A COGNITIVE THEORY DRIVEN NEW ORIENTATION OF INDONESIAN LESSONS

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Abstract

The main focus of this design research was on students' mathematical thinking and skills and on their understanding of mathematical concepts and methods. The mathematical content our project starts with is the introduction of integers. For this content new learning environments have been developed, implemented and evaluated. An important question for the design of teaching and learning activities is how to establish appropriate models of mathematical concepts and methods for students and how they operate in students' minds. In the lecture the use of those methodological concepts and theories in our approach will be explained and justified. The results indicate that the means and methods used in our project have a positive influence on the quality of teaching and learning processes in the class and on students' mathematical thinking and skills.

Keywords: Cognitive Theory, Indonesian Lessons, Mathematical Concept

Abstrak

Fokus utama dari penelitian desain ini adalah pada pemikiran dan ketrampilan matematika dan pemahaman mereka tentang konsep-konsep dan metode-metode matematika. Materi matematika dalam proyek kami dimulai dengan pengenalan bilangan bulat. Untuk konten ini lingkungan belajar baru telah dikembangkan, diimplementasikan, dan dievaluasi. Sebuah pertanyaan penting untuk desain kegiatan belajar mengajar adalah bagaimana membangun model yang sesuai konsep-konsep dan metode-metode matematika untuk mahasiswa dan bagaimana mereka beroperasi dalam pikiran siswa. Dalam perkuliahan, penggunaan konsep-konsep metodologis dan teori dalam pendekatan, akan kami jelaskan dan benarkan. Hasil penelitian ini menunjukkan bahwa sarana dan metode yang digunakan dalam proyek kami memiliki pengaruh positif pada kualitas proses belajar mengajar di kelas dan pada pemikiran matematika dan keterampilan siswa.

Kata Kunci: Teori Kognitif, Pembelajaran di Indonesia, Konsep Matematika

Since 2009 several design research projects for the improvement of the quality of Math lessons in schools on Java and Sumba have been carried out within the scope of our German-Indonesian collaboration (Kaune at al. 2011, 2012). These projects are characterized by an interdisciplinary, theory-driven development, investigation and analysis of the learning environment for a practically oriented purpose. By this complex nature these projects fit well to the view of Math education as a design discipline as it was specified in the European area by Wittmann (1998). In the author's view, the construction and investigation of suitable learning environments for learning and teaching of mathematics is to be seen as one of the specific tasks in the core of mathematics education (ibid, p. 329). This task does not mean, however, the mere development of didactical materials. The complexity of the design of learning environments has been expressed by Cobb at al. (2003, p. 9) with the term “learning ecology”. “Elements of a learning ecology typically include the tasks or problems that students are asked to solve, the kinds of discourse that are encouraged, the norms of participation that are established, the tools and related material means provided, and the practical means by which classroom teachers can orchestrate relations among these elements. We use the metaphor of an ecology to emphasize that designed contexts are conceptualized as interacting systems rather than as either a collection of activities or a list of separate factors that influence learning.”

The specific task of mathematics education can only be updated if the development and research of content-related theoretical concepts and practical teaching concepts with the purpose of an improvement of the real lessons are moved into the core of scientific work (Wittmann 1998, p. 330). From this view of mathematics education, research desiderata result to which nowadays the attention is often given in the approach “Developmental Research”. Thereby the research approach is also meant under different names such as Design research (Eerde 2013), Design experiments (Brown 1992; Cobb et al. 2003) Design-based research (papers in Educational Researcher, Vol. 32, No. 1), Educational design research (Van den Akker et al. 2006) (cf. Eerde 2013, p. 4). The term “Developmental Research” was described by Freudenthal as follows: “Developmental research means: experiencing the cyclic process of development and research so consciously, and reporting on it so candidly that it justifies itself, and that this experience can be transmitted to others to become like their own experience” (Freudenthal 1991, p. 161).

In the variety of the differentiations the intended aim of the particular research method is always the same – design of learning environments for the improvement of lessons and the development of theories which explain this improvement, justify it and – if possible – allow generalizations.

Besides, the focus is laid on selected subject areas or specific mathematical concepts. As a consequence the theories resulting from it also have a local character, i.e. specific for an area or subject. This is also reflected in the notion of “local instructional theories” or “domain specific theories” (Gravemeijer and Cobb 2013, p. 73). The usefulness of this subject-oriented approach in the attempt to explain and improve learning processes in the lessons is undoubted. Nevertheless, if one looks at the lessons from a more global perspective, the question arises how the local changes can contribute to a basic positive and lasting change of individual learning and understanding processes. Therefore considerations which concern the construction, structure and use of mathematical knowledge in the learners' minds are necessary. Such a global cognition-oriented perspective on the lessons is an essential characteristic of the research and development projects managed in the Institute for Cognitive Mathematics (IKM), University of Osnabrück. In the design research projects of IKM (also in Indonesia), a new cognitive theory driven orientation of the lessons is aimed. The implications resulting from this new orientation are taken over for the construction and implementation of the subject-specific learning environments. This is an essential difference distinguishing these projects from many other design research attempts.

In the following, at first the intended cognitive theory driven new orientations of the Indonesian mathematics lessons are explained. Afterwards the resulting consequences for design and empiric research are exemplarily discussed. The introduction of negative numbers and the use of integers serve herby as the exemplary discussion basis. The theory driven considerations are supported with examples from the project lessons. These examples serve on the one hand to record the achieved intended purposes; on the other hand, however, they also make it clear what should be improved furthermore. With the theoretical considerations and the empirically grounded insight into the mental processes of the learners both constitutive parts of a “local instructional theory” (Gravemeijer and Cobb 2013, p. 74) are demonstrated. Nevertheless, the focus does not lie on the design of the didactic materials but on the learners' mental processes in order to stress the aimed re-orientation of the teaching and learning processes in mathematics.

Cognitive Theory Driven New Orientation of Mathematics Lessons

The goal of our design research projects in Indonesia is an improvement of the quality of mathematics lessons, in particular the improvement of their sustainability. What is meant hereby, are not only sustainable calculation skills and use of algorithms, but rather the understanding of mathematical concepts and methods. It appears, among others, in the flexible, understanding use of these concepts and methods and in argumentations. The necessity of an improvement in this area is worked out and justified in Sembiring et al. (2013).

Thus, the theoretical framework of our projects contains theoretical and empirically grounded findings concerning the mental processes of thinking and understanding as well as successful communication processes in the social context of the lessons. Some of these findings are explained in the following.

A broadly known problem of the mathematical knowledge of learners is that it exists only in a fragmented form (Cohors-Fresenborg and Kaune 2005; Mandl et al. 1993; Vinner 1990). Hereby situations are meant “in which two pieces of knowledge (or information) that are known to an individual and that should be connected in the person’s thinking process nevertheless remain unrelated” (Vinner 1990, p. 92). This state results in negative consequences for the use of the available knowledge and thereby also for the understanding: “[...] knowledge compartmentalization causes students to perform meaningless symbol manipulation without understanding the relevance for their everyday life. This leads to the situation that on the one hand real world knowledge is not used in solving arithmetical problems in school, and on the other hand the kind of mathematics taught in schools is not used in everyday activities” (Mandl et al. 1993, pp. 170f).

For a sustainable mathematical education it is necessary to prevent a fragmented construction of mathematical knowledge. The cognitive mathematics uses therefore the cognitive tool of organizing and systematizing of knowledge in an subject realm. Using this tool, the knowledge in this realm is recorded in a form of an axiomatic system (“contract”) and it can be further developed. The availability of this tool enables the introduction of mathematical concepts as parts of a system and makes it possible to explain the connections between these concepts. Consequently a big part of school mathematics receives a structured form. Where an explanation by the access to realistic contexts shows its limitations, an established contract creates the possibilities to promote explanations and justifications. Mathematical relations, so-called formulas and sentences, are to be derived from the given axioms ("paragraphs of the contract") and the sentences are to be derived and proved.

With the new structure of the mathematical contents the essential mathematical activities – axiomatizing, arguing and proving – are put in the foreground. Axiomatizing is meant here as the specific mathematical activity described by Freudenthal (1963, p. 6) as the activity of the local ordering of mathematical knowledge into a system and of searching for an evidence base for it. As such a "humanly activity", it belongs to school mathematics and plays a central role in the systematizing of knowledge (cf. Barzel et al. 2012). Nevertheless, because of the formal and abstract aspects, their educational value finds still little attention.

Implications from the Cognitive Theory Driven Conception of School Mathematics for Design Research

In the cognitive theory driven conception of school mathematics, mental processes and understanding of the learners stand in the center of the lessons. From this point of view consequences result for the design and implementation of didactic materials. These consequences are discussed in the following sections. In a more detailed form these consequences are presented in Kaune at al. (2011, 2012).

From Basic Ideas and Mental Modes to Microworlds and Metaphors

An important question for teaching and learning mathematics is how to establish appropriate models of mathematical concepts and methods for students and how they operate. A common approach is the development of basic ideas (Fischbein 1989) and mental models (vom Hofe 1998). This is to say constituting the meaning of a new mathematical concept through the connection to a known context (vom Hofe 1998, pp. 97f.). Further differentiation occurs in the created new system of mathematical concepts. The term "Grundvorstellung" (basic idea) characterizes fundamental mathematical concepts or methods and their interpretation in the context of real situations (vom Hofe 1998, p. 98). However, the essential didactic function of the basic ideas – the sense-constituting function – is used in the mathematics education also to create substitutes for formal definitions and to practical applications of mathematical concepts to reality (cf. Griesel 1996, p. 18). The approach established in the Institute for Cognitive Mathematics in projects for the improvement of the quality of Indonesian mathematics lessons differs from these positions in the following sense: Mental models for mathematical methods and ways of thinking are used to support the understanding of, communication about and precise formal representation of mathematical concepts: We start elaborating pieces of knowledge that the students are familiar with (from everyday life or from their previous theoretical considerations) and expand them in the context of their experiences into a system, a microworld. In doing so we follow an analysis of Schwank (1995): "Microworlds are the external places where the external actions – caused by the cognitive activities – take place; the effects of these actions give feedback to the mental organizations and the development of mental models" (ibid. p. 104). A microworld has to function and to contain the ideas, theories and procedures that are later on utilised in the mathematical theory (Cohors-Fresenborg and Kaune 2005). To function means, that in such a microworld an intellectual approach can be tested and experience is gained before the mathematisation is carried out. Subsequently, the behaviour which became evident in the microworld, is clarified and formulated, which is to say formed, and then written down, i.e. formalized. Whether what has been said or noted resembles the intended meaning is reflected again and again during this process. Thereby one reflects upon how the formulation is going to be used and understood by the reader. Since this microworld has been developed based on experience, a metaphor system (Lakoff and Johnson 1980) can be created, which can be utilised anytime by the student as an evidence-base. Thus, the aim is not to create sense-constituting mental models for isolated concepts and to motivate the learning of mathematics by applications, but rather to support intellectual behavior which plays important role by the transfer from the context-based relations to formal representations and the abstract world of mathematics. We consider the development of sustainable mental models (for example to deal with numbers and algebraic transformations) more important than the mediation of factual knowledge and the practice of calculation techniques.

From Realistic Scenarios to Mathematical Ways of Thinking and Abstract Representations

For the constitution of adequate mental models, we use external, not mathematical contexts and we follow to some extend the subject-specific teaching learning theory "Realistic Mathematics Education" (REM) (Gravemeijer 1994), going back to Freudenthal (1973). In a critical point we go even further than Freudenthal's followers.

An essential difference consists in the extension of the so-called realistic scenarios (in the sense of Gravemeijer et al. (2003, p. 53)) to microworlds. These have to be experienced by the learners so far and so detailed that in the following reconstruction of mathematics these experiences form the basis for mathematical activities and mathematical theories. For example, the experience of making a contract, keeping to it, and deriving and justifying consequences from it, forms the basis for the development and practicing the competence in dealing with axiomatic theories, proofs and argumentations. This is expected to support the cognitive transfer from the thinking and action in a situation to the thinking and action in mathematics. In the design method „emergent modelling" (cf. Gravemeijer 2002), the intended transfer is described as "[t]he shift from model-of to model-for", i.e. from "*model of* acting in a situation" to "*model for* more formal mathematical reasoning" (Gravemeijer 2002, p. 2).

Thereby we follow Freudenthal's conception of "mathematics as a human activity. That is to say, students should be given the opportunity to reinvent mathematics by mathematizing – mathematizing subject matter from reality and mathematizing mathematical matter" (Gravemeijer 1998, p. 277). In this point, our approach differs from many – in the meantime, also strongly criticized (cf. Landsman 2009, p. 139) – approaches in which realistic contexts are used in school mathematics to remove all abstraction. This has led to the loss of the essential strength of mathematics – to the loss of the interaction between abstraction and application – and to the loss of theoretical basis: “What little theory and abstraction has remained in textbooks is frequently remote from serious mathematics and is sometimes even plainly erroneous” (ibid., p. 139).

From A Discourse to the Metacognitive Discoursive Teaching Culture

From the cognitive theory driven conception of mathematics consequences arise for another lessons characteristic – for the teaching culture. This consequence is one of the aspects of “learning ecology” (Cobb et al. 2003). For a design experiment norms of the intended discourse in the class must be specified.

The construction of suitable mental models does not result automatically from activities in a suitable, sense-constituting context. For this it is necessary to control the reciprocal relationship between conceptions and representations and to reflect on how the relationship can be used and possibly extended. Since we have put the focus on the concept formation and understanding, mental activities must be practiced in the classroom, which can be subsumed under the metacognitive aspect of reflection: Reflection on the adequacy of concepts and metaphors, on the choice of the mathematical approach, on interpretations, conceptions and misconceptions, and on the interplay between what was said, meant, and intended (representations and conceptions). These are activities that relate to the thinking (of oneself or of the others) and thus they belong to metacognitive activities. Another metacognitive activity is monitoring. It includes both the monitoring of the subject relevance and target reference, as well as the adjustment of what is to be achieved and what has been already achieved. In the process of concept formation, controlling of arguments plays an important role in ensuring the insight in the issues to be taught and to prevent the memorization of facts.

Our experience and empirical work that has been carried out in IKM (Cohors-Fresenborg and Kaune, 2007, Kaune and Cohors-Fresenborg 2010) show that a deeper understanding of concepts, the chosen procedures and tools used is only possible if the monitoring and reflection precisely refers to what is discussed in class at the moment. To this end a contribution's reference point has to be made obvious to those involved in the lesson's discourse and understanding of what is said has to be supported by an adequate choice of words. We subsumed the activities essential for this under the notion of “discursive activities”. A discursive teaching culture plays a crucial role when it comes to encouraging metacognitive activities of learners and the construction of mental models.

In order to read and write mathematical knowledge accurately and to reason in the mathematics lesson the ability to realize and articulate the difference between what has been presented and what had been the intention is essential. For this purpose it is necessary to follow the lines of an argument, estimate its applicability and to strategically place doubt and counterarguments. This shows that metacognitive and discursive activities have to be interwoven.

The above explanations show that our design research projects in Indonesia presuppose a cognition-oriented teaching culture. This must be developed gradually. In Kaune and Nowińska (2011) we justify using examples that it is possible to practice metacognitive-discursive teaching culture in Indonesian lessons.

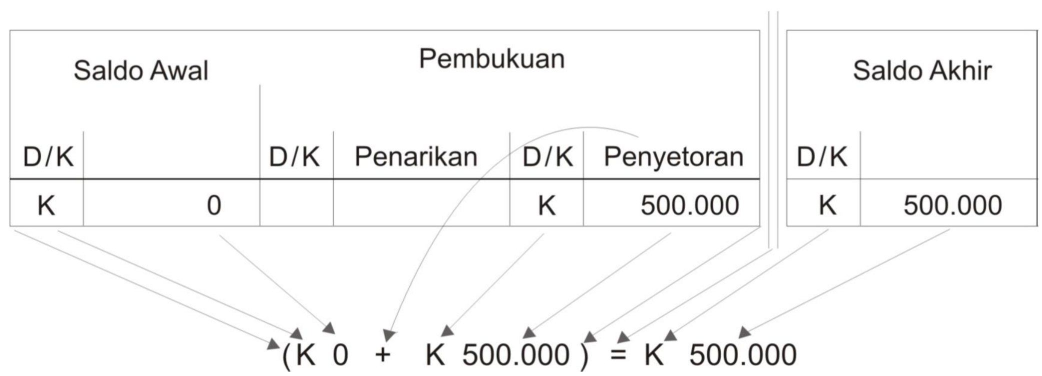
Cognition-Oriented Design of a Learning Environment to Deal with Integers

The following outlines how the theory-based considerations were used in the design of a learning environment for the introduction of negative numbers and for dealing with integers. A detailed explanation can be found in Kaune et al. (2011, 2012).

The Construction of Mental Models in a Realistic Context

For the introduction of negative numbers, we use the intuitive knowledge and experiences of the learners on how to deal with debit and credit in a bank. The learners have the opportunity to expand their experience in a realistic context (in the sense of Gravemeijer et al. 2003, p. 53), to discover properties of and relationships between paying in and paying out, and to describe them. The description is first made in the language of the bank. Here the letters D (debit; indon. - *d*ebet) and K (for credit; indon. – *k*redit) are used to identify the corresponding debit and credit amounts.

Using this language, the transaction recorded in the account card in Fig. 1 has to be written in a shorthand notation as follows: (K 0 + K 500.000) K = 500.000.



(Saldo Awal – opening balance, Pembukuan – transactions, Penarikan – withdrawal, Penyetoran – deposit, and Saldo Akhir – closing balance)

Figure. 1 Symbolic Representation of a Booking Operation

In a next step, this notation is simplified even further. Instead of D and K, only one sign is used to differentiate debit and credit amounts. The debit amounts are characterized with a minus sign:

old notation: (D 50.000 - K 25.000) = D 75.000

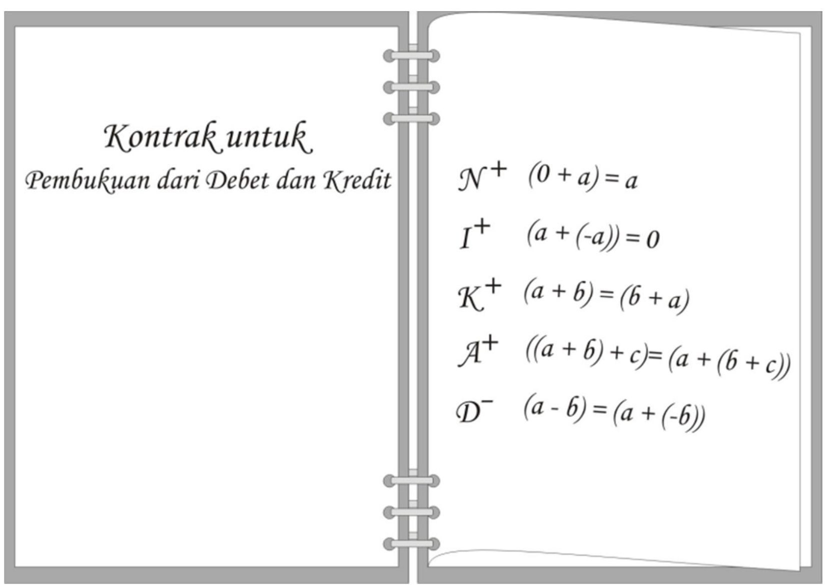
new notation: ((-50.000) - 25.000) = (-75.000)

The difficult case, which is described in the language of mathematics as a subtraction of a negative number, corresponds in the context of the bank world to the operation a paying out of a debt amount. This interpretation seems at first sight limited and for many outsiders also absurd, because no *real* bank pays out a customer a debt amount. What no *real* bank do is, however, conceivable in a realistic scenario. The case of paying out of debts arises in several situations as a solution to a given problem situation. One of these situations is merging of two accounts and another is undoing of an incorrect booked (paid in) debt amount.

The students come easily to the insight that the operation of paying out a debt amount gives the same result as the deposit of the corresponding credit amount, and also that the operation of paying out a credit amount leads to the same balance as the deposit of the corresponding debt amount. This knowledge and experience is used to explain the operation of paying off by recourse to the operation of the deposit (paying in). In the later step of the mathematization, this serves as a basis to reconstruct the definition of subtraction.

Systematization of Conceptions

An important step in the systematization and the development of mental models for dealing with integers is the step of local ordering of intuitive knowledge and experiences of the learners on how to deal with debt and assets amounts in the bank. For this purpose, a situation is created in the classroom, in which a bank customer has to sign a contract with the bank. The contract should specify normatively how the bank handles bookings and how it records them. In the future, all bookings in the bank should be based on this contract. The validity and usefulness of this contract is evident. The result looks like this:



(Designations of the names of paragraphs prepare the future use of the contract for calculations: N^+ : additive identity element; I^+ : inverse element; K^+ : commutative law; A^+ : associative law; D^- : definition of subtraction)

Figure. 2 Contract for Dealing with Debit and Credit

The next step is the abstraction from the idealized bank to the "Math Bank". The new contract arising in this step is the contract for dealing with numbers. Hereby, the numbers with the sign "-" are interpreted as negative numbers, and the unsigned as positive numbers. The plus sign for the two-digit operation of the deposit is interpreted as the two-digit addition sign, and the two-digit minus sign for paying out as the subtraction sign.

In the step of the reconstruction of the mathematical knowledge, the learners use their experience in dealing with contracts in order to control step-by-step mathematical calculations and their correctness in accordance with the new contract. From the existing paragraphs of the contract further conclusions are drawn, i.e. – from the mathematical point of view - new sentences are to be proved. In this way, the learners are guided into the abstract mathematics and in mathematical activities. That what is eliminated in some approaches to RME (abstraction and formalism), gets a special status in our approach. Mental models are used to understand formal and abstract aspect rather than to abolish them and to use these aspects as a means in communicating mathematical knowledge. The following shows examples of how the mental models constructed in a realistic context are used for learning mathematics.

An Interpretative Analysis of Students' Productions and Transcripts

Traces of Mental Models in Students' Productions

The effectiveness of the mental models constructed by the learners is reflected in particular by the fact that the learners fall back automatically to them, in order to interpret and explain mathematical concepts. In Kaune et al. (2012), we investigated the question of whether the students, who were taught according to our concept, actually use the microworlds offered to make calculations with integers. Using exemplary students' works we were able to show already 2011 that this is the case in the group of these students. Without being asked to do that, they provided comments for their calculations with integers. These comments indicate the use of a microworld. Also in the following years of the implementation of the teaching concept in other schools we could find evidence for this statement.

The student Jessica who participated in our project on Java 2011 left in her solution to the task *Calculate: a) (239 + (-39)) and d) (54 + (192 - 54))* traces of using the microworld "Banking":

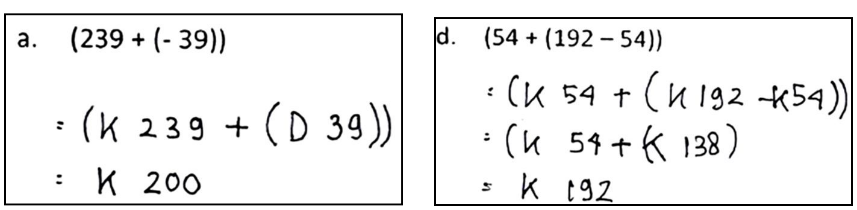


Figure. 3 Traces of Mental Models in Calculation Tasks

This task was a part of a test in which no explicit references to the microworld "Banking" have been predefined. At the time of the test, the students have not been working explicitly with this microworld in class. Jessica interprets the mathematical expressions as notations of banking operations in a bank. Using the letters D and K she distinguishes between the debt and the credit amount. Then she calculates the correct balance of the booking. Her solution to part a. violates the agreed rules of syntax. The debt amount D 39 should not be written in brackets here. Presumably, the bracket pairs have been taken over from the given mathematical expression.

It is worth noting that, at first glance, the numbers noted in the task cannot be associated with amounts of money in the currency of Indonesian Rupiah, because none of these numbers can be construed in this currency in everyday life. The fact that the mental models have been applied here, shows that they are viable and sustainable. Instead of being just tools for application tasks, they have been used here as a "*model for* mathematical reasoning" – as a means to identify mathematical structures and to work with them in a familiar way.

This transfer of the constructed mental models to new situations reveals a special achievement. The transfer can be achieved if the relationships between representations and conceptions are discussed in the classroom, practicing thereby reflection and monitoring. It must be, however, accepted that in this process of discussion and of the transfer some mistakes happen. They may have their origin in inadequate representations of the right ideas.

Reflection on the Interplay between Representations and Conceptions

The following transcript documents a teaching sequence in which the teacher and the students discuss the interplay between representations and conceptions. This lesson was given 2012 in grade 7, in the third week of the implementation of the design research project on Sumba. From the incorrect students' solutions, the teacher selected the following as the basis for a classroom discussion:

Calculate:

C) ((-250) + 300) = 50.000

D) ((-500) + (-200)) = (-700.000)

E) (250 – (-100)) = 350.000

[1] Ibu Olif Ibu would like to talk about this. That is the way how Mira and the others did it. It happened again as in this one here *[here the incorrect written thousands digit are meant]*. *(9 sec)* Try to explain to me how your friends *[in the meaning of "classmates"]* think about what is written here if they solve this task as you see it here! Look at this first, is there a mistake or not? Is there a mistake or not, Fandika?

[2] Fandika There is one, Ibu.

[3] Ibu Olif What is the mistake?

[4] Fandika There is written 250 minus 250 plus 300. It must be equal to 50, not 50.000. And the same by the next numbers *[in the meaning of "next lines"]*.

[5] Ibu Olif Okay. So, what do you think, what do your friends *[in the meaning of "classmates"]* think who answered in the same way as here, how do they conceive the result written here? Alfredo!

[6] Alfredo He thinks this way Ibu, he thinks it is 250.00. Though (…).

[7] Ibu Olif He thinks this is 250.00 and then?

[8] Alfredo Minus 220 minus 220 er minus 200.

[9] Schüler Hehehe.

[10] Alfredo 300.000 is added to minus 250.000. So he thinks minus *[in the meaning of "subtract"]*, 50.000.

[11] Ibu Olif He probably thinks that these are shortened, he probably thinks like this, yes. 250 is a short form of 250.000 or is there someone who made the same mistake? Mira also made the same mistake like this here. So, in each of yesterday's tasks Ibu has often seen this case. So, how can you avoid this case? You can imagine this 500.000, 700, minus 700.000, 350.000 only in your mind. You imagine that this 250 is 250.000 because we are still dealing with yesterday's booking with yesterday's account. Okay, can you understand, Mira? Mira?

[12] Schüler It works *[in the meaning of "Yes, we can".]*

[13] Mira It works *[in the meaning of "Yes, we can".]* Ibu.

In the given task, the values of the terms are to be calculated. The task does not give any hints that here a microworld can be used. At the time of the test in the classroom the learners have often interpreted such terms as notation of bookings in a bank. The structure of the terms corresponds here to the structure of a booking. However, the numbers are so small that it is unusual to interpret them as amounts of money in the currency Indonesian Rupiah. That is the reason for mistakes that occur at the beginning while projecting the metaphor "booking" on mathematical terms. In the mind, thousands are added to the given numbers. This way, (-250) is conceived as the debit amount of 250.000 Rupiah und 300 as the credit amount of 300.000 Rupiah. Consequently, the credit amount of 50.000 Rupiah arises as the result of the booking imagined on the basis of the term ((-250) + 300). This answer has to be regarded as erroneous. This is the consequence of the lack of monitoring of the interplay between the imagined banking story and the representation of the final mathematical notation.

The transcript indicates the efforts of the teacher to guide her students to reflect on what has been written and meant. Her question [5] "So, what do you think, what do your friends think who answered in the same way as here, how do they conceive the result written here?" directs the attention of the learners on thinking about what is written and meant. This way she leads the learners to metacognition. Alfredo explains what the mistake was caused by. He formulates a conjecture concerning a misconception about what is written. His answer is not elaborated, but it shows that he tries to verbalize a plausible explanation. Noteworthy is also the answer of Fandika [4]. She refers to a mistake, makes a correction and even suggests a commonality with another mistake ("And the same by the next numbers.").

The learners' behavior indicates an agreement of the didactic-social contract (Sjuts 2003) in this class. This agreement says that mistakes made by the classmates have to be discussed, explained and corrected, and it must be found out what the mistakes are caused by. In her final contribution, the teacher compliments the pupil's contributions with a hint explaining how such errors can be avoided in the future. She puts the cognitive processes of the learners in the center of her statement [11]: "So, in each of yesterday's tasks Ibu has often seen this case. So, how can you avoid this case? You can imagine this 500.000, 700, minus 700.000, 350.000 only in your mind. You imagine that this 250 is 250.000 because we are still dealing with yesterday's booking with yesterday's account."

Thus, the transcript demonstrates the transition from thinking and acting in the microworld "banking" to thinking and acting in mathematics, and it points to the effectiveness of the conceptions constructed in the microworld.

The precise responsiveness to conceptions and to the interplay between conceptions and external representations is an important variable influencing the success and effectiveness of our design research projects. Our experience shows that it is quite possible to motivate and to educate teachers and learners to such metacognitive considerations. However, the effectiveness of the attempts to practice metacognition in the class often failed due to the lack of precision in the contributions made by the teachers and their learners.

The teachers have frequently violated the precision of the language in their own statements and they did not pay enough attention to the accuracy in the statements of their pupils. An improvement in this area could positively affect the effectiveness of the microworlds offered and also help low-performing students to follow lessons.

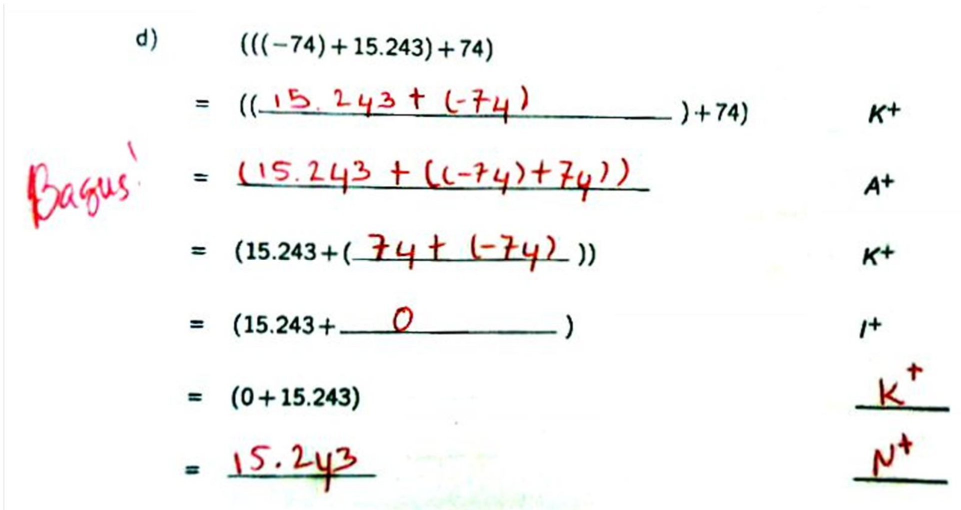
We have gained this insight through an intensive, interpretive lesson analysis. As a consequence of this insight, the research desideratum results in the application of an interpretive analysis of transcripts in order to get more insight in the interplay between mental models and representations. In this investigation also metacognitive and discursive activities should be identified and evaluated.

Stepwise Controlled Calculations and Reasoning

The intended effectiveness of the mental models does not concern only simple calculations, as in the above transcript. With the contract for dealing with debit and credit, the pupils should learn and practice how contracts are to be used as a basis for reasoning and how further agreements are to be derived from them. This experience of a meaningful, rigorous application of a contract in the banking context causes the learner to the same rigor in dealing with mathematical structures and to justify each transformation step.

An example of such a stepwise controlled approach is documented in the following. This is a transcript of a lesson in grade 7. After the transition of the contract for dealing with debit and credit to the contract for dealing with numbers, the learners were for the first time confronted with a challenging task of stepwise controlled calculations. The student Tasya has solved part b of this task correctly. The teacher (Ibu Misseri) marked her solution with the comment “Bagus” (correct!):

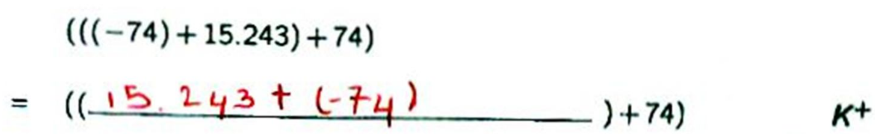
Fill in the gaps!



In the following part we show a transcript to the teaching scene, in which Tasya presents her solution. The aim of the transcript it is not to provide a “proof” of Tasya's autonomy in solving this task. The focus of the transcript interpretation is on the student-student interactions and socio-didactical norms in the teaching culture. The transcript indicates that the learners' thinking processes (and not only the correctness of the recorded solution) are in the center of this lesson sequence.

[1] Ibu Misseri Please pay attention to what is happening in front of you! Now, er, Tasya's group will read out the result of their work and they will justify, why they have written the following. Yes, Tasya, yes. Please let us have your justification without being asked, how you have come to this way of understanding. So, now Tasya is going to explain her answer. Everyone please pay attention to what is happening in the front!

[2] Tasya *[She explains the first line which has to be filled out. Doing this she shows with her pen on the correspondent parts of the line above it.] Here K plus is written. Here is the last minus 74, they write here minus 74, I imagine it being a, and I imagine 15.243 being b. Thus, if, if it is K plus, then we turn it, we change it, we turn it. Thus, here I write minus 74 behind it and 15.243 here. [She uses her pencil to point out the numbers.] How do you find it, my friends, is my answer correct or false?*



[3] students *Tasya Tasya Tasya Tasya sssstttttt.*

[4] Tasya Randy.

[5] Randy Correct.

[6] Tasya Who thinks it is false?

[7] Ibu Misseri So, do you all agree with Tasya's answer?

[8] students We do.

[9] Ibu Misseri So, the next step.

[10] Tasya *[She explains the second line which has to be filled out. Doing this she shows with her pen on the correspondent parts of the line above it.] Here A plus has already been written down. When we see A plus, only the brackets are to be changed. Thus, here it is already, it is already there in it, there are already brackets. We write this down here below, we exchange them, these (...). (3sec) Excuse me, no, wrong. There is already something written in these brackets. Thus, we change this 15.243, we write it down here, and then we write minus 74.000 down here, but before we pull minus 74.000 down, we, first of all, open a bracket. So we write it like this here. Thus, the 74, which is over there, is moved over here, then we close the bracket, first bracket, the one around 74, which means that these both numbers belong to one operation and the last bracket belongs to the bracket which is over there. What do you think about this, friends? Is my answer correct or false?*

$$= ((15.243 + (-74)) + 74) \qquad K^{+}$$

$$= (15.243 + ((-74) + 74)) \qquad A^{+}$$

[11] students Tasya Tasya Tasya Tasya.

[12] Tasya The one who is sitting next to Mira.

[13] Schüler Correct.

[14] Tasya Who thinks it is false?

[15] students Hihihi.

[16] Tasya Where is the mistake?

[17] Schüler In this explanation, 75.000 was said, but actually it is minus 75, not minus 75.000 er 74.000.

[18] Tasya Sorry, I said it incorrectly.

[19] Ibu Misseri Yes, Tasya said it incorrectly, it is actually minus 74. So, do you all agree to Tasya's answer?

[20] students We do.

[21] Ibu Misseri And now the next step.

Results of the cognitive-oriented interpretation of the transcript:

- It is noteworthy that Tasya explains in her comments to the first transformation how she thinks. Her response indicates an adequate understanding of the term rewriting. Tasya refers to the commutative law (a + b) = (b + a) and conceives of the two numbers (-74) and 15.243 as numbers that were put in the variables a and b. Originally, this formal notation was used to express that the order of bookings to be executed in order to combine two accounts does not affect the final balance. Tasya shows that she can properly apply this formal notation of the commutative law in a context-free example.

Here again the intended goal of our teaching design can be indicated – the microworld serves not only to demonstrate and to motivate applications of mathematics but to build a metaphor system supporting the practice of mathematical ways of thinking and behavior supporting the understanding and systematizing of context-free structures.

- Tasyas explanation to the second transformation is very detailed; she explains very well the changes in the structures of the two terms. However, her contribution also shows some problems in the verbalization of what she means. Instead of 74 she says 74.000. This may be another example of the kind of error already discussed above, which has its origin in the lack of monitoring of what is meant and written down. Here, the error refers only to what is said. The written solution is correct.

  Noteworthy are the reactions of the students who reveal and comment (unfortunately still not precise enough) on the difference between what is written down and what is said. Tasyas questions concerning the agreement are part of the didactical-social contract and they are taken seriously by the learners. According to this contract, solutions and argumentations have to be controlled and mistakes have to be corrected. The student-student interactions in the interpreted lesson scene are not only an expression of forced *social* norms that determine the external behavior. It seems that there are socio-didactical norms that have been internalized by the learners.

- A detailed analysis of the transcript shows what can be achieved in the classroom when implementing the designed learning environment (in math performance and in the classroom culture) and what needs a further development.

  Practicing metacognition plays a special role in our design research projects – not only as an internationally recognized feature of effective teaching, but rather as a necessary condition for the construction and functioning of mental models. In contrast to approaches focused on the metacognitive behavior in problem solving (Schoenfeld 1992, Mevarech et al. 2010), we see the special role of metacognition in the processes of constructing, systematizing, extending and applying a conceptual system. These processes include reflection on the individual elements of this conceptual system and the related representations and conceptions. In the above transcript, metacognition can be indicated in the calls to control (monitor) the given argument, in Tasyas self-control and in her reflective analysis of the structure (representation) of the individual terms as well as in controlling and correcting the written and the verbal responses given by Tasya.

Stepwise Controlled Calculation and Prove

The mentioned examples from our design research projects are related to the aims which are still closely associated with calculation. The new orientation of the math lesson aims, however, also goals that go beyond the step by step calculation. In the following some of these goals are explained on the basis of selected tasks.

The stepwise controlled computations in which every step is to be justified with the corresponding paragraphs of the contract for dealing with numbers can in some cases lead to very long calculations. These are used in the classroom to motivate the introduction and proofs of new paragraphs (new theorems).

The following solution to a task from the student book (Kaune, Cohors-Fresenborg and Paetau 2012) shows how the use of computational advantages can be justified with stepwise controlled calculations (Fig 4.1):

Calculate the term and justify each step of your calculation!

$(((-25)+175)+25)$
$= ((175 + (-25))+ 25)$
K^+
$= (175 + ((-25)+ 25))$
A^+
$= (175 + (25 + (-25))$
K^+
$= (175 + 0)$
I^+
$= (0 + 175)$
K^+
$= 175$
N^+

Fig. 4.1

Proof of the theorem

T1: $(a + 0) = a$

for each arbitrary value of a.

$(a + 0)$
$= (0 + a)$ K^+
$= a$ N^+

Fig. 4.2

Transformations of the term $(((-25)+175)+25)$ with the use of the theorem T1:

$(((-25)+175)+25)$
$= ((175 + (-25))+ 25)$
K^+
$= (175 + ((-25)+ 25))$
A^+
$= (175 + (25 + (-25))$
K^+
$= (175 + 0)$
I^+
$= 175$
T1

Fig. 4.3

There are several ways to determine the value of the term (((-25) + 175) + 25). The numbers given in the term suggest to first add (-25) and 25 as the sum of the two opposite numbers results zero. The solution reveals, however, the complexity of the calculation when this computational advantage is used. Especially in the last three lines, it is notable that the calculation of the sum (175 + 0) cannot be justified by just one agreement of the contract. For the future one wants to have the possibility to justify the result of each sum of the form (a + 0), for any value a, in a more efficient way. The performed calculation gives a hint how to cope with this sum. It can be justified that the following theorem T1 applies: (a + 0) = a, for any value a (Fig 4.2). Using this theorem, the previously executed calculation can be shortened (Fig. 4.3).

The first proof for the theorem T1 shows how the activities of the stepwise controlled calculation and proving are related with each other. The experience, to justify each step of a booking conducted by a bank with the adequate agreement from the contract for dealing with debit and credit, was extended into a metaphor of stepwise controlled calculation in mathematics. In a further step, the learners transferred their experience of justifying consequences derived from the contract with the bank to proof activities in mathematics.

In the student's book (Kaune, Cohors-Fresenborg and Paetau, 2012), tasks requiring proving activities play an important role. These activities are fostered also in other contents (multiplication, division, calculations with numbers in a root representation, calculation with numbers in a power representation, equations). They are a prerequisite for a systematic construction and extension of mathematical knowledge and for the local systematizing of it. With the practice of proofs in other subject areas and with the use of contracts constructed for these local subject areas the students gain insight into mathematical theories and the ways how they function.

In the following we present students' solutions to two tasks from a test in grade 7. The test was written 2013, after three months of conducting our design research project in this class. The tasks are related to the content taught during this period of time. The task is:

Task 5: Write a proof for the theorem T13: (b + (b – b)) = b!

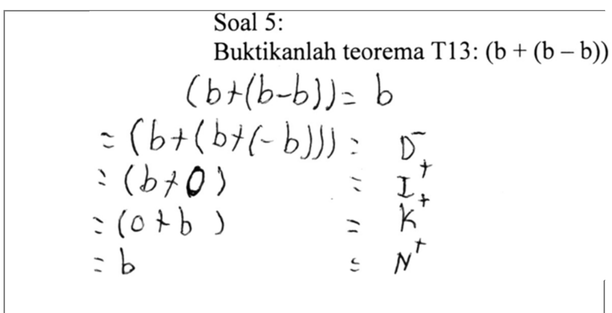


Fig. 5.1 Solution of Karolus

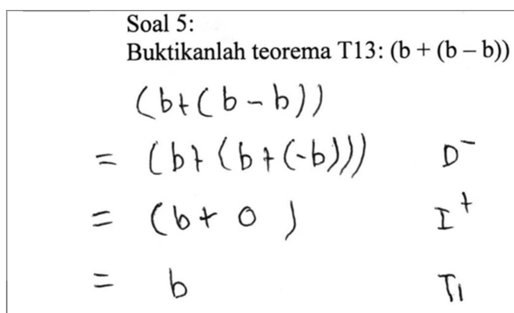
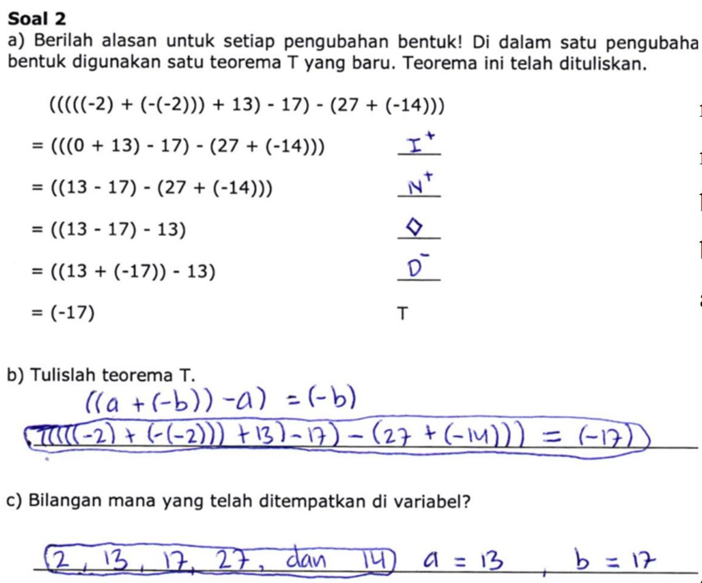


Fig. 5.2 Solution of Emilia

The solution of Karolus is syntactically incorrect. Since Karolus rewrites only the term (b + (b - b)), he should have written the first line without the signs “= b”. In each of the next lines all the signs “=” standing on the right side cause syntactical errors. Despite the errors, the solution indicates a correct thinking process of the student. Each term transformation is justified with the correct agreement.

Emilia's solution is correct. Unlike Karolus, Emilia uses in her proof the theorem T1 proved before and therefore her reasoning is shorter than this of Karolus. Since only the contract for dealing with numbers was available on the test sheets, it can be assumed that Emilia had learnt the theorem T1 by heart.

Another task in the test asked for agreements justifying the given calculation. The rhombus-symbol is used to justify calculations for which there is no agreement in the contract. These calculations are based on arithmetical facts known since the elementary school. Here this sign is to be used to justify the result of the addition of 27 and (-14). The theorem T used as a justification for the last calculation step has to be reconstructed by the students. There are several different ways to record this theorem. One of them is presented in the correct solution of Modestus:



Soal 2

a) Berilah alasan untuk setiap pengubahan bentuk! Di dalam satu pengubaha bentuk digunakan satu teorema T yang baru. Teorema ini telah dituliskan.

(((((-2) + (-(-2))) + 13) - 17) - (27 + (-14)))

= (((0 + 13) - 17) - (27 + (-14))) I^+

= ((13 - 17) - (27 + (-14))) N^+

= ((13 - 17) - 13) ◇

= ((13 + (-17)) - 13) D^-

= (-17) T

b) Tulislah teorema T.

$((a + (-b)) - a) = (-b)$

$(((((-2) + (-(-2))) + 13) - 17) - (27 + (-14))) = (-17))$

c) Bilangan mana yang telah ditempatkan di variabel?

(2, 13, 17, 27, dan 14) $a = 13$, $b = 17$

Fig. 6 Solution of Modestus

Task 2

a) Justify each term rewriting. For one term rewriting a new sentence T has been applied. This sentence has been already written down as an argument.

b) Write down the sentence T.

c) What numbers have been substituted for the variables?

The circled parts of the solution have been identified and marked by the student as incorrect. The circling (instead of erasing) of an incorrect answer was agreed in our project classes as a method for identifying and highlighting errors. The goal of applying this procedure is to ensure that errors can serve as a basis for a discussion fostering students' understanding.

The examples from the project lessons, transcripts and student solutions presented in this article indicate what is potentially achievable in grade 7. The aim of the designed teaching concept is not to rehearse the mathematical rigor in making proofs. We see prove rather as an activity that support mathematical discourse, communication and systematization of mathematical knowledge. In this respect, we agree with Hanna and Jahnke (1996, pp. 878ff) that prove is an important tool in communicating mathematical knowledge and understanding, and not just an end goal in itself.

Discussion

One aim of the paper was to present the intended cognitive theory driven reorientation of teaching mathematics in Indonesia. For this purpose, a theoretical framework was presented. It was explained why this new orientation is necessary. Implications were formulated for the practical goal of improving instruction using the approach called design research. Based on the example of the teaching concept "Introduction of negative numbers", it was shown how these implications can be operationalized to design a new learning environment. There were also examples from the class interpreted in order to explain the intended improvement of teaching and learning processes.

With the theoretical considerations and the empirically grounded insight into the thinking processes of the learners, the two constitutive parts of a "local instructional theory" (Gravemeijer and Cobb 2013, p. 74) were presented. The focus was on the cognitive processes of the learners. The overarching theoretical framework for a cognition-oriented teaching and learning can be used in further design research experiments in order to design adequate "learning ecologies".

The detailed discussion and analysis of selected students' productions and transcripts reveals an important didactical value of the project carried out. We do not try to convince the reader about the possibility of replication of the results obtained. Because of the complexity of the real circumstances this even seems to be not always possible. "Design research aims for ecological validity, that is to say, (the description of) the results should provide a basis for adaptation to other situations. The premise is that an empirically grounded theory of how the intervention works accommodates this requirement" (Gravemeijer and Cobb 2013, p. 103). With the selected documents, we try to meet the requirements for "ecological validity". This is also a way to give the reader an opportunity to gain insight in how the theoretical considerations and the designed learning environment work in a real situation. Here we see an important didactic value of this paper for practice-oriented readers. "By describing details of the participating students, of the teaching learning process, and so forth, together with an analysis of how these elements may have influenced the whole process, outsiders will have a basis for deliberating adjustments to other situations" (Gravemeijer and Cobb 2013, p. 103).

CONCLUSION AND SUGGESTION

A review of the theoretical framework and the interpretation of the analyzed examples from our project classes explain how the introduction of negative numbers can be used to guide students to practice challenging mathematical activities. We have assigned a special role to the activity of local ordering of pieces of mathematical knowledge (the role of systematic construction and organization of mathematical knowledge). However, this activity can only be useful when teaching mathematics is not restricted to dealing with context-related activities. These activities should become an object of reflection. Only in this way the transfer to thinking and acting in mathematics and mathematical reasoning are possible to achieve.

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