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JUSTIFICATION FOR THE SUBJECT OF CONGRUENCE AND SIMILARITY IN THE CONTEXT OF DAILY LIFE AND CONCEPTUAL KNOWLEDGE

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Abstract

This study aims to examine prospective elementary mathematics teachers' conceptual knowledge level for congruence and similarity in triangles subject and to examine their ability to represent the knowledge, to associate the knowledge with daily life, and to justify and solve the geometry problems about this subject. The study is designed in a characteristic pattern. Total of 46 prospective elementary mathematics teachers were selected using purposive sampling method. The instruments used to collect data in this study are: GJP (Geometry Justification Problems), GCKQ (Geometry Conceptual Knowledge Questions) and GQDLE (Geometry Questions of Daily Life Examples). The data were analyzed using descriptive statistics method. The results of the study show that 1) the prospective teachers are successful in geometry conceptual knowledge questions but had difficulty in the justification problems; 2) there is a relationship between the theoretical knowledge levels and the argument standards of the prospective teachers; 3) the prospective teachers had difficulty in the daily life examples of congruence and similarity in triangles subject.

Keywords: Congruence and Similarity, Justification, Conceptual Knowledge, Daily Life Association

Abstrak

Tujuan dari penelitian ini adalah untuk menguji tingkat pengetahuan konseptual dalam hal kesesuaian dan kesamaan dalam segitiga, kemampuan untuk mewakili pengetahuan tentang konsep kongruensi dan kesamaan, untuk mengasosiasikan mereka dengan kehidupan sehari-hari, dan untuk membenarkan dan memecahkan masalah geometri pada subjek kongruensi-kesamaan dari calon guru matematika SD. Penelitian ini dirancang dalam pola karakteristik dan sampel penelitian terdiri dari 46 calon guru matematika SD melalui metode purposive sampling. Sebanyak tiga instrumen pengumpulan data yang digunakan dalam penelitian ini, yaitu GJP (*Geometry Justification Problems*), GCKQ (*Geometry Conceptual Knowledge Questions*), and GQDLE (*Geometry Questions on Daily Life Examples*). Statistik deskriptif digunakan dalam analisis data. Hasil penelitian menunjukkan bahwa calon guru berhasil dalam pengetahuan konsep geometri; Namun, mereka memiliki kesulitan dalam menentukan suatu masalah. Studi ini menemukan hubungan antara tingkat pengetahuan teoritis dan standar argumen dari calon guru. Selanjutnya, salah satu hasil yang diperoleh dalam penelitian ini adalah bahwa calon guru mengalami kesulitan dalam membuat contoh pada permasalahan kehidupan sehari-hari untuk materi kongruensi dan kesamaan segitiga.

Kata kunci: Kongruensi dan Kesamaan, Justifikasi, Pengetahuan Konseptual, Asosiasi Kehidupan Sehari-hari

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Mathematics is a multifaceted science with an unlimited span and depth (İlgar & Gülten, 2013). Geometry, one of the oldest branches in mathematics, is known as one of the important learning areas in mathematics, involving the concepts of planar and spatial shapes (Fidan & Türnüklü, 2010). People encounter the shapes and concepts in geometry in daily life and professional life in most situations and these shapes and concepts enable us to overcome many simple problems in our lives (Turgut & Yılmaz, 2007). From the perspective of the fields of application, there are three cases in mathematics as follows: practical information, real life problems and internal discussions in mathematics (İlgar & Gülten, 2013). The use of mathematics in order to maintain the daily chores can be regarded within the scope of practical activities. However, the attempt to learn or teach mathematics, specifically geometry, when not associated with the daily life, would create problems in the comprehension of the subject matter. Students would be interested in and the effectiveness of learning would be enhanced to the extent a similarity between the subjects and the daily life is established. Utilizing mathematics in daily life, individuals indeed attempt to mathematize the nature in some way (Olkun & Toluk, 2007).

Representation in Mathematics Education

The use of different representations is considered to be as important as the integration to the daily life in facilitating the comprehension of mathematics and geometry and in ensuring the retention (Elia, Gagatsis & Deliyianni, 2005). Though the concept of representation is defined as the shape of the signs, characters or objects (Goldin & Shteingold, 2001), the place of this concept can be limited when it is defined as the means of representing mathematical ideas (Gagatsis Elia & Mougi, 2002). In the recent times, the different types of representations in mathematics learning have been widely acclaimed, and these representations indicate the same concepts in different ways; it is remarkable that the concepts are manipulated and there is a transition from one representation in a concept to another, and the verbal and figural representations are of a great importance particularly in geometric concepts (Panaouraa, 2013). One of the goals of the mathematics education is to teach the skills of problem-solving (MoNE, 2013). To have information on the relevant subject matter is the leading factor necessary for students to acquire the skills of problem-solving. In fact, the quality of such information has a particular importance in the issue of problem-solving. Problem-solving only by means of a series of formulas in books, without an intellectual process, does not achieve success in general. In this regard, it is concluded that the acquisition of the skills of questioning is paramount for students and the improvement of these skills is related to the comprehension of why and for what reason on the solutions that they develop, which highlights the importance of the concepts of proof/justification not only in mathematics but also in geometry.

Justification in Mathematics Education

Justification is at the core of doing and learning mathematics. In fact, certain deductive proofs, namely justification, which substantiates mathematical claims, are required in order to establish a new conclusion in mathematics. There is a lack of a single definition of the concepts such as proof or informal proof and justification on which theorists and researchers have come to agree (CadwalladerOlsker, 2011; Jaffe, 1997). Staples and Truxaw (2009) define justification as the process in which people get rid of the doubts by means of logical reasoning. Moreover, Staples, Bartlo and Thanheiser (2012) define and utilize justification as an argument that demonstrates (or refutes) the truth of a claim that uses accepted statements, and as a mathematical forms of reasoning. De Villiers (1999) reports that there are many purposes of justification such as to validate claims, illuminate or provide insight into a result or phenomenon, and systematize knowledge. The previous studies involving mathematical justification seem to focus primarily on proof (Komatsua, Tsujiyamab & Sakamaki, 2014; Sears, 2012; Simon & Blume, 1996).

Proof, with the purpose of convincing the other person of the truth of a proposition, involves the processes such as to reason on the proposition, to make an inference, to validate and advocate the inference

(Harel & Sowder, 1998; Stylianou, Chae & Blanton, 2006). Individuals are able to justify their arguments during the discussions in everyday life by means of observation and experiences; however, the justifications in the process of proving are required to be of an axiomatic nature since mathematical proofs are deductive (Esty, 1992). The comprehension of mathematical justification requires the reflection on what and how we know as well as how we evaluate the validity of the claims based on the other information that we have (Staples et al., 2012). The questions that promote mathematical justification include the questions of "How did you come to that conclusion?", "Could you explain what you did?", "What would you do to be persuasive to others while describing the methods in your study?" (Simon & Blume, 1996).

In terms of the types of justification, Bell (1976) divided mathematical justification into two categories as experimental justification where the claims are justified by means of examples and as deductive justification where the inferences are made in connection with the results. Balacheff (1988) examined the concept of proof in two different levels as pragmatic justification and conceptual justification. According to Balacheff (1988), pragmatic justifications are based on the use of examples and representations whereas conceptual justifications are based on the use of abstract formulas. The purpose of proof (justification) in mathematics was examined by Villers (1999, 2002) and Hanna (2000). The purposes of proof were presented as a set in mathematics communities. These are validation (concerning the accuracy of the statement), explanation (concerning an insight why the statement is accurate), systematization (basic concepts and theories), exploring (exploring or finding new results), communication (transfer of mathematical knowledge), and integration (comprehending well-known findings in a new framework).

Bell (1976) states that proof refers to three dimensions as validation or advocacy, explanation and systemization of the results. Harel and Sowder (1998) classify justification into three general categories as external, experimental and analytical proof schemes. In the external proof schemes, the formulas are exactly applied to the problems, and students generally prefer to memorize the rules. The experimental proof schemes are examined in two dimensions as deductive and perceptual proof schemes. In the analytical proof schemes, proofs are based on logical deductions.

Whereas De Villiers (1999) and Hanna (2000) utilized the concept of proof in both of the studies, Staples and Bartlo (2010) preferred to use the concept of justification since justification in the mathematics community reflects the aims that they adopt and there is a reflection that proof is a special form of justification. Some studies utilized mathematical proof as both an informal justification and a type of justification in mathematical proofs (Harel & Sowder, 2007). There are certain studies in which the concept of justification was utilized since the concepts of proof and justification have different meanings on behalf of teachers and proof is considered to involve a higher level of formality whereas there are the studies where proof and justification are considered to serve the same purpose (Staples et al, 2012). As it is acknowledged that justification comprises proof as well, this study utilized the concept of justification more often.

The interviews by Knuth (2002a, 2002b) with the mathematics teachers on the evaluation of proof, revealed that teachers undertake certain roles such as improving the skills of logical thinking, introducing thinking, and explaining why the statements are accurate. In the study by Staples and Truxaw (2009) asked the teachers which one of the justifications provided by students is better and for what reason. The teachers reflected that justification allows the students to explain their answers in detail, to define the method and to enable them to utilize the key relationships. The researchers stated that the teachers prefer the justification of these characteristics since they acknowledge that this kind of justification provides an insight for students. Staples and Bartlo Thanheiser (2012) reported that one of the important roles of justification in the classroom is to provide a profound perspective on certain subject matters for students. Besides, the teachers reflected that the available justification processes provide ever more profound understandings as they require the students to deal with the key ideas that enable them to acquire new insights and to make connections.

One of the important purposes of justification, probably the most global one acknowledged by teachers, is to contribute and to enhance the learning of students. A teacher stated that "justification prompts a student to go beyond the procedure to comprehend mathematics more profoundly. The student needs to advocate how he did or did not, in other words, why he did what he did, in order to support his or her own idea." In this regard, it seems that justification is considered as a practice enhancing the comprehension in terms of the opinions of teachers (Staples et al., 2012).

The practice of proof and justification in schools first begins with the courses of geometry (İpek, 2010). The process of proof and justification in geometry enables learning geometry theorems and problems more permanently rather than memorizing (Fujita & Jones, 2014; Yachel & Hanna, 2003). In this regard, it is considered important to ensure that students deal with proof and justification at an earlier age (Otten, Gilbertson & Males, 2014).

Lee Clark aFurthermore, the conclusion that there need to be the teachers with this skill in order to enable students to acquire this skill is inevitable. However, most teachers do not likely to teach justification properly since their knowledge level on this concept is restricted (Harel & Sowder, 2007).

Justification in Regard to Pedagogical Content Knowledge

Justification is known to enable us to understand how the ideas are interrelated in a logical way and to be a complex method requiring content knowledge (Staples & Truxaw, 2009). Thus, the importance of content knowledge and pedagogical content knowledge of teachers has become apparent in this regard. Shulman (1986) argued that an effective teacher is required to have the knowledge of subject matter as well as other types of knowledge involving the knowledge of specific strategies to be used in teaching. There are a number of studies in which the development of the mathematical competence of the students is linked to the mathematical knowledge of the teacher (Ball, Hill & Bass, 2005). Since it is believed that the mathematical knowledge of teachers has an influence on the achievement of students in mathematics (Ball et al., 2005), the improvement of the content knowledge of teachers may enhance the achievement of students in geometry. The geometry knowledge of teachers comprises of the content knowledge and relevant knowledge. The content knowledge in geometry refers to the facts, the information in geometry curriculum and the geometrical concepts. On the other hand, the relevant knowledge can be described as the relationship between certain subject matters with geometry and mathematics or with the subjects of other lessons. The geometry knowledge of teachers consists of the knowledge of facts, concepts and principles in geometry as well as the relationships between these facts, concepts and principles (Rossouw & Eddie Smith, 1997).

Congruence and Similarity

Traditionally, similarity is defined as the same shape which is not necessarily in the same size. The conceptualization of another similarity which is more based on mathematics involves the observation of a proportional relation when the relevant lengths in similar shapes or within a shape are compared (Seago, Jacobs, Heck, Nelson & Malzahn, 2013). Baykul (2009) stated that the shapes that can be overlapped are congruent whereas the shapes with the same forms are similar shapes. According to Baykul (2009), two planar shapes are congruent if their sizes and forms are same, and congruence is a special case of similarity. Uppal, John, Gill and Chawla (2006) reflected that two similar shapes, although they have the same shape, do not need to be in the same size, and all congruent shapes are similar; however, similar shapes may not be congruent. The comprehension of the concept of similarity underlies the comprehension of several geometry features. In this context, this study deals with the subject of congruence and similarity in triangles.

Current Study

The purpose of this study is to examine the conceptual knowledge level in regard to congruence and similarity in triangles, the ability to represent the knowledge on the concepts of congruence and similarity, to associate them with the daily life and to justify and solve the geometry problems on the subject of congruence-similarity of the prospective elementary mathematics teachers. In this regard, the following questions are to be answered:

1. At what level are the conceptual knowledge of the prospective teachers in regard to the congruence and similarity in triangles and their ability to respond these questions by means of different representations?
2. At what level is the association of the questions in the subject of congruence and similarity with daily life suggested by the prospective teachers?
3. At what level are the ability of the prospective teachers in solving and justifying the questions on similarity and congruence?
4. How is the relationship between the conceptual knowledge of the prospective teachers on the subject of congruence and similarity, their association of the knowledge with daily life and justification?

METHOD

Research Design

This study falls under the category of non-experimental research. In order to categorize non-experimental research, there are two main dimensions as time and the purpose of researcher (Johnson & Chritensen, 2000). This study is a cross-sectional and descriptive study given that it was conducted within a time period to describe or illustrate the characteristics of a condition or phenomenon in an accurate way. The practice of justification was the research subject in the dimension of time and a suitable time period was determined.

Participants

A total of 46 students in the department of elementary school mathematics in a state university participated in this study. As the sample of the study, criterion sampling, which is one of the purposeful sampling methods, was utilized. The criterion of the study was whether the students enrolled in geometry course or not. The prospective teachers receive education on congruence and similarity under the headings of congruence axioms and theorems in triangles, the practices in regard to congruence in triangles, similarity theories on triangles, the practices in regard to similarity in triangles during the geometry course that they enrolled in the second semester (YÖK, 2015). Given that the course involves proof and justification, the prospective teachers are equipped with the sufficient knowledge on this subject. Therefore, the prospective teachers in the second year of mathematics teaching were selected as the participants.

Data Collection Tools

A total of 3 different data collection tools (Appendix 1) developed by the researchers were utilized in this study. The opinions of two experts in the field of mathematics as well as the information obtained by means of literature review were used in the preparation of these data collection tools, and these tools were finalised following the required revisions.

1. **Geometry Justification Problems** (GJP): It consists of 8 geometry problems including the problems on congruence and similarity in triangles. The participants were asked to answer these questions with their justifications.
2. **Geometry Conceptual Knowledge Questions** (GCKQ): It consists of 6 questions prepared on the concept of the subject of congruence and similarity in triangles. These questions were on the definitions of the concepts of congruence and similarity and the prospective teachers were asked to provide certain examples from daily life and mathematics field in regard to these concepts in order to evaluate the knowledge level of the prospective teachers.
3. **Geometry Questions on Daily Life Examples** (GQDLE): It consists of 4 questions on congruence and similarity in daily life. These questions aim to examine the reflections of the knowledge of the prospective teachers in regard to congruence and similarity in triangles on the daily life.

Data Collection Process and Data Analysis

In the data collection process, GJP, GCKQ and GQDLE were utilized as the data collection tools. These tests were performed at three different times. Figure 1 indicates how these tests were performed to the participants. The tests of GCKQ, GJP and GQDLE were carried out respectively. The reason that GCKQ was initially performed was to ask the students to provide any information and example on congruence and similarity without showing any material. Subsequently, in the test of GQDLE, the prospective teachers were provided with certain examples on the concepts of congruence and similarity and asked whether these examples were congruent or similar. In the event that the test of GQDLE was initially performed, the prospective teachers might have had an insight while answering the test of GCKQ, which resulted in this order in regard to the implementation of the tests.

Figure 1. Aplication Sequence of Data Collection Tools

Geometry justification problems (GJP) were scored in two stages. Initially, the answers were evaluated in terms of correctness. A score of “1” was given for correct answers and a score of “0” was given for incorrect answers. When a question was answered correctly, a score of “2” was given for completely correct justifications and a score of “1” for partly correct justifications (Reasoning is accurate but conclusion is incorrect), and a score of “0” for incorrect justifications. In the analyses, the total scores for each question were calculated by adding these two types of scores. The questions in geometry conceptual knowledge test (GCKQ) were evaluated in two stages. Initially, a score of “1” was given for correct answers and a score of “0” was given for incorrect answers. The types of representation (verbal, algebraic, figural) used in order to solve the problem were determined for each answer. In geometry questions on daily life examples, a score of “2” was given for completely correct answers and a score of “1” for partly correct answers, and a score of “0” for incorrect answers. Table 1 illustrates the scoring in the data collection tools.

Table 1. Scoring criteria for GJP, GCKQ, GQDLE

GJP Scoring Method		**GCKQ Scoring Method**		**GQDLE Scoring Method**	
Points	**Description**	**Points**	**Description**	**Points**	**Description**
0	Left blank or a justification provided inaccurately and without the mathematical symbols, explanations and algorithms.	**0**	Inaccurate Answer	**0**	Inaccurate Answer
1	Incomplete (Partially Accurate) Justification, the answer is accurate but not based on rationality by providing intuitive justification through inferences not based on fully developed mental images and by establishing the limited connections between the previous arguments.	**1**	Partially Accurate Answer (The answer is accurate but there is a lack of mathematical explanation and the concepts of congruence and similarity)	**1**	Accurate Answer
2	Totally Accurate Justification (The answers are detailed, easy to understand, expressed by	**2**	Totally Accurate Answer (explained based on mathematics)		

	means of a special mathematical method (e.g. AAA (angle) similarity theorem) by a person who comprehended the key mathematical relationships required to solve the problem or made an inference, provided consistent explanations, reached the formal level.	

Following the data collection, the data collection tools were scored separately by two researchers. In order to enhance the reliability of the study, the data obtained by the prospective teachers were examined by two field experts and categorized as "Consensus" and "Dissidence". The formula of Reliability= [(Consensus) / (Consensus) + (Dissidence)] x 100, developed by Miles and Huberman (1994), was utilized; and the compliance to the formula should be 70% at minimum so as to consider the scoring reliable (Yıldırım and Şimşek, 2008), and the value of this study was found as 88%.

Each question in the test of GCKQ was analyzed in terms of the accurateness as well as the type of representation. Certain examples from the answers provided by the prospective teachers in the test of GJP were discussed. Furthermore, the scores of the prospective teachers in each of the three tests were divided into low, moderate and high according to the scoring classification by Alamolhodaei (1996). According to this formula, the quarter of a standard deviation is added to the average, and the prospective teachers with the higher scores than the resulting number have a "high" (S_3) level of knowledge whereas those with the lower scores than the number obtained by subtracting the quarter of a standard deviation from the average have a "low" (S_1) level of knowledge; and, those with the scores between these two scores have a "moderate" (S_2) level of knowledge ($S_1 = \bar{X} - \frac{s}{4}, S_3 = \bar{X} + \frac{s}{4}, S_1 < S_2 < S_3$; *s: standard deviation*, $\bar{X}$: *average*).

RESULTS AND DISCUSSION

This chapter consists of the findings of the data obtained by the data collection tools in the study. The findings are demonstrated based on the order of the sub-problems.

Descriptive Analysis on the Conceptual Knowledge of the Prospective Teachers on the Subject of Congruence and Similarity and the Ability to Answer the Questions by means of Different Representations

Table 2 indicates the examination of the answers by the prospective teachers for the test of GCKQ.

Table 2. Descriptive Statistics of the Test of Geometry Conceptual Knowledge

Question No:	Correct/ Incorrect	Verbal	Visual	Algebraic	Verba/ Algebraic	Visual / Algebraic	Verbal /Visual	Verbal / Visual/ Algebraic	Total
Question 1	Correct	40	0	0	0	0	1	1	42
	Incorrect	4	0	0	0	0	0	0	4
	Total	44	0	0	0	0	1	1	46
Question 2	Correct	34	1	0	0	0	1	1	37
	Incorrect	9	0	0	0	0	0	0	9
	Total	43	1	0	0	0	1	1	46
Question 3	Correct	31	0	0	0	0	6	0	37
	Incorrect	9	0	0	0	0	0	0	9
	Total	40	0	0	0	0	6	0	46
Question 4	Correct	24	0	0	0	0	5	0	29
	Incorrect	17	0	0	0	0	0	0	17
	Total	41	0	0	0	0	5	0	46
Question 5	Correct	9	15	3	1	8	7	1	44
	Incorrect	2	0	0	0	0	0	0	2
	Total	11	0	3	1	8	5	0	46
Question 6	Correct	9	15	1	0	8	9	1	43
	Incorrect	2	0	1	0	0	0	0	3
	Total	11	15	2	0	8	9	1	46

Table 2 indicates how many of the prospective teachers answered accurately and which types of representations were utilized by them in the questions. As it can be seen in Table 2, the majority of the prospective teachers accurately answered the questions. The fifth question was the most correctly answered question by a total of 44 prospective teachers. This question involved a mathematical example for the concept of congruence. The fourth question was the question with the least number of correct answers by a total of 29 prospective teachers and involved the daily life examples in regard to the concept of similarity. The types of representations utilized by the prospective teachers in these questions were analyzed. It is remarkable that the verbal representation was used most in all of the questions except for the fifth and sixth questions. In the fifth and sixth questions, the type of representation used most was verbal representation. In these questions, the prospective teachers were asked to provide a mathematical example for the concepts of congruence and similarity. Therefore, it can be stated that the prospective teachers tend to use visual representations in the mathematical examples. There was a student that utilized all of the three representation types in all of the questions except for third and fourth questions.

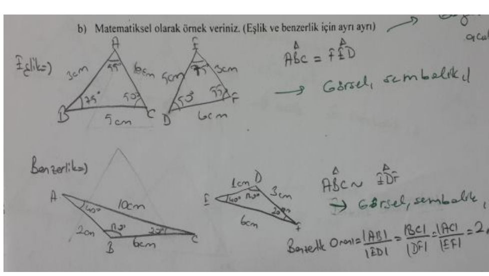

Figure 2

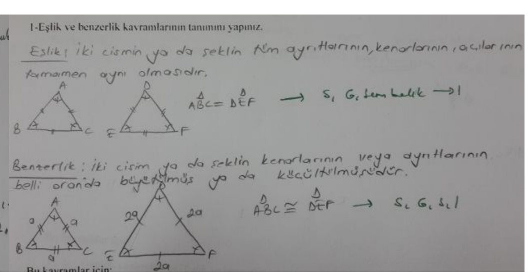

Figure 3

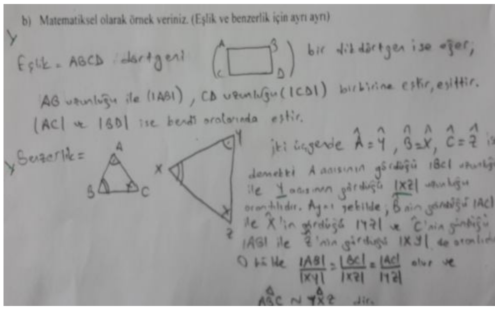

Figure 4

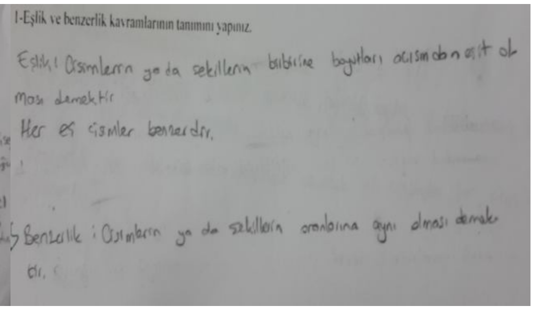

Figure 5

Some of the answers in the test of geometry conceptual knowledge were demonstrated as examples. In Figure 2, there is a question asking for a mathematical example in regard to the concepts of congruence and similarity and the mathematical description of these concepts. It can be seen that the prospective teachers utilized both visual and algebraic representations. In Figure 4, it is remarkable that all of the three representations (visual, verbal and algebraic) were used for the same question. In regard to the concepts of congruence and similarity, Figure 3 demonstrates that all of the three representation types were used whereas the definitions were provided only by means of verbal representation in Figure 5.

Descriptive Analysis on the Ability of the Prospective Teachers in the Justification of the Problems on the Subject of Congruence and Similarity

Table 3 indicates the examination of the answers by the prospective teachers for the test of GJP.

Table 3. Descriptive Statistics of Geometry Justification Problems

Question No:	Correct/Incorrect	Correct Justification		Partly Correct Justification		Incorrect Justification		Total
		f	%	f	%	f	%	
Question 1	Correct Answer	23	62,16	13	35,1	1	2,7	37
	Incorrect Answer	0		0		9		9
	Total	23		13		10		46
Question 2	Correct Answer	17	50	15	44,1	2	5,9	34
	Incorrect Answer	0		0		12		12
	Total	17		15		14		46
Question 3	Correct Answer	15	45,45	17	51,5	1	3	33
	Incorrect Answer	0		0		13		13
	Total	15		17		14		46
Question 4	Correct Answer	13	39,4	16	48,5	4	12,1	33
	Incorrect Answer	0		0		13		13
	Total	13		16		17		46
Question 5	Correct Answer	14	56	9	36	2	8	25
	Incorrect Answer	0		0		21		21
	Total	14		9		23		46
Question 6	Correct Answer	15	46,9	15	46,9	2	6,25	32
	Incorrect Answer	0		0		14		14
	Total	15		15		16		46

Question 7	Correct Answer	13	43,3	15	50	2	6,7	30
	Incorrect Answer	10		0		16		16
	Total	13		15		18		46
Question 8	Correct Answer	11	44	12	48	2	8	25
	Incorrect Answer	0		0		21		21
	Total	11		12		23		46

Table 3 indicates the number of the prospective teachers who answered the Geometry Justification Problems accurately and who did not as well as the number of those whose justifications were correct and incorrect. In general, more than half of the prospective teachers answered each question accurately. The question that was most correctly answered was the first question answered by a total of thirty seven participants whereas the most challenging question was the ninth question answered by a total of twenty five participants incorrectly. The prospective teachers were also asked to proof/justify the answers that they provided for each question. In this regard, the justifications of the prospective teachers were analyzed as well. As Table 3 demonstrates, not all of the participants who answered the questions accurately provided a totally correct justification. The first question had the highest rate of the correct answers and totally correct justifications, 62.16%. A total of 23 prospective teachers both answered the question accurately and provided the correct justification as seen in Table 3. The fourth question was one of the correctly answered questions with the highest rate of incorrect justification by 12.1%. Furthermore, the fourth question was correctly answered by a total of four participants who provided an inaccurate justification. The third question was one of the correctly answered questions with the highest rate of partially accurate justification by 51.5% and a total of 17 participants. As it can be seen in the table, the rate of the participants who both accurately answered the questions and provided an accurate justification for them is below 50% for each question.

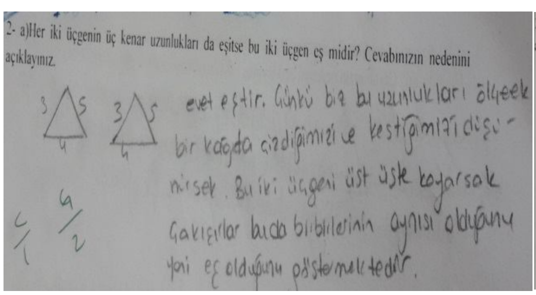

Figure 6

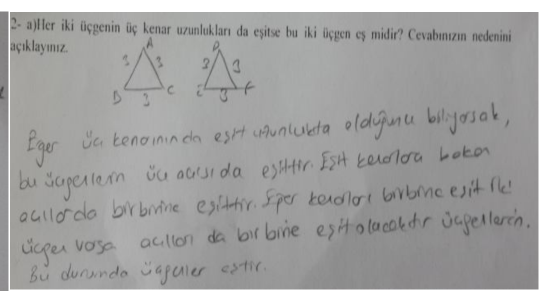

Figure 7

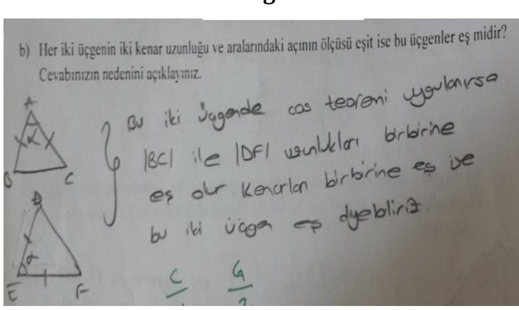

Figure 8

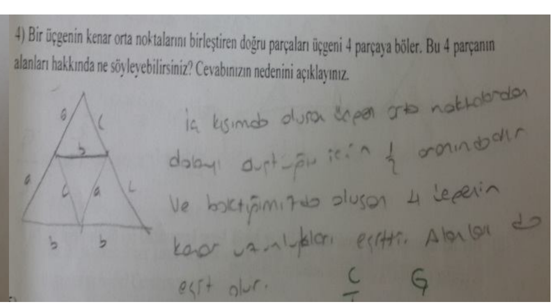

Figure 9

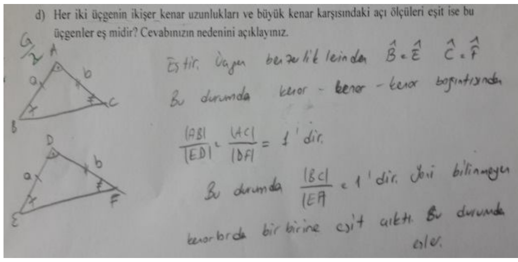

Figure 10

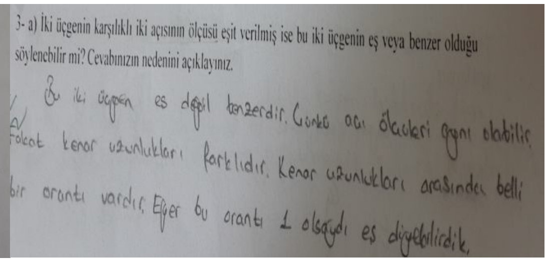

Figure 11

The question in Figure 6 asked if the three side lengths of two triangles are equal, whether these two triangles are congruent or not. A prospective teacher stated that these two triangles are congruent and this congruence can be indicated by means of an experiment. Thus, the teacher claimed that the accuracy of the answer that he or she provided can be justified by experiment and demonstration. Therefore, this type of justification can be an example of experimental justification according to the types of justification proposed by Bell (1976). In Figure 7, there is the answer of another prospective teacher who answered the same question. The teacher provided the correct answer that the triangles are congruent. It is remarkable that the prospective teachers were aware of the correlation of edge/edge/edge and triangle congruence in providing the justifications but did not express the correlation with terminological statements. It was observed that the prospective teacher utilized the data and attempted to have an inductive conclusion. Figure 8 demonstrates the question of “If the two side lengths of and angle between both triangles are equal, are these triangles congruent?” This question attempted to determine whether the prospective teacher is acquainted with the relation of edge-angle-edge equality. The prospective teacher utilized the fact that the three side lengths of a triangle must be congruent in order to classify the triangles as congruent. The teacher, accordingly, reflected that it would be necessary to determine the length of the third side, which may be calculated by means of cosine law. Subsequently, the teacher stated that since the two sides and the measure of the relevant angle are equal to each other, the length of the third side would be congruent and the triangles are congruent accordingly. The justification in the answer of the prospective teacher is an inductive justification and can be an example of the conceptual justification according to the classification by Balacheff (1988). In Figure 9, there is the question of “are the areas of the triangles in a triangle formed by connecting the edge midpoints of the triangle congruent to each other?” The prospective teacher demonstrated that the lengths of the edges of the triangles are congruent by the principle of similarity and concluded that the four areas of the triangles are congruent on the grounds that the triangles with the same edge lengths are congruent triangles. As it can be seen in Figure 10, the prospective teacher attempted to justify his or her answer by establishing equality relations. This type of justification can also be an example of the conceptual justification according to the classification by Balacheff (1988). However, certain abstract formulas are utilized in this type of justification. In Figure, there is the question of “If the two opposite angles of two triangles are equal, can you conclude that these two angles are congruent or similar?” The prospective teacher provided a justification stating that they may not be congruent since the rate between the lengths of their sides might not be equal to each other.

Descriptive Analysis on the Association of the Questions on the Subject of Congruence and Similarity of the Prospective Teachers to the Daily Life

Table 4 indicates the examination of the answers by the prospective teachers for the test of GQDLE.

Table 4. Descriptive Statistics of the Test of Geometry Daily Life Examples

	Correct Answer	Partly Correct Answer	Incorrect Answer	Total
Question 1	6	37	3	46
Question 2	33	9	4	46
Question 3	5	3	38	46
Question 4	3	1	42	46

The first question in the test of GQDLE was on how the fingerprints of a person change with age. The expected answer from the prospective teachers in that question was that the shape of the fingerprints does not change but the size of the fingerprints may change; in other words, that the fingerprints would be similar to each other with age. The use of the concept of "similar" was taken into consideration in the analysis of the answers by the prospective teachers. As Table 4 indicates, only a total of 6 prospective teachers answered the first question completely accurately. The majority of them intuitively comprehended that the fingerprints are similar to each other but did not utilize the concept of similarity. There were merely 3 prospective teachers that answered the question completely inaccurately.

The second question in the test of GQDLE was what the prospective teachers can conclude about Matryoshka dolls. The expected answer was that these dolls are similar to each other as well. As Table 4 indicates, a total of 33 prospective teachers answered to the question accurately and a total of 9 teachers did not utilize the statement "they are similar" but intuitively attempted to describe the same situation; therefore, their answers were considered as partially accurately. The second question, compared to the first question, was a more explicit question for the prospective teachers to observe the similarity.

The third question in the test of GQDLE was whether the lead sizes of mechanical pencils (propelling pencils) such as 0.5, 0.7, 0.9 mm are similar to each other or not. There is an increase of 0.2 mm between the radii of these three leads, which is an additive increase. However, there is not an increase at the same rate in terms of the rate between them (multiplicative increase). Thus, there is not a similarity between the leads. Nevertheless, the majority of the prospective teachers, specifically a total of 38 participants, reflected that the leads were similar to each other, and thus did not answer the question accurately. Only a total of 5 prospective teachers stated that the rate between the leads was not similar, and thus answered the question accurately.

The last question in the test of GQDLE was on the shoe sizes. In this question, it was stated that for each increase in shoe size, there is an increase by 2/3 of a centimetre in the length of the shoes and asked whether there is a similarity between the consecutive sizes or not. There was an additive increase in this question. In other words, there is an increase by 2/3 of a centimetre in the length of the shoes, which leads that there is not a same rate between the sizes. For instance, the similarity rate between the shoes with the size of 35, 36, and 37 would not be same. The expected answer was that the sizes are not similar to each other. However, as Table 4 indicates, a total of 42 prospective teachers, namely the majority of them, did not answer the question accurately. The prospective teachers perceived an additive increase as a multiplicative increase.

Descriptive Analysis on the Relationship between the Conceptual Knowledge of the Prospective Teachers on the Subject of Congruence and Similarity, Their Association of the Knowledge with Daily Life Association and Justification

In this chapter, Table 6 demonstrates the relationship between the three data collection tools divided into three levels as low, moderate and high levels in terms of these levels according to the scoring method by Alamolhodaei (1996) in a descriptive way.

Table 5. Descriptive Statistics Demonstrating the Relationship between GQDLE, GCKQ, GJK of the Prospective Teachers

		GQDLE High Level	GQDLE Moderate Level	GQDLE Low Level
GJK High Level**	GCKQ High Level	4*	3	3
	GCKQ Moderate Level	2	3	0
	GCKQ Low Level	0	3	0
GJK Moderate Level	GCKQ High Level	1	2	0
	GCKQ Moderate Level	0	2	1
	GCKQ Low Level	1	1	0
GJK Low Level	GCKQ High Level	3	2	3
	GCKQ Moderate Level	1	2	1
	GCKQ Low Level	2	3	3

***The numbers given are the numbers of the prospective teachers.**

**A total of 18 prospective teachers had a high level of GJK; a total of 10 prospective teachers had high level of GCKQ and there are 4 prospective teachers with a high level of GQDLE and 3 teachers with a moderate level of GQDLE and 3 teachers with a low level of GQDLE.

As Table 5 indicates, there were a total of 10 teachers with a high level in the justification problems and in the conceptual knowledge problems whereas a total of 3 prospective teachers had a low level in the conceptual knowledge. While there were a total of 8 prospective teachers with a low level in the justification problems and a high level in the conceptual knowledge problems, there were a total of 8 prospective teachers with a low level in these categories. There were a total of 5 prospective teachers with a high level in justification and a moderate level in the conceptual knowledge whereas a total of 4 prospective teachers had a low level in the justification and a moderate level in the conceptual knowledge. The increase in the level of the conceptual knowledge along with the increase in the level of justification supports the idea that justification requires a sufficient level of knowledge on the relevant subject.

Whereas there was a total of 4 prospective teachers with a high level in all of three data collection tools, a total of 3 prospective teachers had a low level. As demonstrated in Table 5, there were 3 prospective teachers with a high level in the justification and conceptual knowledge questions and a low level in the daily life examples there were also 3 prospective teachers with a low level in the justification and conceptual knowledge and a high level in the daily life examples. It may be concluded that the ability in providing the daily life examples does not have an impact on the levels of justification and conceptual knowledge.

The purpose of this study was to examine the conceptual knowledge level in regard to the congruence and similarity in triangles, the ability to represent the knowledge on the concepts of congruence and similarity, to associate them with the daily life and to justify and solve the geometry problems on the subject of congruence-similarity of the prospective elementary mathematics teachers. In this regard, given the conceptual knowledge level of the prospective teachers on the subject of congruence and similarity in triangles, that most of them answered the questions correctly indicates that their conceptual knowledge level are at a quite good level, and according to the classification, more than half of them have a moderate or high level of knowledge. The questions involving mathematics examples in regard to congruence and similarity are the questions with the highest rates of correct answers whereas those involving daily life examples are the questions with the lowest rates of correct answers. According to the ways of representations in the questions, it is remarkable that most of the participants utilized verbal representations in all of the questions except for two questions in particular. The use of visual representation were only remarkable in the last two questions, which is believed to result from that the question involves a mathematical example on congruence and similarity, that is, the nature of the question requires the use of visual representation. Similarly, the wide use of verbal representation may arise from the nature of the question. On the other hand, since there was a participant who utilized all of three representation types (verbal, visual and algebraic) in all questions except for two, it is apparent that the questions could be answered not by a certain type of representation but by various representations.

In the geometry justification problems, more than half of the prospective teachers accurately answered nearly all of the questions. However, it is remarkable that there were some prospective teachers who accurately answered the questions but could not justify the answer with a rational reason. Although the majority of the prospective teachers were aware of the reason of their answers, they were not able to fully explain them. This finding demonstrates the fact that the prospective teachers answered the questions by heart and did not know the reason of the solution that they came up with. The study by Özer and Arıkan (2002) revealed a similar finding that the proof levels of high school students were quite low. Jones (2000) reflected that the difficulty of the students in the process of proof results from the lack of a general view on the subject of the proof. Furthermore, many other studies concluded that the levels of proof of the prospective teachers were not sufficient (Moralı, Uğurel, Türnülü & Yeşildere, 2006). The prospective teachers who provided an accurate justification preferred to utilize verbal expressions rather than mathematical expressions. The majority of their justifications were based on a correlation related to similarity; however, they did not explicitly specify the name of the correlation. The most preferred method by the prospective teachers in providing a mathematical justification was inductive justification. The ability of the prospective teachers in explaining the justification of the solutions that they developed indicates that they have the sufficient and required knowledge on the relevant subject. Indeed, having the necessary content knowledge by teachers is one of the elements that have an effect on the quality of teaching. In order to improve their justification level, the students require an extensive pedagogical repertoire supported by comprehensive and relevant class norms and content knowledge (Knuth, 2002; Staples, 2008). That said, the profound knowledge of the prospective teachers on proof and justification may be an indicator that they can teach the subject in an efficient way. In this regard, the introduction of the required course contents in their education at university enables the prospective teachers to improve their justification skills.

The prospective teachers were asked the questions on congruence and similarity related to daily life. Nevertheless, it is remarkable that the success of the prospective teachers was quite low, for which the reason might be that the prospective teachers were asked the questions that they constantly encountered in the daily life but did not consider as a case of congruence and similarity. However, the answers of the prospective teachers revealed that they did not comprehend the concept of similarity rate properly. Indeed, they considered additive increase as multiplicative (rational) increase and obtained the result that there was a case of similarity in the situations where there was not. In this regard, it was concluded that there is not a direct relationship between the concepts of similarity in mathematics and the daily life.

The prospective teachers were divided into three levels in terms of the total scores that they obtained in the three data collection tools. These three levels were high, moderate and low, and the relations between these levels were examined. As the tables analyzing the information and the scores in the justification problems indicate, the lower number of the teachers with a high justification level and a high conceptual knowledge level than that of the teachers with a high justification level but a low conceptual knowledge level reveals that justification requires a high knowledge level on the same subject. In this regard, as Staples and Truxaw (2009) renders, it can be stated that justification, a complex method requiring content knowledge, enables the comprehension of how the ideas are related with each other in a logical way. Jones (2000) carried out a study on proof with mathematics teachers and reveals that the skills related to proof are at a low level. However, a remarkable finding in this study by Jones (2000) was that the mathematics teachers with a high level of skills did not have the required mathematical knowledge. Although the prospective teachers at a high level performed the proof in a smoother way, it was found that they could not perform this task with extensive mathematics knowledge (Jones, 2000). Fawcett (1938) reflects that the competencies of students in proof improve their competencies in mathematics (Citation, Stylianides, Stylianides & Philippou, 2007). In this regard, if one wants to improve the skills of proof and justification of students starting from primary school level, he or she should primarily teach the information on the subject area and, moreover, utilize justification while teaching them. It goes without saying that a teacher who is supposed to teach the subject of justification properly should improve himself or herself and receive a good education in that subject. Accordingly, the institutions for teacher training are required to place a particular importance on teaching the concepts such as justification and proof.

As there was not any change in the number of the prospective teachers with a high level in terms of justification and conceptual knowledge as well as a low level in daily life, the number of the prospective teachers with a high level in terms of justification and knowledge as well as a low level in terms of daily life examples and that of those with a low level in terms of justification and conceptual knowledge as well as a high level in terms of daily life knowledge, it may be concluded that the knowledge of daily life examples might not depend on the level of conceptual knowledge and justification. On the other hand, it is believed that the comprehension of the concept of congruence and similarity in the daily life is associated with the concept and knowledge on that subject. However, since the prospective teachers are quite unsuccessful in the questions of congruence and similarity in daily life, such relationship does not seem to exist.

CONCLUSION

The study revealed the findings that the conceptual knowledge level of the prospective teachers on the subject of congruence and similarity in triangles was high, that although the number of the prospective teachers who accurately answered the justification problems was high, the number of those who completely accurately answered them was low, that the prospective teachers did not consider justification as the concept of proof, that the prospective teachers had difficulty in the daily life questions, in other words, could not adapt the subject of congruence and similarity into the daily life. Among others, the findings that the level of knowledge influences not the level of the daily life examples but the level of justification and that the level of the daily life examples does not have an impact on the level of justification are among other findings of the study. Furthermore, the level of the conceptual knowledge and the level of justification in together do not have an influence over the daily life examples.

Since the deficiencies in the ability of teachers in regard to mathematical proof and justification have an impact on their frequency of the use of mathematical proof and justification in courses, it is concluded that they should be underlined in both pure mathematics courses and mathematics education courses in the institutions for teacher training (Moralı et al., 2006). A process of learning in which the students are also actively engaged in the process of justification, advocate and criticize the proofs that they provide rather than the teacher-centered justifications while teaching proof and justification in the programs for teacher training, is believed to enable the prospective teachers to comprehend the concepts of proof and justification better.

The relation of justification between the conceptual knowledge level and daily life examples level, in regard to congruence and similarity, was examined in this study. A similar study on a different subject as well as more detailed studies may be carried out to deal with different variables such as geometric thinking levels, which have an effect on justification. This study conducted with university students may be performed with different groups in terms of grade. The courses on proof and justification might be reshaped by means of available schemas in the literature and further studies on proof schemas of students.

REFERENCES

Alamolhodaei, H. (1996). A study in higher education calculus and students' learning styles. *Doctoral dissertation.* Glasgow: University of Glasgow.

Balacheff, N. (1988). Aspect of proof in pupils' practice of school mathematics. (Eds. D. Pimm). *Mathematics Teachers and Children*, 216-235. London: Hodder & Stougtoni.

Ball, D.L., Hill, H.C. & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator (Fall)*, 14-46

Bell, A.W. (1976). A study of pupils' proff-explanations in mathematical situations. *Educational Studies in Mathematics*, *7*, 23-40.

CadwalladerOlsker, T. (2011). What do we mean by mathematical proof? *Journal of Humanistic Mathematics, 1(1),* 33–60.

Ceylan, T. (2012). Investigating preservice elementary mathematics teachers' types of proofs in GeoGebra environment. *Master Thesis.* Ankara: Ankara University.

de Villiers, M. (1999). *Rethinking proof with the Geometer's Sketchpad. Emeryville.* CA: Key Curriculum Press.

de Villiers, M. (2002). Developing understanding for different roles of proof in dynamic geometry. *Paper presented at ProfMat*, Visue.

Elia I., Gagatsis, A., & Deliyianni E. (2005). A reviewof the effects of different modes of representations in mathematical problem solving. In: Gagatsis A, Spagnolo F, Makrides Gr., Farmaki V, editors, *Proceedings of the 4th Mediterranean Conference on Mathematics Education, 1,* 271-286. Palermo, Italy: University of Palermo, Cyprus Mathematical Society.

Esty, W.W. (1992). Language concepts of mathematics. *Focus on Learning Problems in Mathematics, 14(4)*, 31-54.

Fidan, Y., & Türnüklü, E. (2010). Examination of 5th grade students' levels of geometric thinking in terms of some variables. *Pamukkale University Journal of Education, 27*, 185-197.

Fujita, T., & Jones, K. (2014). Reasoning-and-proving in geometry in school mathematics textbooks in Japan. *International Journal of Educational Research, 64*, 81-91.

Gagatsis, A., Elia, I., & Mougi A. (2002). The nature of multiple representations in developing mathematical relations. *Scientia Paedagogica Experimentalis, 39(1),* 9-24.

Goldin, G., & Shteingold, N. (2001). System of mathematical representations and development of mathematical concepts. In: Curcio FR, editor, *The roles of representation in school mathematics: 2001 yearbook.* Reston: National Council of teachers ofMathematics.

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics, 44*, 5-23.

Harel, G. & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *American Mathematical Society, 7*, 234-283.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. *Second handbook of research on mathematics teaching and learning, 2*, 805-842.

İlgar, L., & Gülten, D. Ç. (2013). Matematik konularının günlük yaşamda kullanımının öğrencilere öğretilmesinin gerekliliği ve önemi. *İstanbul Zaim Üniversitesi Sosyal Bilimler Dergisi,* Güz.

Jaffe, A. (1997). Proof and the evolution of mathematics. *Synthese, 111(2)*, 133-146.

Jones, K. (2000). The student experience of mathematical proof at university level. *International Journal of Mathematical Education in Science and Technology, 31(1)*, 53-60.

Johnson, R.B., & Christensen, L.B. (2000). *Educational research: Quantitative and qualitative approaches.* Boston: Allyn and Bacon

Knuth, E.J. (2002a). Secondary school mathematics teachers' conceptions of proof. *Journal for Research in Mathematics Education, 33(5),* 379-405.

Knuth, E.J. (2002b). Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teacher Education, 5(1),* 61-88.

Komatsu, K., Tsujiyama, Y., & Sakamaki, A. (2014). Rethinking the discovery function of proof within the context of proofs and refutations. *International Journal of Mathematical Education in Science and Technology, 45(7),* 1053-1067.

Milli Eğitim Bakanlığı [MoNE] (2013). *Ortaokul matematik dersi (5,6, 7 ve 8 sınıflar) öğretim programı*. Ankara: Talim Terbiye Kurulu Başkanlığı.

Moralı, S., Uğurel, I., Türnüklü, E., & Yeşildere, S. (2006). Matematik öğretmen adaylarının ispat yapmaya yönelik görüşleri. *Kastamonu Eğitim Dergisi, 14(1),* 147-160.

Olkun, S. & Toluk, Z. (2007). *İlköğretimde Etkinlik Temelli Matematik Öğretimi.* (3.Baskı). Ankara: Maya Akademi Yayın Dağıtım.

Otten, S., Gilbertson, N. J., Males, L. M., & Clark, D. L. (2014). The mathematical nature of reasoning-and-proving opportunities in geometry textbooks. *Mathematical Thinking and Learning, 16(1),* 51-79.

Özer, Ö. & Arıkan, A. (2002). Lise matematik derslerinde öğrencilerin ispat yapabilme düzeyleri, V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, 16-18 Eylül, Ankara, *Bildiriler Kitabı Cilt II,* s.1083-1089.

Panaoura, A. (2013). Using representations in geometry: a model of students' cognitive and affective performance. *International Journal of Mathematical Education in Science and Technology, 45(4),* 498-511.

Rossouw, L., & Smith, E. (1997). Teachers knowledge of geometry teaching-two years on after an ınset course. *African Journal of Research in Mathematics, Science and Technology Education, 1(1),* 88-98.

Seago, N.M., Jacobs, J.K., Heck, D.J., Nelson, C.L., & Malzahn, K.A. (2013). Impacting teachers' understanding of geometric similarity: results from field testing of the Learning and Teaching Geometry professional development materials. *Professional Development in Education, 40(4),* 627-653.

Sears, R. (2012). The impact of subject-specific curriculum materials on the teaching of proof and proof schemes in high school geometry classrooms. *Short Research Report Presentation at the 12th International Congress on Mathematical Education* (ICME-12), Seoul, Korea.

Shulman, L. (1986). Paradigms and research programs in the study of teaching: a contemporary perspective. In M, Wittrock (Ed.), *Handbook of Research on Teaching.* NY: Macmillian Publishing Company.

Simon, M.A., & Blume, G.W. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior, 15*, 3-31.

Staples, M., & Bartlo, J. (2010). Justification as a learning practice: Its purposes in middle grades mathematics classrooms. *CRME Publications.* Paper 3.

Staples, M., & Truxaw, M. (2009). A journey with justification: Issues arising from the implementation and evaluation of the Math ACCESS Project. In *Proceedings of the thirty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 5, 827-835.

Staples, M.E., Bartlo, J., & Thanheiser, E. (2012). Justification as a teaching and learning practice: Its (potential) multifacted role in middle grades mathematics classrooms. *The Journal of Mathematical Behavior, 31(4),* 447-462.

Stylianides, A.J., Stylianides, G.J., & Philippou, G.N. (2004). Undergraduate students' understanding of the contraposition equivalence rule in symbolic and verbal contexts. *Educational Studies in Mathematics, 55*, 133-162.

Stylianou, D., Chae, N., & Blanton, M. (2006). Students' proof schemes: A closer look at what characterizes students' proof conceptions. In *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Mérida, México: Universidad Pedagógica Nacional.

Turgut, U.M., & Yılmaz, S. (2007). Geometri derslerine nasıl giriş yapardık? İlköğretim matematik öğretmen adaylarının görüşleri. *Bilim Eğitim ve Düşünce Dergisi, 7(4).*

Uppal, S., John, M., Gill, J., & Chawla, A. (2006). Triangles. Yackel, E. & Hanna, G. (2003). Reasoning and proof. J. Kilpatrick, W.G. Martin ve D. Schifter (Ed.), A *research companion to principles and standarts to school mathematics*, 227-236. Reston, VA: National Council of Teachers of Mathematics.

APPENDIX-1

Geometry Questions on Daily Life Examples

1- Do the fingerprints of a person change? How do fingerprints change with age? Please explain your answer.

2- Matryoshka dolls are a series of wooden dolls that nest into one another, unique to Central Europe. What can you conclude about these dolls? Please explain your answer.

3- The lead sizes of mechanical pencils (propelling pencils) are 0.5, 0.7, and 0.9 mm, etc. In your opinion, are these leads similar? Please explain your answer.

4- For each increase in shoe size, there is an increase by 2/3 of a centimeter in the length of the shoes. In your opinion, is there any similarity between these sizes? Please explain your answer.

Geometry Conceptual Knowledge Questions

1- Please define the concept of congruence.

2- Please define the concept of similarity.

3- Please give examples on the concept of congruence from daily life.

4- Please give examples on the concept of similarity from daily life.

5- Please give examples on the concept of congruence in mathematics.

6- Please give examples on the concept of similarity in mathematics.

Geometry Justification Questions

1- If the three side lengths of two triangles are equal, are these two triangles congruent? Please explain your answer.

2- If the two side lengths of and angle between both triangles are equal, are these triangles congruent? Please explain your answer.

3- If the two angles of two triangles and the side length between these two angles are equal, are these triangles congruent? Please explain your answer.

4- If the two side lengths and their angles opposite to the longer edges are equal, are these triangles congruent? Please explain your answer.

5- If the two opposite angles of two triangles are equal, can you conclude that these two angles are congruent or similar? Please explain your answer.

6- If the lengths of the opposite sides of two triangles are proportional, can you conclude that these two triangles are similar? Please explain your answer.

7- If the lengths of the two sides of two triangles are proportional and the angles opposite to the longer edges of these two sides are equal, can you conclude that they are similar? Please explain your answer.

8- The line segments connecting the edge midpoints of a triangle divide the triangle into 4 parts. What can you conclude about the areas of these 4 parts? Please explain your answer.