when the instructional activities are employed in the classroom (Gravemeijer & Cobb, 2006, p. 48). The design research in this study is an iterative process which refers to the research incorporates a cyclic approach of preparing for the experiment, teaching experiment, and retrospective analysis (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006).

The cyclic process of developing the LIT to learn decimals in this study is prototyped by a Hypothetical Learning Trajectory (HLT) which is elaborated and refined when conducting the design. The initial HLT of this study is developed based on the analysis of key areas of decimals from literature review, the analysis of Indonesia mathematics curriculum, and the analysis of the potential use of contexts and model based on the framework of RME.

Data collection consists of observation, interview, and written work. What kind of data in observation are lesson observations of students and teacher: video registrations of whole class discussions, video registrations of group work, and participating observations like field notes by researcher. What kinds of data in interviews are individual or group works of students like critical learning moment, students' opinions, and teacher's pre-post experiment and between lessons of teaching the topic of decimals. What kind of data in written work are students' notations, calculations, and explanations.

This paper explains about the teaching and learning process of teaching experiment phase for three meetings which is about one-digit decimals.

RESULT AND DISCUSSION

Here is reported a description of teaching experiment of learning one-digit decimal numbers of students in the fourth grade of SDIT Al-Ghilmani Surabaya and more focus to the focus group students: student 5, student 6, student 7, and student 8. The classroom teacher (T) acted as a teacher and the researcher (R) acted as an observer who also got involved in learning process.

Meeting 1: Acquisition of Tenths

The first meeting aims to encourage students to use tenths in notating length of things before they turn into a decimal quotient later. There are two main activities: first, students measure table with their span and investigate why the measuring would give different length result for same objects, so the question to discuss is how to get the same measuring result. The discussion would end by introducing the paper strip as the standard length of their span to do measuring. Second, students explore measuring with the paper strip and discuss what would you do if the paper strip does not fit for many times? This leads them to symbolize the length as unit-fractions.

In the first activity, not as expected in the HLT students would get various length when measuring a table with a span, in the class, the students measured various things with various non-standard measurements: span, thumb, and foot. Although so, they who measured a table compared, in the whole class, the different result of their measurement: 3 spans, 1 step, 34 thumbs, 3 feet, 50 little fingers, and 3 caps. Being asked why different, the focus group students argued it was because of different ways of measuring and different children who measured. Other groups discussed why different, because the measurement tool they used was different; there was a long span and a short span. One group suggested using a measurement tool which has the same length.

In the next activity, as the students suggested using the same measurement tool, expected in the HLT, hence the paper strip was introduced, representing a span. They in group measured things in the classroom. They were struggling to say the rest length; the length which was not exact whole paper strip. In the focus group, student 6 initially measured the length of a table and found that the table did not fit completely the paper strip. He and friends discussed how to say it and started to use unit-fractions (see fragment 1).

1	S_5	It is one-fifth *(told the rest measuring table)*
2	R	One-fifth. How? How do you show that it was one-fifth?
3	S_5	*(S_5, used his hand, estimated dividing the paper strip into 5)*
4	R	You show this to your friend. Hey, look at your friend! He got it.
5	S_5	One-fifth.
6	S_6	Three-fifth? (confused because the table was 2 paper strips with rest)

7	S_5	One-fifth.
8	T	How is the length? What did you measure?
9	S_5	Two-fifth (changed from one-fifth to two-fifth because the table was 2 paper strips with rest).

Fragment 1: Discussion on notating length with fractions

In fragment 1, the students showed their mastery of naming fraction parts but confused because the length was mixed fraction (fragment 1 line 5-9). The following discussion showed they used both fraction and decimal numbers to say the length (see fragment 2).

1	S_6	Two-fifth. How we would write it?
2	S_5	**Two comma five.**
3	T	Just write what your group discussed, how do you think, if you found three, three and what?
4	S_6	Ok, it is two comma five?
5	T	Two comma five what [unit]?
6	S_5	This one (showing the paper strip), what paper is this?
7	S_6	Paper strip.

Fragment 2: The students started using decimals

The teacher then conducted classroom discussion and invited the students to compare their approach dividing the paper strip. The different fractions are hoped would puzzle the students whether the length is actually the same, so they need a refinement of fractions they need to use. However, the class seemed confused how to notate the rest: only in terms of halves, quarters, making incorrect fractions and even pretended that the length was exact the paper strip. Thus, the teacher directly asked the students to divide the paper strip into 10 parts. The teacher explained this decision was made because of our ten-base system.

The teacher drew on the whiteboard the paper strip divided in 10 parts and discussed the fraction name of each part. The students practiced to say the tenths. Then the teacher measured the height of the whiteboard and guided the students to name the length in tenths. The plan to ask the students to measure again with divided paper strip was cancelled due to time limit.

From the findings so far, it was observed that the focus group students were in their phase developing the fraction language to represent the parts of the paper strip. Another fact, they performed decimals in incorrect manner. Fragment 1 and fragment 2 give information how they misunderstood naming with fractions and decimals: 2 paper strips and $\frac{1}{5}$ was said as $\frac{2}{5}$ and was written 2,5. It was also noticed the other students had problems naming fractions. Overall, the students still had problem: 1)

name the fractional pieces, especially writing mixed fraction and 2) make incorrect equivalent relation of fraction and decimal number.

In conclusion, however, this lesson 1, with the paper strip, gave experience for the students measured things, divided the paper strip, gave estimation and discussed its rest with fraction language. Although it was the teacher telling them to divide the strip in 10, from the process gaining fraction it is known that they developed the sense of tenths. The instructional activities were set for students' understanding of decimal notation by its relation with tenths. Therefore, although the decimal numbers were already mentioned by students at initial phase, it was decided, as in HLT, to begin with students' good sense of tenths.

Meeting 2: Notation of One-Digit Decimal Numbers

In the second lesson the context of measurement was again presented to generate decimal notation. They were asked to measure things in the classroom with divided-10 paper strip and notate the result in terms of fractions. Then, the question asked to them was to leave out the fraction tenth, what would you write?

At the beginning of the class, the students divided the paper strip into 10 parts. Some folded it and some used ruler making each part 2 cm (the paper strip was 20cm). It was noticed that students of our focus group used long names for the fractions that came up, but improved their fraction understanding, like the length of a stake was "8 more $\frac{2}{10}$ paper strips". Meanwhile, there were some students excluding the focus group struggled making the fractions in term of the length of a paper strip, for instance the length of a cap was written as *1 and 5 paper strip* (the students intended to say $1\frac{5}{10}$), and the height of a table was written $\frac{20}{20}$ = *2* (the students counted the small part of the paper strip, so 2 paper strips was 20 small parts).

The problem 'what do you write for the length if we do not use tenths?' was supposed to be group discussion, but became whole class discussion. The teacher asked 'if we erase tenth in 8 more $\frac{2}{10}$ paper strips'. The solutions proposed by some children written on the boards were discussed in whole class. They answered with informal and long names: 8 and more 12 paper strips, 8 and more 2 parts from 10 parts of the paper strip, 8 and more $\frac{1}{5}$ paper strip (student 5 tried to make different answer by using equivalent fraction), and 8,2 paper strip. The teacher took their responses into a discussion leading to decimals (see fragment 3).

1	T	If we don't use the word 'more' like this, in mathematical language, whose answer we could use?
2	Ss	*(pointed the answer 8,2 paper strip)*
3	T	What does 8,2 mean?
3	S_{15}	Its rest.

4	S_{14}	I know, ustadz *(teacher)*. It is equal with $8\frac{2}{10}$.
5	T	Can you relate 8,2 with the paper strip? what is 8, what is comma, what is 2?
6	S_{14}	Comma is decimal fraction
7	S_5	It was 10 *(moving his hand as if drawing paper strip)*, and the rest was 8.
8	T	Who measured the stake? What does 8 mean?
9	S_5	(student who measured the stake) 8 paper strips
10	T	And the rest was $\frac{2}{10}$ means 2 parts of 10 parts. So, when we deleted $\frac{}{10}$, Abil wrote 8,2. What is 8?
11	Ss	Paper strips
12	T	8 means ones, and what is 2?
13	S_6	2 parts of 10 parts
14	T	Why do we have comma?
15	S_5	Comma means more, because there is more
16	S_{14}	Decimal fraction
17	T	Why it was decimal fraction? Why we use comma?
18	S_{10}	To differ the rest
19	T	To differ the rest and the ones

Fragment 3: Whole class discussion about decimal notation

Soon, the students in the focus group were able to communicate their measuring result now, using fraction tenth before, in decimal numbers (see figure 1).

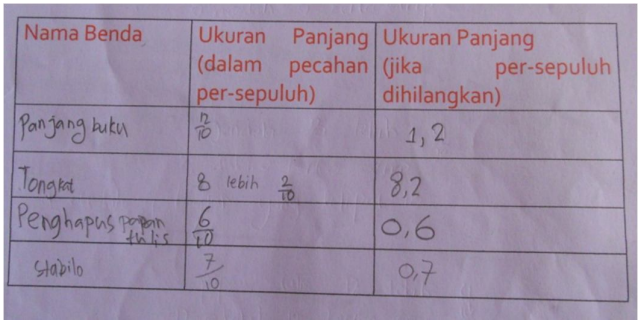

Figure 1: Focus group students transformed tenths into decimal numbers

While discussing tenths to decimal numbers, the confusion of focus students appeared how to name lengths with decimal number for small things, like the length of an eraser, $\frac{6}{10}$ (see fragment 4).

1	R	If $\frac{6}{10}$ what is the decimal form of it?
2	S_8	Six comma ten [6,10]
3	R	S_7, $\frac{6}{10}$ is already 1 or not?
3	S_7	Not yet.
4	R	So, how do you notate it?
5	T	*(Came and gave explanation)* show me where is$\frac{7}{10}$? (referred to the paper strip), is it already one paper strip?
6	S_8	Not yet.
7	T	What number is before number 1?
8	S_5	Zero [0]
9	T	So how do you write$\frac{6}{10}$?
10	Ss	Zero comma six [0,6]
11	T	Write it.

Fragment 4: The focus group discussion about decimal number of $\frac{6}{10}$

It was noticed the focus students, and the rest, struggled to transform from informal to formal notation and from tenths to decimal numbers. The conjecture in HLT students would use strip, apostrophe, as separator wholes and tenths did not appear. How the students could come with the idea using comma symbol quickly? They were already familiar with decimal numbers. As appeared in meeting 1, if the length could not be said in ones, the students tended to say in decimals, though in incorrect manner. In this second lesson, their prior knowledge of decimals was intervened by the idea that the decimal is another way to write fraction tenth.

The fragment 3 shows clearly now the students could reason what they meant by 8,2 and why comma was used. Surprisingly, they saw the number relationships, which the ones and the tenths. Moreover, the confusion notating length of small things with decimal numbers (shown in fragment 4) actually shows how the students were gradually building idea of the meaning of decimal numbers.

Meeting 3: Number Line as a Model of One-digit Decimals

This meeting is aimed to introduce the decimal numbers on number line. The students are asked to play SMS game (predicting length game), in which they list not-yet measured things in the classroom, send the prediction of its length in terms of the length of the paper strip to each other, and verify in group the real length with the paper strip. Following this activity, the students decide who wins the game by comparing who make closer or exact prediction with the real measurement.

The teacher reminded the students not to use ruler and to predict based on the paper strip. Students' responses were varied: use a span (in lesson 1, the paper strip was told as the standard of the length of a span), use the length of a pencil, and make length by the hand like length of the paper strip.

Student 6 played the game with student 7 and student 5 was with student 8. They made their predictions immediately in decimal numbers (see figure 2). They said the length of the paper strip was as long as the length of student 8's span.

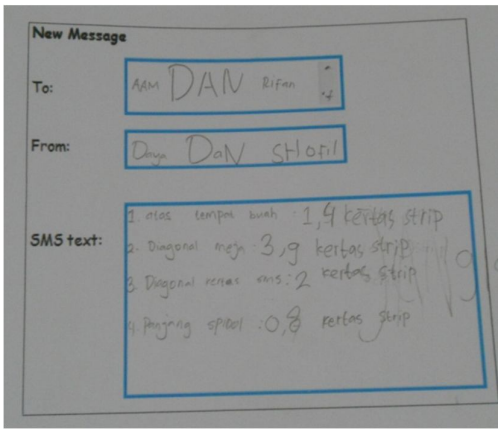

Figure 2: Student 6 and Student 7's message of lengths in decimal numbers

The students used already-marked paper strip to verify its real length. It was seen then that the students found no problems in deciding the winner because most of their predictions were exactly correct. The argument arose when both groups made wrong predictions of the length of the diagonal of an A4 paper. Student 5 argued the winner was who made closer prediction, but who, 1,7 or 2 to 1,8? (See the discussion in fragment 5).

1	R	How do you compare 1,7; 2; and 1,8? Why did group A win? [group who answered 1,7]
2	S_5	Because mine was 1,7 and the answer was 1,8; it was closer, only one number.
3	R	What do you mean by one number?
3	S_5	Plus one more number.
4	R	To 1,8?
5	S_5	Yes!
6	R	What about 2 [to 1,8]? How many more numbers?
7	T	2
8	R	Plus 2 more?
9	S_5	No, negative.
10	T	You mean subtract?
11	S_5	Yes.

Fragment 5: Discussion which number was closer to 1,8

The fragment showed how students could see the arithmetic and manipulate **t**he decimal numbers as whole numbers. This idea was also strengthened by what the group wrote on the worksheet (see figure 3). They explained with their words what they meant by plus one more number and subtract 2 more numbers. Their reasoning was from 1,7 to 2 they needed 0,1; and from 2 to 1,8 they needed 0,2.

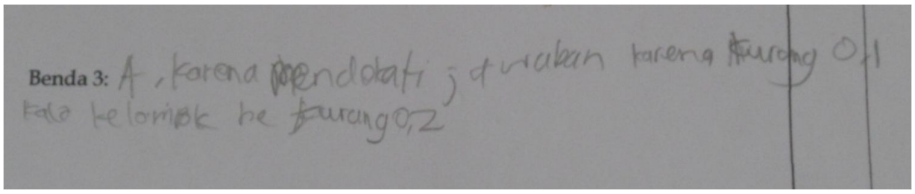

Figure 3: The focus group students' reasoning whether 1,7 or 2 was closer to 1,8 (Group A because they guessed closer to the answer, because they needed 0,1, but group B needed 0,2)

Another group able to think arithmetically on decimals was student 16's group. Their problem was even higher level, jumping number, 4,1 and 4,2 to 3,4. They explained on the worksheet 4,1 to 3,4 needed 0,7 while 4,2 to 3,4 needed 0,8. There was no information how they did this. There was no scrap on their worksheet paper. But, their answer on worksheet shows that, in their mind, the students could see the magnitude of the decimal numbers; they thus could explain the number backward and forward.

The struggle in comparing was seen in student 10's group. They took a long time to understand how to compare. The students actually already could decide the winner, but it took longer time for them to write their reason on their worksheet. Different with student 5's and student 16's group that could play with decimal numbers, student 10, in comparing, still thought in terms of small pieces of the paper strip, like 'the length 1,4 only needed 1 part from 1,5 while 1,9 needed 4 parts from 2,5'. This was as conjectured in the HLT. The other groups reasoned by using the term "closer" or "further" as predicted in the HLT. The prediction in HLT that did not appear was there would be some students compare by using the fraction.

On the whole, the impression, when observing the students discussed their prediction, was how the predicting context, the paper strip, and decimal numbers could work in the students' mind. That the students could estimate and decided the rest using decimal numbers informs that 10 partitions of the paper strip and the decimal number of it were abstractly figured in their mind. Additionally, the focus group students showed that they even did indirectly simple arithmetic of decimal numbers (figure 3). Also, generally, the students in the class could play arithmetic of the decimal number by this activity.

It is true none expressed a number line in paper when comparing, but probably the students had in their mind something like a number line or a visualization of order of decimal numbers. There would be none of the students made number line in comparing was predicted in the HLT. To react, the teacher was planned to draw the students an empty number line on the board and to discuss their answers in regard to the number line. However, the plan to ask the students to discuss some of group's

answer in whole class did not run due to limited time. So, the introduction about number line did not happen.

Furthermore, it is noticed that the students eased higher understanding of decimals. If previous meeting they represented lengths in decimals, now they communicate a decimal number was as a certain length. However, while discussion, there again appeared students' tendency to: notate with exact number although their measuring did not fit one paper strip; and say incorrect equal fraction and decimal, for example they meant 3,9 but said $\frac{3}{9}$. It was not known whether their problem was confused about fractions and decimals or was only mistake in naming the decimal numbers.

Some practical remarks in this activity, generally the students could make a good working group. Like the focus group students decided whose span to be used to measure and while one measured, the other marked the position. Another thing, the teacher must encourage the students harder to write their thinking on the worksheet unless the worksheet would be left empty. Equally important, it was noticed that the students confused about the instruction of the game. Therefore the teacher should give an example to the students how to play the SMS game. Also, more time was needed to finish this game. The students consumed long time in predicting part. So, in next meeting, the teacher would review about decimal numbers on number line.

CONCLUSION AND SUGGESTION

The lessons in this study were constructed with an idea understanding of fractions as a foundation to relate to decimals. Therefore, the context measuring lengths of things in classroom with a paper strip was chosen. By the context, the students divided the paper strip when the lengths did not fit many times and would start using fractions. Furthermore, the context of game is chosen to elicit number line as a model for decimal numbers.

During lesson 1, however, it was found that still many students faced difficulties in fractions, especially mixed fractions. Here was the role of class discussion. Together with the students, the teacher discussed the name of the lengths the students made and asked them to use tenths and discussed the meaning of it.

The fact that the lessons started with fractions prior to decimals was not to complicate the students by fraction things. But to make them have a good sense of the numerical relations in the notations they made. The context measuring, with its tool paper strip, provided this chance. The students experienced to investigate lengths, give estimation, divide the paper strip into parts, and discuss its rest with fraction language and be guided by the teacher to divide the strip in ten parts and to use fraction tenths. This resulted in the students using fraction tenths in a proper way after considerable efforts.

Later in lesson 2, still with the context of measuring, the students were asked to leave out 10 in tenths. The students faced confusion naming the lengths with other notations instead of fractions. They

started using long names, like '1 paper strip plus 7 small parts' and finally suggest using comma. Letting the students to change tenths notation into their own notations at the beginning to say lengths means giving them chance building an understanding of decimal numbers.

The SMS game in lesson 3 asked the students to make prediction of lengths of some objects in the classroom, verify the lengths with the paper strip, and compare the lengths. The lesson was aimed at students develop into representation of decimal numbers on number line. However, none of the students initially used number line to compare the lengths. This case was already predicted in the HLT. So it was the teacher who introduced the number line for decimal numbers. Besides the finding the students started to manipulate decimal numbers like natural numbers; the context comparing decimal numbers also gave chance to the students to see the magnitude of numbers on number line. Another finding in this lesson was that when the teacher tried to ask the students relation tenths and decimals, the students fluently related that $\frac{1}{10}$ is 0,1; $\frac{2}{10}$ is 0,2; and so on.

Furthermore, seeing the students make lengths prediction in decimal numbers through SMS game in lesson 3, there were two points: they got used to using decimal numbers and improved the decimal sense. What those mean is when students made prediction of lengths, they used decimal numbers and the decimal numbers they mentioned were based on their estimation for example by hand representing the length of the paper strip. So, it was not true if one said the students made predictions in decimal numbers just simply by writing decimal numbers without consideration. It seemed like the students were while measuring thinking of how much lengths was a decimal number. In other words, this shows that the students knew they used decimal numbers as a notation to communicate lengths; furthermore, they knew how much length the decimal number represents.

The general conclusion to answer the research question 'How can students develop an understanding of one-digit decimals?' is that the students gradually developed their understanding of decimal one-digit. The context brings the students from misunderstanding of decimal numbers, puzzling naming with incorrect fractions, here in the situation of measuring result, to using tenths, to using decimal numbers, then to model decimal numbers with number line. This learning should be supported by the discussion among the students themselves. Let the students argue and give their opinion; also, the teacher should guide them by giving good question like, why you use this? What does it mean?

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