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INVESTIGATING STUDENTS' DEVELOPMENT OF LEARNING INTEGER CONCEPT AND INTEGER ADDITION

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Abstract

This research aimed at investigating students' development of learning integer concept and integer addition. The investigation was based on analyzing students' works in solving the given mathematical problems in each instructional activity designed based on Realistic Mathematics Education (RME) levels. Design research was chosen to achieve and to contribute in developing a local instruction theory for teaching and learning of integer concept and integer addition. In design research, the Hypothetical Learning Trajectory (HLT) plays important role as a design and research instrument. It was designed in the phase of preliminary design and tested to three students of grade six OASIS International School, Ankara – Turkey. The result of the experiments showed that temperature in the thermometer context could stimulate students' informal knowledge of integer concept. Furthermore, strategies and tools used by the students in comparing and relating two temperatures were gradually be developed into a more formal mathematics. The representation of line inside thermometer which then called the number line could bring the students to the last activity levels, namely rules for adding integer, and became the model for more formal reasoning. Based on these findings, it can be concluded that students' learning integer concept and integer addition developed through RME levels.

Keywords: integer concept, integer addition, Realistic Mathematics Education

Abstrak

Penelitian ini bertujuan untuk menginvestigasi perkembangan siswa dalam mempelajari konsep bilangan bulat dan penjumlahan bilangan bulat. Investigasi ini didasarkan pada analisis karya siswa dalam memecahkan masalah matematika yang diberikan dalam setiap kegiatan pembelajaran yang dirancang berdasarkan tahapan pembelajaran menurut Pendidikan Matematika Realistik (PMR). Desain penelitian dipilih untuk mencapai dan memberikan kontribusi dalam mengembangkan teori instruksi lokal untuk mengajar dan belajar konsep bilangan bulat dan penjumlahan bilangan bulat. Dalam penelitian desain, Hipotetis Belajar lintasan memainkan peranan penting sebagai instrumen desain dan penelitian. Ini dirancang dalam tahap desain awal dan diuji kepada tiga siswa kelas enam OASIS International School, Ankara - Turki. Hasil percobaan menunjukkan bahwa suhu dalam konteks termometer bisa merangsang pengetahuan informal siswa tentang konsep bilangan bulat. Selanjutnya, strategi dan alat yang digunakan oleh siswa dalam membandingkan dan mengkaitkan dua suhu secara bertahap dikembangkan menjadi matematika yang lebih formal. Representasi dari garis dalam termometer yang kemudian disebut garis bilangan bisa membawa siswa ke tingkat aktivitas terakhir, yaitu aturan untuk menjumlahkan bilangan bulat, dan menjadi model untuk lebih penalaran formal. Berdasarkan temuan ini, dapat disimpulkan bahwa pembelajaran siswa mengenai konsep bilangan bulat dan penambahan bilangan bulat berkembang melalui tahapan pembelajaran menurut PMR.

Kata kunci: konsep bilangan bulat, penjumlahan bilangan bulat, Pendidikan Matematika Realistik

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Many studies have been conducted regarding the development of integers. It shows students' struggles with integers, particularly with what it means to have numbers less than zero (Gallardo, 2002). Students often view negative numbers as "absurd" in students' early conception because they have not developed a way to understand number less than zero. In particular, they often question why negating a negative number will result in a positive quantity. Moreover, students mostly have difficulties in conceptualizing numbers less than zero, creating negative numbers as mathematical objects, and formalizing rules for integer arithmetic (Stephan & Akyuz, 2012). To overcome these difficulties, several studies have attempted to determine which real-world context and model will be the most useful for supporting students' construction of integer concept and operations.

In this study, we designed a Hypothetical Learning Trajectory (HLT) served as a research instrument containing the instructional activities. Later, this HLT for learning integer concept and integer addition proposed a potentially viable instructional theory.

In our instructional activities, we built on Realistic Mathematics Education (RME) developed by Freudenthal (Gravemeijer, 1994). This method is based on the idea of mathematics as a human activity and as a constructive activity. Therefore, to facilitate the meaning of negative numbers and the addition operation of it, we used temperature in the thermometer context. Besides, the line inside thermometer was used as a provoking idea to bring the students to the idea of number line model to solve integer addition. As explained by Linchevski & Williams (1999), "situations and models must describe a reality that is meaningful to the student, in which the extended world of negative numbers already exists and the students' activities allow them to discover it".

To summarize, the aim of this study was to investigate students' development of learning integer concept and integer addition. The investigation was based on analyzing students' work in solving the given mathematical problems in each instructional activities designed based on Realistic Mathematics Education (RME) levels of mathematical activity. The model used in this study was the number line as it was engaged in the context of temperature in the thermometer. Thus, the research question was formulated as follows.

"How does students' learning of integer concept and integer addition develop through RME levels of mathematical activity?"

Context for Exploring Integers

Almost every day, students have interactions with negative numbers or experienced phenomena that negative numbers can model, for instance the temperature, altitude (above and below sea level), money, and other phenomena (Van de Walle, 2010). In fact, almost any concept that is quantified and has direction probably has both positive and negative values.

In learning the concept of integers, it is important to start with familiar context so that the students can use their prior knowledge to build the meaning. At first, students are often confused as to which number is bigger or which direction they are moving when they do the operations with integer (Liebeck, 1990). Therefore, having a context as a starting point is particularly important. Besides, it is also important to include visuals with the context to support language development (Swanson, 2010).

There are many real contexts of negative numbers that are linear. In addition, number line provides good tool for learning the operations and relate well to what the students have done with whole numbers and fractions operations (Van de Walle, *op.cit.,* p. 479). In this research, we used thermometer context as a starting point for learning integer concept. Besides, we also connected the line inside thermometer with the idea of number line as a model to support learning integer concept and integer addition. There are also some prior researches in learning integers which used context to bridge them to a more formal mathematics. For instance, a research conducted by Stephan and Akyuz (*op.cit.,* p. 433), they use a context of a net worth statement that lists the assets and debts that a person has. Research result showed that the students could interpret positive and negative signs as having the unary function, negative, and positive, from the context of assets and debts. Furthermore, Stephan and Akyuz also used vertical number line as the students came to the problem of comparing the assets and debts of two people and be asked to find and compare their net worth (all unary functions of negative and positive values).

Another research which used context as a starting point for learning integer is a research conducted by Liebeck (op.cit., p. 222). She developed a game named *Scores and Forfeits* which use black counters ("scores") and red counters ("forfeits") as manipulative. Through this game, the students could perceive the equivalence of different combinations with counters such as three black being equivalent to four black and one red.

As we could learn from prior study explained above, it can be seen that the contexts used in learning integer were supported by model such as vertical number line and counters. Therefore, in the next section, the models for teaching integers and its deficiency of each model according to prior research will be discussed.

Models for Teaching Integers

Modeling in mathematics education has been a central focus for instructional designers. According to some studies, there are two models for integer instruction, namely *counters* and *number lines* (Battista, 1983; Liebeck, 1990; Stephan & Akyuz, 2012).

For counters model, it consists of two color counters, namely one color for positive counts and another for negative counts. For instance, as in Battista (1983) study, he used white chips as the positive counts and red chips as the negative counts. Another, Smith (1995) constructed a computer micro world called *Cy-bee Chips* as the counters. Two counters of each type result in zero (i.e., 1 + -1 = 0). For instance, if we use the context of money, if reds are credits and yellows are debits, 6 reds and 4 yellows is the same as 2 reds or 2 credits and is represented as -2 (see figure 1).

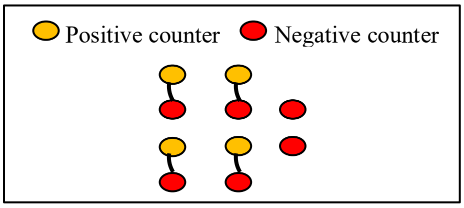

Figure 1. A model of negative 2 by using yellow and red counters

In using this counters model, the students should also understand that it is always possible to add and to remove from a pile any number of pairs consisting positive and negative counters without changing the value of the pile. As described by using the context of money, it is like adding equal quantities of debits and credits.

The number line is the second model for integer instructions. Based on research conducted by Liebeck (1990), number line model have the advantage for modeling integer operations. Because it showed the distance from 0, the students can see that integer moves to the left go to smaller numbers and moves to the right go to larger numbers. Moreover, the number could help the students to explore non-integer negative and positive values (e.g., $-5\frac{1}{2}+4\frac{1}{4}$) that cannot be modeled very well by using counters (Van de Walle, *op.cit.,* p. 481). In this number line, arrows can be used to show distance and direction. For instance, to show 3 can be modeled with an arrow three units long pointing to the right. To show -4 can be modeled with an arrow four units long pointing to the left (see figure 2). The arrows will help the students think of integer quantities as directed distances. Furthermore, each arrow is a quantity with both length (magnitude) and direction (sign).

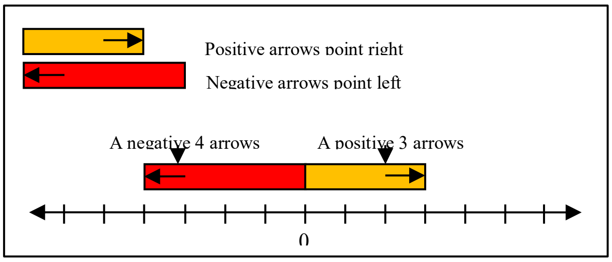

Figure 2. Number line model for integer

Related to the two aforementioned models, very few studies have compared these two models. Liebeck (1990) showed that the students who used counters model performed slightly better on addition problems. However, the students have difficulty in subtraction problems, especially ones that include different signs. She also found that the number line models have the advantage that ordering and symmetric understandings of negative numbers can be supported. Therefore, according to prior research, in this study we used number line as a model to support students' developing of integer concept and integer addition.

Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) is a theory of mathematics education which has been developed in the Netherlands since 1970s. This theory is strongly influenced by Hans Freudenthal's concept of '*mathematics as a human activity*' (Freudenthal, 1991). To shift from the situational activities to the more formal mathematics, the tenets of Realistic Mathematics Educations (RME) is offered clues and design heuristics (Treffers, 1991), namely:

1. The use of contextual problems.

   Contextual problems figure as applications and as starting points from which the intended mathematics can come out. The mathematical activity is not started from a formal level as students usually face with, but from a situation that is experientially real for students. Consequently, this study will use changing temperature in the thermometer context to guide the students to the linear context.

2. The use of models or bridging by vertical instruments.

   Broad attention is paid to the development models, schemas and symbolizations rather than being offered the rule or formal mathematics right away. Students' informal knowledge in observing the temperature in the thermometer needs to be developed into formal knowledge of integers. It would lead the students to the idea of number line when the thermometer is viewed horizontally. Moreover, the number line model would guide the students in finding the solution of integer addition.
3. The use of students' own creations and contributions.

   The biggest contributions to the learning process are coming from student's own constructions which lead them from their own informal to the more standard formal methods. Students' strategies and solutions can be used to develop the next learning process. The use of thermometer serves as the base of the emergence model of number line.
4. The interactive character of the teaching process or interactivity.

   The explicit negotiation, intervention, discussion, cooperation and evaluation among students and teachers are essential elements in a constructive learning process in which the students' informal strategies are used to attain the formal ones. Through discussions about the changing temperature and the difference between two temperatures, students may communicate their works and thoughts in the social interaction emerging in the classroom.
5. The intertwining of various mathematics strands or units.

   Generally, negative numbers are introduced with integers, namely the whole numbers and their negatives or opposites – instead of with fractions or decimals. However, not only know about integer values, students should also understand where numbers such as $-3\frac{1}{4}$ and -6.5 belong in relation with integers. The learning activities of integer negative numbers are intertwined with non-integer negative numbers.

Emergent Modeling

Gravemeijer suggested that instead of trying to help students to make connections with ready-made mathematics, we should try to help students construe mathematics in a more bottom-up manner (Gravemeijer, 1994). This recommendation fits with the idea of emergent modeling. Modeling in this conception is an activity of the students. Gravemeijer elaborated the *model of* and *model for* distinction by identifying four general types of activity, namely:

1. *Situational activity*. The setting in this study was the context of temperature in the thermometer. In this level, through the problem of "*placing the given temperatures on the thermometer*", students would explore their informal knowledge of reading temperature in the thermometer.
2. *Referential activity*. In this study, the activity of "*comparing two temperatures*" serves as referential activity in which the students use the vertical line in the thermometer as a representation of the horizontal number line model. In this activity, the line in the thermometer becomes the *model of*.

3. *General activity*. The *model for* more mathematical reasoning in this present study was the number line. In this activity, number line was introduced as a generalization tool of line in the thermometer. Students were asked to describe the relation between two temperatures which are compared. In addition, it is expected that the activity of "*finding the relation between two temperatures*" will lead students to the idea of integer addition.
4. *Formal mathematical reasoning* which is no longer dependent on the support of *models for* mathematical activity. The focus of the discussion moves to more specific characteristics of models related to the concept of integer addition.

METHOD

The type of research that we used is design research (Gravemeijer & Cobb, 2006). Design research consists of three phases, namely developing a preliminary design, conducting pilot and teaching experiments, and carrying out a retrospective analysis (Gravemeijer, 1994; Bakker, 2004; Gravemeijer & Cobb, 2006). Three students of grade six in OASIS International School, Ankara – Turkey, were involved in this research. The students were about 11 to 12 years old and they have learnt about operations with positive whole number in previous grades. The data collected in this research were interviews with the students, classroom observations including field notes and students' works. After we collected the data, we analyzed these data in the retrospective phase. Finally, we made conclusion based on the retrospective analysis. The instructional activities of learning integer concept and integer addition were as follows.

Table 1. The instructional activities of learning integer concept and integer addition

Learning Goal	Activity	Tools	Conjecture of Students' Strategies	Conjecture of Students' Difficulties
Students will observe the line in the thermometer which shows negative and positive numbers.	Observing positive and negative numbers in the thermometer.	Thermometer	Students will notice the positive and negative numbers directly from the line inside the thermometer.	Students may ask about two different measurers (the F for Fahrenheit and the C for Celsius) if we use such thermometer.
Students will place the given temperature from the problem in the made-thermometer.	Placing the given temperature on the thermometer.	Artificial thermometer made from cardboard.	- Students will put the given temperature in the thermometer. - Students will relate the line inside thermometer with number line.	Students probably will have difficulties in recognizing the small lines without numbers.
Students will compare two temperatures and find the relation between two temperatures. It is expected that they also will relate it	Comparing two temperatures and finding the relation between two temperatures.	Artificial thermometer made from cardboard.	- Students will find the differences between two temperatures by counting the small lines manually. - Students will use addition. For instance when they are asked to compare -5^{0}C with	- Students may do a mistake when placing the temperature on the thermometer so that they will make wrong interpretation. - Students may have difficulties in finding the difference between

with integer addition.			3^0C, they may come to the idea of -5 + ? = 3.	two given temperatures.
Students will investigate the integer addition on the number line.	Investigating integer addition by using number line.	The number line model.	- Students will connect their prior knowledge about the relation of two temperatures. - Students will come to the idea of *positive arrows point right and negative arrows point left.*	Students may confuse how to draw the representation of, for instance, -5 + 3. And, they will come to another difficulty in finding the solution of -5 + 3.
Students will find the solution of integer addition on the number line.	Solving integer addition by using number line.	The number line model.	Students will use number line model together with the arrows to help them to think of integer quantities as directed distance.	Students may confuse that each arrow is a quantity with both length (magnitude or absolute value) and direction (sign). These properties are constant for each arrow regardless of its position on the number line.

RESULTS AND DISCUSSION

Observing Positive and Negative Numbers in the Thermometer

As mentioned in the first tenets of Realistic Mathematics Education (RME), contextual problem figured as starting point from which the intended mathematics could come out. For that reason, the thermometer context was chosen as the context in which the students could connect their real life experience with the idea of integer concept, namely the positive and negative numbers.

In the first activity, pictures of two types of thermometer were shown to the students (figure 3). They were asked whether they familiar with these tools and the use of these tool. From the first figure, it was shown that the thermometer used two types of measurement, namely Celsius and Fahrenheit. Connor said that he often used Fahrenheit in his country instead of using Celsius. Connor is an American student. While, Kirin, a Japanese, and Kelly, an American, said that they often used Celsius. This conversation was in line with what had been predicted in our HLT, namely the students might ask about different measurers shown in the thermometer.

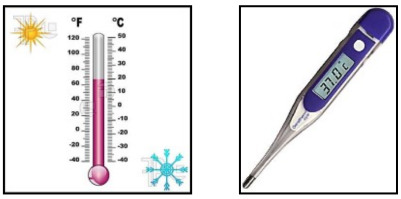

Figure 3. Thermometer used as a context for introducing positive and negative numbers

When we were discussing about the temperature, Kirin came up with the idea of positive and negative number and it influenced her friends, Kelly and Connor. She said, "*Oh, I know, this negative number (pointing out -10 degrees) is below zero and this positive number (pointing out 13 degrees) is above zero*". From this moment, we could see that Kirin noticed the positive and negative number directly from the line inside the thermometer. Besides, Kirin also used 0 (zero) as the base of deciding positive and negative numbers. After asking Kirin to explain more, we found that she used her own experience and prior knowledge about thermometer when she was taking science class in the 3rd grade. It can be seen that the context of thermometer indeed supported students to build the meaning of integer. Besides, the visual of thermometer also supported students' language development of integer. As they later concluded that *positive numbers are above zero* and *negative numbers are below zero*. This concept of integer will be used by the students in the next activity, namely placing the given temperature on the thermometer.

Placing the Given Temperature on the Thermometer

In this activity, the students were asked to place some temperatures, namely 5 degrees, 10 degrees, and -8 degrees on the artificial thermometer made from cardboard. In general, they did not have any difficulties in giving marks of given temperatures. They could recognize the small lines on the thermometer and counted it as one degree. From the conversation with Connor, he noticed that the line inside thermometer could be linked with a number line. Below is the conversation of Connor's idea about number line.

Researcher : *Can you explain how can you find -8 degrees on the thermometer, Connor?*
Connor : *Yes, I just remembered about number line. It just looked like a number line for me.*
Researcher : *Number line? What is it? From where and when did you learn it?*
Connor : *I learn from school. Hmm..but I don't remember in which grade. I think I used it for addition and subtraction of numbers. Hmm..in grade..hmm..I'm sorry, I forget.*
Researcher : *How can you notice that this is number line? Does the representation of number line shown as seen in this thermometer?*
Connor : *Hm..not really, it should be like this* (he showed horizontal line by using his index finger)*, but this is vertical.*
Researcher : *Ok, so, the line is shown vertically instead of horizontal?*
Connor : *Um..ya..*
Researcher : *And how can you decide that this is exactly -8 degrees?* (pointing out -8 degrees mark made by Connor in his artificial cardboard)
Connor : *Well, I know that this thermometer jump 5, and I count these small lines between 0 and 5, and there are 4 small lines. Then I know that this small line equal to 1. So, I just looked at -5, then add 3 more small lines down.*
Researcher : *Ok, Connor. Good job! How about the others?*
Kirin : *I don't have any difficulties. It is easy.*
Kelly : *Yes, me too*

From the conversation above, it can be seen that the students could gain insight about the concept of integer, namely the positive and negative numbers by using the context of thermometer. They could also link the line inside thermometer to the idea of number line which is very useful to guide them to reach the goal of using number line to support their learning of integer addition. Below are students' works of placing the given temperatures on the thermometer activity (figure 4).

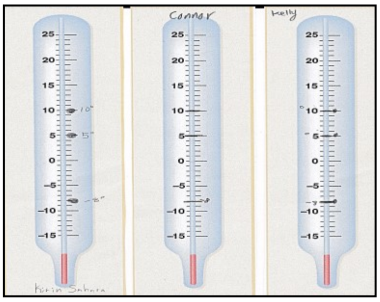

Figure 4. Students' works of placing the given temperatures on the thermometer activity

Comparing Two Temperatures and Finding the Relation between Two Temperatures

In this activity, the students were asked to give marks on the line for the given temperatures and compare them. They were also asked to give argument about the relation between two temperatures. In this moment, this problem could be considered as *referential* when students were initially used the representation of line inside thermometer. As mentioned in the second tenet of Realistic Mathematics Education (RME), namely the use of models or bridging by vertical instruments, the use of line in the thermometer served as the base of the *emergence model* of number line.

From students' work, it is found that Kelly and Connor related two temperatures based on its multiple. For instance, in question number 1, they gave arguments about the relation between 5^0C and 15^0C (see figure 5) that "they are both multiples of 5 or divisible by 5". Moreover, Connor added an argument that they are both odd numbers. Nor for question number two, Kelly and Connor argued the relation between 2^0C and 14^0C was that "they are both multiples of 2 or divisible by 2". Moreover, Connor also gave additional note that 2 and 14 are even numbers.

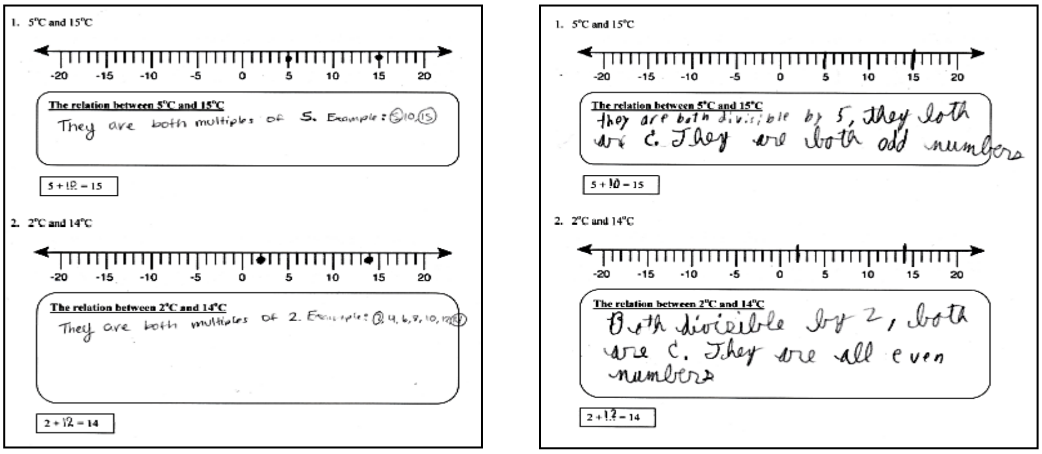

Figure 5. Kelly and Connor's idea about the relation of 5^0C and 15^0C and 2^0C and 14^0C

However, when they came to the third (compare -5^0C and 5^0C) and fourth (compare -8^0C and 12^0C) questions, they could not use the same idea of multiples as they used in the first and second question. Instead of using multiples, they said "they are opposites", for the relation between -5^0C and 5^0C. In this problem, we also asked the students to find -5 + … = 5 and they both could not find the answer.

Surprising idea came from Kirin. For question number 3, she gave argument that "5 goes to the right and -5 goes to the 5. The difference is 10". Nor for question number 4, she said that the difference between -8^{0}C and 12^{0}C is 20. By finding the difference, she then could find the answer of -5 + … = 5 and -8 +… = 12 as shown in the figure 6.

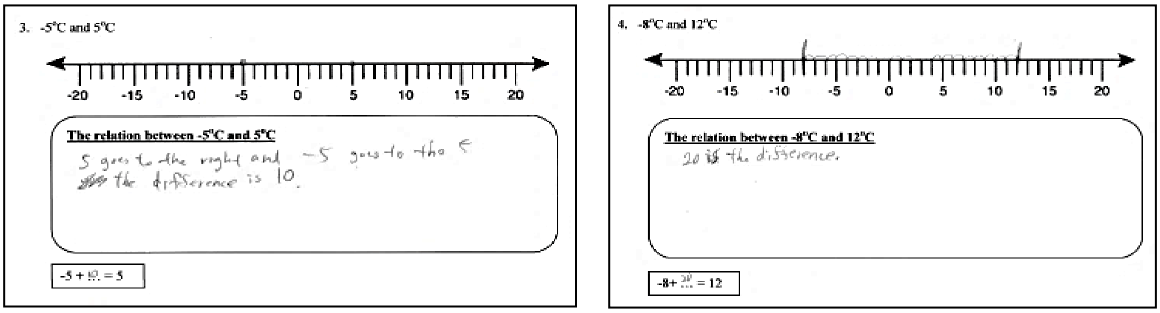

Figure 6. Kirin's idea about the difference between two temperatures

To follow-up Kirin's idea, we asked Kirin how she came up with the idea of difference between two temperatures and related it with integer addition as shown in the following excerpt.

Researcher	:	*Kirin, can you explain how do you come up with the idea of difference?*
Kirin	:	*Well, there is a box ask about negative 8 add etcetera so that the result is 12. I just counted the points.*
Researcher	:	*How?*
Kirin	:	*Like one, two, three, four, …, twenty* (pointing out the point and counting on until the line of 12).
Researcher	:	*Hm, so you count it manually.*
Kirin	:	*Yes.*

From the conversation above, it is observed that Kirin could make relation between two temperatures which include negative and positive numbers by using the idea of differences (figure 7). She also came up with the idea of difference which could be used in the next activity, namely investigating integer addition by number line. Her strategy of counting the small lines manually to find the differences between two temperatures was in line with our conjecture of students' thinking in our HLT.

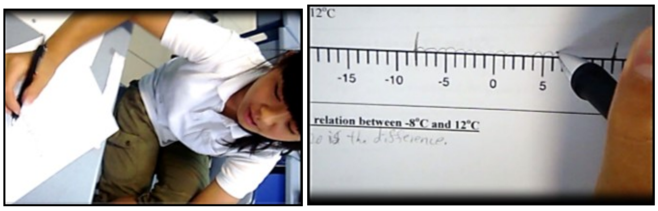

Figure 7. Kirin showed how she counted the lines to find the differences between -8^{0}C and 12^{0}C

Investigating Integer Addition by Using Number Line

By using problem in students' worksheet 1, we then discussed the integer addition. We took one example of 5 + 10 = 15. We asked students to help us making representation of integer addition using the number line. We first provoked them by making an arrow start from zero to 5 to represent 5 in the question. We then asked one of them to represent + 10 by using arrow. Connor drew arrow above the number line start from 5 and stop until 10. Kelly argued Connor's drawing. Kelly then revised Connor's drawing and said, *"to make 5, you jump 5 times to the right, so, adding 10, you need to jump 10 times to the right. So the result is 15"*. Connor's and Kelly's drawings of representation + 10 are shown in figure below 8.

Figure 8. Connor's and Kelly's drawings of representation + 10.

We then continued by giving another example of integer addition which included negative and positive numbers. This moment was critical since it was expected that the students could come up with the idea of the direction of the arrow to represent positive and negative numbers also to represent the operation of addition. We took example of -5 + 10 = 5. As we discussed the problem, Kirin came up with the idea of the direction of the arrow. She argued that "*for positive number, the arrow goes to the right*" and "*for negative number, the arrow goes to the left*". However, the students struggled to decide from where they should start when they add the first number with the second number. For instance, to add -5 with 10, first they made an arrow to the left as much as 5 jumps to represent -5. Then added 10, start from the point in -5, they needed to draw the arrow to the right and jump as much as 10 times to represent adding 10. The result was the difference between the first arrow and the second arrow as shown in the figure 9.

Through discussion, the students may communicate their thoughts to grasp the idea of using number line for solving integer addition. The role of the teacher here only as a guide and let the students to construct their own learning. As mentioned in the fourth tenet of RME, namely the *interactivity*, negotiation, discussion, cooperation and evaluation among students and teachers are essential elements in a constructive learning process. Students' informal strategies, as founded by Kirin about the idea of the direction of the arrow for representing positive and negative numbers, were used to attain the formal ones.

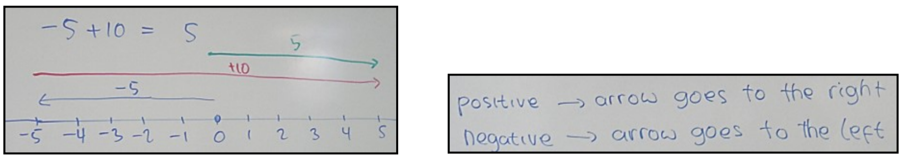

Figure 9. The idea of the direction of the arrow for positive and negative numbers.

The direction of the arrow, the starting point for drawing the arrow, and the difference between two arrows are the crucial part that the students should comprehend through the discussion among their peers and the teacher, when number line model used in the classroom, for learning integer addition. The use of number line here transformed into a *model for* more formal reasoning of mathematics in learning integer addition. Related to the RME's levels of mathematical activity, this stage is on the *general activity* level.

Moreover, at this level, the students were expected to be able to solve problems in a more and more refined manner at the symbolic level in which the number line used as tool to support their development of learning integer addition. As mentioned in the third tenet of RME, the biggest contributions to the learning process were coming from *student's own creations and contributions* which led them from their own informal to the more standard formal methods.

Solving Integer Addition by Using Number Line

As mentioned in RME emergent modeling, the last level, the formal level, focuses on the discussion which moves to a more specific characteristics of models related to the concept of integer addition. Throughout the activity of solving integer addition by using number line, the students reflected on the rule for integer addition as it had been decided together through discussion from previous activity. The transition to a more formal integer addition was preceded by stimulating students to contribute their own informal ways of working which led by Kirin's idea about the direction of the arrow for representing positive and negative numbers and it was used to solve integer addition. In solving problems in the worksheet 2 and 3, Kirin and Connor seemed like did not have any difficulties in solving integer addition by using number line model. However, Kelly struggled in deciding the jumps and the arrow direction for representing positive and negative numbers including the addition operation.

For instance, to solve problem -3 + 5, she first drew an arrow to the left as much as 3 jumps to represent -3. But then, start from the point of -3, she drew arrow to the right until point 5 to represent + 5. She used the same strategy for all question in worksheet 2. However, opposite to her drawing, her answer was correct. She could answer -3 + 5 = 2. But it was not in line with her drawing on the number line. When we tried to ask her, she said "*I use my own strategy. I try to link it with the idea of temperature. I have -3 degrees and then the temperature increases 5 degrees. Now, the temperature is 2 degrees.*" From her explanation, it can be observed that number line had not support her understanding in solving integer addition. She used the context of thermometer instead of using number line and showing the process by using arrow to find the result. This moment can be the limitation of the use of number line as model for solving integer addition. It may confuse the student that each arrow is a quantity with both length (magnitude) and direction (sign). We have already conjectured this student's difficulty in our designed HLT. By guiding her also receiving helped from Kirin, Kelly could understand how far the length of the second arrow that she had to draw as explained in the following conversation.

Researcher	:	*Well Kelly, we start from -3, 0 to -3, you jump 3 times to the left. And then you start from here* (pointing out -3)*, how many jumps adding 5? To the right or to the left?*
Kelly	:	*Umm..to the right.*
Researcher	:	*And, how many jumps? Can you show me?*
Kelly	:	*Mm..we start from here, one, two, mm..two jumps, but how is it negative?* (she looked confuse and suddenly Kirin wanted to help her)
Kirin	:	*Can I help her?*
Researcher	:	*Yes, of course you can help her.*
Kirin	:	*You were too much.* (Kirin gave explanation to Kelly). *It until 2 because you add 5* (pointing out the second arrow). Kelly was erasing her drawing of second arrow and correcting her drawing.
Researcher	:	*Kirin, how do you help her?*

Kirin : *I just said that she drew that line, I think too long.*
Researcher : *Why is it too long?*
Kirin : *Because she added 5, but she did the line until 5 where has to be the second place* (pointing out 2 on the number line) *since she started from negative 3* (pointing out -3 on the number line).
Researcher : *So, how long she draws the line for here (pointing out -3) to here* (pointing out 5).
Kirin : *She jumps 8.*
Researcher : *Hm, do you know why she draws an arrow start from -3 to 5?*
Kirin : *Because she added 5, so she draws the arrow until 5, I think she thought like that.*
Kelly : *Is it true Kelly? Because you think you add 5, so you draw the arrow from -3 to 5?*
Kelly : *Um..yes, at my first thought. But now I understand.*
Researcher : *Now you understand, very nice. Thank you for your help Kirin!*

From the discussion above, it can be seen that by getting explanation from her peer, Kelly could manage her confusion in drawing the arrow to represent the addition. Kelly's work is shown in figure 10.

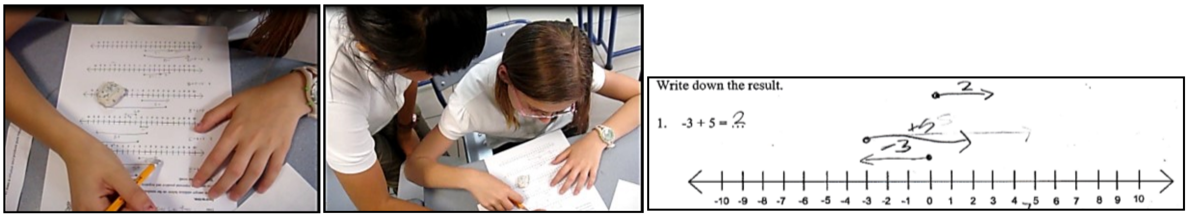

Figure 10. Kelly's work of problem -3 + 5.

CONCLUSION

To answer the research question, this study has shown students' development of learning integer concept and integer addition through RME levels of mathematical activity. The levels are situational activity (i.e., the use of thermometer context to come up with the idea of integer concept), referential activity (i.e., the use of the line inside thermometer which served as *model of* comparing two temperatures, relating two temperatures, and leading the students to the idea of integer addition), general activity (i.e., the use of number line model to investigate integer addition), and formal mathematical reasoning (i.e., the use of number line to solve integer addition). In this study, some ideas and concepts from RME theory has underpinned the designed instructional activities. The context used was temperature in the thermometer and we found that this is a good context that has allowed students to structure and to mathematize following the RME levels of mathematical activity.

In the first level, students started to observe the line in the thermometer which shows negative and positive numbers. Related to the first tenet of RME, namely *the use of contextual problems*, thermometer context served as the source for the mathematics to be produced. In the second level, the students were comparing and finding the relation between two temperatures. Problem situation in which the students noticed the differences between two temperatures on the line had a strongly generative nature. It led them to the idea of integer addition as they were also asked to solve the problem, for instance, -8 + … = 12. Connected to the second tenet of RME, namely *the use of models*, the use of line inside thermometer here was as a bridge to the number line model which was in more abstract level. At this level, students could construct and produce their own mathematical knowledge. In the third level, the students were investigating integer addition by using number line model. This level was a crucial moment for student to comprehend the strategy for solving integer addition by using number line. The direction of the arrow, the starting point for drawing the arrow, and the difference between two arrows were the crucial part that the students should comprehend through the discussion among their peers and the teacher. In the last level, namely formal mathematical reasoning, it gave opportunity to the students to use their strategy for solving integer addition as it summed up together through discussion with researcher and peers. Connected to the third tenet of RME, namely *the use of students' own creations and contributions*, the representation of line inside thermometer as *model of* transformed into the number line as a *model for* more formal reasoning. This transformation is another important learning moment for students where they can use the model to move from concrete context to a more formal mathematics.

In this study, we also included discussion in which the students could share their thinking and strategies. Connected to the fourth tenet of RME, namely *interactivity,* the classroom settings was elaborated where the students could share their thinking in the class discussion. At this moment, teacher plays an important role in orchestrating the flow of the discussion. Although the study reported here is a relatively small scale study and only worked with three students, not all findings of which can be generalized, this study has some implications for educational practice. Realistic Mathematics Education (RME) can be used as an approach to teach mathematics, or in specific, on the topic of learning integer concept and integer addition. Considering the last tenet in RME, namely *intertwinement*, some activities used in this research could be developed to reach other mathematical topics by intertwining it with other mathematics topics. We indicated the intertwined of integer negative numbers with non-integer negative numbers such as $-3\frac{1}{4}$ and -6.5. There are also, of course, even more intersections with all kinds of other lines of learning, such as those for integer subtraction, multiplication, and division. Therefore, it is recommended that these topics be a focus for further studies.

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