



## **CULTIVATING UPPER SECONDARY STUDENTS' MATHEMATICAL REASONING-ABILITY AND ATTITUDE TOWARDS MATHEMATICS THROUGH PROBLEM-BASED LEARNING**

E. Elvis Napitupulu<sup>1</sup>, Didi Suryadi<sup>2</sup>, Yaya S. Kusumah<sup>2</sup>

<sup>1</sup> State University of Medan (Unimed), Jl. Pasar V Medan Estate, Medan

<sup>2</sup> Indonesia University of Education (UPI), Jl. Dr. Setiabudi No. 229 Bandung  
E-mail: elvisnapit@gmail.com

### **Abstract**

This paper reports some results of a research attempted to cultivate students' mathematical reasoning ability (MRA) by means of implementing problem-based learning (PBL) model. The population was upper secondary students of public schools in Bandung. Cluster sampling technique applied to take sample of amount 158 students from middle-level schools. Data were teacher's assessment on students' performance in last two months, prior mathematical knowledge (PMK) and MRA test, observations, and interview. Two-factors ANOVA then used to analyze the data. Findings showed that overall students in PBL-classroom achieved better MRA than their counterparts in conventional one though the category in both classrooms was low. Interaction between PBL and PMK factor towards MRA did not emerge. Most part of the participants viewed PBL neither positive nor negative. On the contrary, they assessed that the tasks given made and demanded them thinking and learning more actively than ever. The findings showed adopting PBL in schools to facilitate students getting their higher order thinking skills is a reasonable choice, especially schools having middle or high ability students.

**Keywords:** problem-based learning (PBL), mathematical reasoning ability

### **Abstrak**

Artikel ini melaporkan hasil penelitian yang bertujuan untuk menyelidiki efek pembelajaran berbasis masalah (PBL) terhadap kemampuan penalaran matematis (MRA) siswa SMA. Populasi penelitian adalah seluruh sekolah menengah atas negeri di Kota Bandung. Sampel dicuplik dengan teknik sampling kluster. Sebanyak 158 orang siswa dari sekolah kluster menengah terlibat dalam penelitian. Data diperoleh dari penilaian kinerja siswa dua bulan terakhir sebelum pelaksanaan penelitian, uji pengetahuan awal matematik (PMK), uji kemampuan penalaran matematik (RMA), pengamatan dan wawancara. Data dianalisis menggunakan Anova dua jalur. Hasil penelitian mengungkap secara keseluruhan RMA siswa di kelas PBL unggul dibandingkan rekan mereka di kelas biasa meskipun capaian itu di kedua kelompok terbilang rendah. Tidak ada interaksi antara PMK dengan PBL terhadap MRA siswa. Sebagian besar siswa tidak memandang PBL positif dan tidak pula negatif. Hanya saja, mereka menilai tugas-tugas yang diberikan selama proses pembelajaran berlangsung membuat dan menuntut mereka berpikir dan belajar lebih aktif dibanding sebelumnya. Temuan tersebut menunjukkan penerapan PBL di kelas untuk memfasilitasi siswa meraih kemampuan berpikir tingkat tinggi menjadi pilihan beralasan, khususnya di sekolah kluster menengah dan atas.

**Kata kunci:** pembelajaran berbasis masalah, kemampuan penalaran matematis

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Many researches (Kilpatrick, Swafford, and Findell, 2001; Schönfeld, 1992; Sumarmo, 2005), Ministry of Education of Indonesia (Depdiknas, 2006), and Ministry of Education of Singapore (MoE, 2006) mentioned the importance of two main goals of mathematics education. First, short-term goals, is about coping contents, skill, and process of mathematics and to solve problems emerged either in mathematics itself or in other disciplines. Second, long-term goals, is about inculcating and catering higher order thinking ability such as analytic, critic, and creative, generating mathematical disposition,

and cultivating social skills. Those goals should be achieved to ensure students be able to develop their potentialities optimally to become future productive citizens.

Elsewhere, National Council of Teachers of Mathematics [NCTM] (1989, 2000); MoE of Indonesia (2006); MoE of Singapore (2006) emphasized the importance of mathematical reasoning and problem solving as part of doing mathematics. It is likely that problem solving became focus of school mathematics. Moreover, National Council of Supervisor of Mathematics [NCSM] (Wilson, *et al.*, 1997) stated that learning to solve problems is the main reason why students should learn mathematics in school. The NCSM's view looks reasonable since by solving problems, the students have much chance experiencing and involving in the process of constructing meaningful knowledge and skills, which in its turn applying and transferring to solve new and more complex problems in mathematics or beyond.

If problem solving takes role as the heart of mathematics, mathematical reasoning takes the same role in problem solving. Children who are engaging in problem solving activities automatically are building their reasoning skills. Since in solving problems, the main tool one uses is his reasoning. His reasoning works when trying to understand problem, making connections and representations between concepts in the problem to his previous knowledge, making conjectures and generalization, and trying to prove conjectures he made. Reasoning and making sense are two interdependent mathematical activities on which other mathematical processes based. By experiencing all the processes, it expects that self-confidence and positive attitude towards mathematics emerge in the part of children.

Though the ultimate role that reasoning plays in learning mathematics and solving mathematical problems, facts revealed that most of lower and upper secondary students lack of this skill (Hamzah, 2003; Herman, 2006; Martin, *et al.*, 2008; Sumarmo, 1987; Suryadi, 2005; Zulkardi, 2001). Particularly in upper secondary school, Sumarmo found that 55% of them were not able to think deductively. Our Initial investigation at tenth grade in one of secondary public school in Bandung Barat showed that in general the students performed poorly on mathematical reasoning. They lacked conceptual understanding, which implied lack of ability to grasp problem proposed completely. In addition, they failed to connect their previous knowledge and their experiences working on it to the context of the problem. They also failed interpreting words-problems into mathematical representation or model, looked difficult solving problem with multiple-steps solution, and lacked of reflection while solving problem (Authors, 2011).

Concerning to the poor students' performance particularly in high-order thinking, researchers presumed the discourse developed by teachers in classroom while learning and teaching being conducted to be one of the responsible. Most teachers are accustomed to using traditional instruction for all of their time with the students. Teacher-centered learning still dominates almost all of mathematics classrooms, and emphasis is heavily laid on grasping basic skills without any stressing on applying mathematics in daily life, communicate and reasoning mathematically (Hulukati, 2005; Minarni, Napitupulu, and Husein, 2016; Pomalato, 2005; Shadiq, 2007; Wahyudin, 1999).

To help students meet the standards in mathematical reasoning (Kilpatrick, Swafford, and Findel, 2001; NCTM, 2000), many researchers have applied varies approaches or models of teaching.

Kramarski and Mevarech (2003) applied cooperative learning and metacognitive training. The two writers investigated students' performance on three measures, i.e. graph interpretation, graph construction, and metacognitive questionnaire. Dahlan (2004) applied open-ended to enhance students' reasoning. Suryadi (2005) used indirect and combination of direct and indirect instruction to enhance students' higher order thinking ability. All of the studies took participants from lower secondary school and harvested positive results.

Among others, Problem-Based learning (PBL) is an instructional approach, which use problem to trigger learning. Students are in-group to work collaboratively to search resolution of the problem. Teacher plays his role to facilitate learning with scaffolding technique by giving indirect hints or posing stimulated questions to help students make use their reasoning and experiences to search for possibilities ways to get intermediate or even final solution. Particularly for upper secondary students, theoretically speaking they have been in formal deductive period. It is time for them to grasp varieties of reasoning skills to solve problems either in mathematics or in other disciplines even in everyday life. PBL with all of its characteristics seems fit to help and facilitate students achieve the cognitive and affective target mentioned earlier. Therefore, it is particularly important to examine the effect of PBL on mathematical reasoning ability of upper secondary students.

The present study addresses to seek the effect of PBL on upper secondary students' mathematical reasoning ability either in overall or in accordance to their mathematical prior ability (high, middle, and low). Theoretically, this study contributed and enriched the domain of mathematics education especially to what extent PBL holds and powerful to help students develop their reasoning ability. Practically, the study proposed a constructivism-based instruction model in the framework of constructing habit and skills in higher-order thinking.

Artzt dan Yaloz-Femia (1999) formulate reasoning as part of thinking constitutes generalize and draw valid conclusion on ideas and how the ideas intercorrelated. *Curriculum and Evaluation Standards* (NCTM, 1989) describe important components of reasoning process which should be parts of measuring mathematical reasoning ability. Their components are using inductive reasoning for recognizing patterns and constructing conjecture, developing various mathematics arguments, using spatial and comparison reasoning to solve problems, using deductive reasoning to verify conclusion, justify argument validity, and to construct valid argument, and analyzing situation to determine properties and general structure.

## **METHOD**

### ***Research Design***

This experiment used group-static comparison design. Besides applying PBL, the study also involved prior mathematical knowledge (PMK) factor. Combining with teachers' assessment within two last months on students' achievement (MPA), students' PMK then categorized as high, middle, and low. The students took MPK test before the instruction began. It also played role to examine the

homogeneity of all of the classrooms. At the end of instruction, all of the participants engaged in mathematical reasoning test.

### ***Research Participants***

The participants were 158 students (79 for each group) embedded in natural sciences eleventh-grade four classrooms from middle level public school. PBL was conducted in two of the four classrooms and conventional instruction in the other twoes. Within schools, classes were normally distributed in terms of mathematical prior ability and prior knowledge. First author participated as teacher-facilitator in all of the classrooms.

All classes studied trigonometry and circle. These units were selected since they fit to the purposes of using PBL as an instructional approach. It is perceived that trigonometric materials in the natural sciences eleventh-grade consist of formula to be derived and many trigonometric identities to be proven and applied. Similarly, we can find many tasks in the topic of circle which classified as high order thinking demanding.

### ***Instrumentation***

Two kinds of instruments were used to gather data, i.e. quantitative and qualitative. MRA-test, MPK-test, MPA document, and Attitude toward Mathematics (AM) scale were quantitative. Observation on teaching and learning processes and the students' performance on the tests were sources of qualitative data. The MRA-test was validated in terms of construct and content by five collegial experts before being used. The MPK-test, adopted from National Examination (UN), consisted of 20 items with materials from tenth-grade and has duration sixty minutes to work with.

The AM-scale contained 18 items, which is valid, selected from 23 items with each has five choices. The items were derived from three components: self-potentiality to learn and success in mathematics, values, and mathematics teacher. The reliability of the AM-scale was categorized high and moderate for the MRA-test. Four of the six items of the MRA-test was classified difficult and the other two was moderate. We used four measures to assess students' mathematical reasoning ability i.e. 1. Draw logical conclusion; 2. Give explanation on model, fact, properties, relationship, or pattern exists; 3. Make conjecture and proof; 4. Use of relationship pattern to analyze situation, or to make analogy, or to generalize.

### ***Data Analyzed Technique***

Cochran-Q statistics was used to test the homogeneity of the result of the validators' validation on the MRA-test. Other tests used were t-test, one-way and two-way ANOVA. All of statistics test done under significance level  $\alpha = 0.05$ . Software used to run all of the tests were SPSS version 17 and Microsoft-Excel. Criteria for attributing students' performance on MRA and AM listed in Table 1.

**Table 1.** Criteria for MRA and AM Performance

MRA-Mean ( $\bar{x}$ )	$\bar{x} < 12$	$12 \leq \bar{x} < 15.6$	$15.6 \leq \bar{x}$	Maximal score: 24
Standard Score ( $T$ )	$T < 45$	$45 \leq \bar{T} < 55$	$55 \leq T$	Maximal score: 90
Criteria	Low	Moderate	High	

## RESULTS AND DISCUSSION

### *Mathematical Reasoning Ability (MRA)*

Table 2 revealed general information on students' MRA in accordance with factors involved. Either overall or by MPA categories, PBL group got better mark than their counterparts linearly. Using of K-S Z test and Levene-test showed that samples were drawn from population that normally-distributed and have homogenous variance.

**Table 2.** Mean Score of MRA

MPA Category	PBL			Conventional			Total		
	N	Mean	SD	N	Mean	SD	N	Mean	SD
High	21	<b>13.81</b>	3.75	20	<b>12.65</b>	3.22	41	<b>13.24</b>	3.51
Middle	39	<b>8.64</b>	3.54	41	<b>6.51</b>	3.36	80	<b>7.55</b>	3.59
Low	19	<b>6.37</b>	2.83	18	<b>5.61</b>	3.62	37	<b>6.00</b>	3.22
Total	79	<b>9.47</b>	4.40	79	<b>7.86</b>	4.38	158	<b>8.66</b>	4.45

Note: SD stands for standard deviation; maximum score =24

One-way ANOVA test (Table 3) and t-test (Table 4) consecutively showed that PBL and MPA of middle category group outperformed their counterparts on MRA.

**Table 3.** Test of MRA Mean Difference based on Instruction

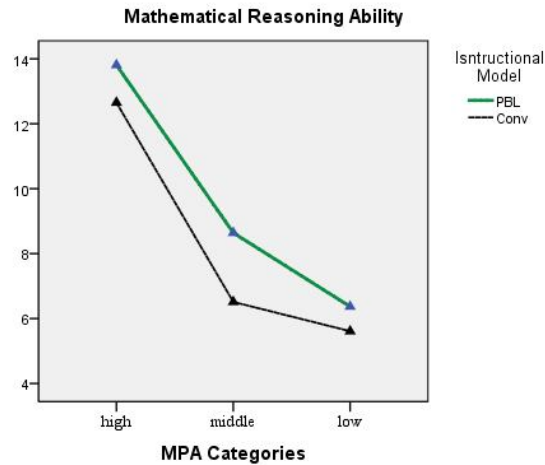
Source of Variance	Sum of Squares	df	Mean Square	F	Sig	$H_0$
Between Group	102.082	1	102.082	5.303	0.023	Rejected
Within Group	3003.139	156	19.251			

From Table 3 we got the significance value 0.023 which clearly less than 0.05. Similarly, from Table 4 we got the significance value 0.007 for MPA middle category which again is less than 0.05.

**Table 4.** Test of MRA Mean Difference for Each MPA Categories

MPA Categories	Test of Variance Homogeneity		Mean Difference Test		
	F	Sig	t	dk	Sig
High	0.211	0.649	1.060	39	0.296
Middle	0.169	0.682	2.757	78	0.007
Low	1.722	0.198	0.711	35	0.482

Moreover, two-way ANOVA test revealed the non-existence of interaction between instructional with MPA factor towards MRA. This means we did not find union influence of PBL and MPA on MRA. The graphic of the non-existence of interaction between factors was shown in Figure 1.



**Figure 1.** Interaction between Instruction and MPA on MRA

Table 5 showed students’ lowest achievement occurred on the second indicator, meanwhile they performed best on the third either in PBL or in conventional group.

**Table 5.** Students’ Mean Score of Each of MRA Aspect Based on Teaching Approach

Aspect to measure	Prob. number	Teaching Approach			
		PBL	%	Conv	%
1. Draw logic conclusion	5	1.54	38.50	1.59	39.75
2. Give explanation on model, fact, properties, relationship, or pattern exists	2	0.60	15.00	0.38	9.5
3. Make conjecture and proof	3 and 4	2.27	56.75	2.04	51.00
4. Use of relationship pattern to analyze situation, or to make analogy, or to generalize	1 and 6	1.44	36	0.91	22.75

Note: Maximal score of each aspect is 4; Conv stands for Conventional

**Attitude towards Mathematics (AM)**

1. AM Comparison Based on Instruction (PBL and Conventional)

Table 6 illustrated the AM-mean of the two groups. From the values in the table, practically we did not see their significant difference.

**Table 6.** AM Description of the Two Groups

Statistics	PBL	Conventional
N	79	79
Mean	65.97	66.73
Standard Deviation	6.92	6.86

Test on data of AM showed that samples were drawn from population that normally-distributed and have homogenous variance. By one-way ANOVA test as shown in Table 7, we concluded that no difference on AM existed between the two groups. Similarly, from Table 8, the significance value for each MPA category is greater than 0.05. This means that no difference on AM existed between each MPA category of the two groups.

**Table 7.** Test of AM Mean Difference based on Instruction

Source of Variance	Sum of Squares	df	Mean Square	F	Sig.	H <sub>0</sub>
Between Group	22.785	1	22.785	0.480	0.490	Accepted
Within Group	7407367	156	47.483			

**Table 8.** Test of AM Mean Difference for Each MPA Category

MPA Category	Instruction	N	Mean	Mean Difference	T	df	Sig.	H <sub>0</sub>
High	PBL	21	69.62	1.07	0.521	39	0.605	Accepted
	Conventional	20	68.55					
Middle	PBL	39	65.28	0.33	-0.217	78	0.829	Accepted
	Conventional	41	65.61					
Low	PBL	19	63.37	3.91	-1.751	35	0.089	Accepted
	Conventional	18	67.28					

2. AM Comparison Before and After Instruction

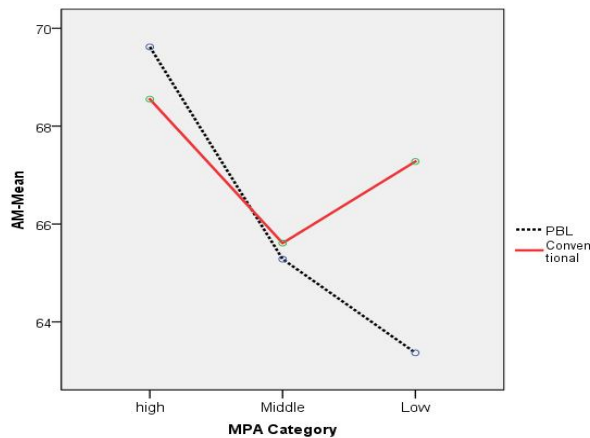
From Table 9, it can be seen that the mean of AM after instruction held is slightly decrease. By using t-test as shown in Table 10, we concluded that students' AM before and after instruction remained unchanged.

**Table 9.** AM Description Before and After Instruction

Statistics	PBL	
	Pre	Post
N	79	79
Mean	66.81	65.97
Standard Deviation	7.17	6.92

**Table 10.** Test of AM Mean Difference Before and After Instruction

T	df	Mean Difference	Sig.	H <sub>0</sub>
0.745	156	0.835	0.457	Accepted



**Figure 2.** Interaction between Instruction and MPA on AM

As Figure 1 did, Figure 2 showed the non-existence of interaction between instruction and MPA on AM. However, it is interesting to note that the MPA low category in conventional group had higher score on AM than their counterpart, even than the MPA middle category, in PBL.

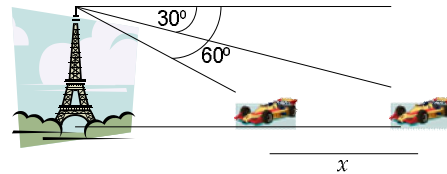
Table 2 showed MRA-mean score 9.47 for PBL and 8.66 for conventional group. This achievement categorized low (Table 1). Students with high MPA in both groups have moderate ability in MRA whereas the others categorized low. This result did not surprise us for the students' MPA criteria are moderate or low while their MPK is low. All of the test-item was difficult for the students.

Of the four indicators used to explore the students' MRA, they lack most on the ability to give explanation on model, fact, property, relationship, or pattern which exist. Problem number 2 measured this indicator as shown below.

### Problem number 2

Observe the picture aside.

The height of the tower is 100 m. Calculate the distance the car moved on toward the tower's feet ( $x$ ) if the angle of depression changed from  $30^\circ$  to  $60^\circ$ .



$$\frac{a}{\sin A} = \frac{x}{\sin B}$$

$$\frac{100}{\sin 30^\circ} = \frac{x}{\sin 60^\circ}$$

$$\frac{100}{\frac{1}{2}} = \frac{x}{\frac{1}{2}\sqrt{3}}$$

$$200 = \frac{x}{\frac{1}{2}\sqrt{3}}$$

When interviewed, the student told he directly related the situation illustrated in the picture to Sinus formula without either rethinking on what he thought and concluded or the accordance of using it in this situation. Though he perceive Sinus concern with the right side and hypotenuse of a right triangle, he continued working to find the value of  $x$ .

**Figure 3.** Sample of student's work

It seems the students lacked the ability on seeing relationship between facts exist and related it to relevant trigonometric knowledge in order to translate the problem representation into trigonometric representation and then into mathematics equation. This situation demands students to create some mathematics equations and it was very difficult for them to fulfill. However, in the teaching and learning process, they had done some problems alike. It indicated that the tasks they did had not traced heavily in their cognitive structure such that they were not fluent when asked to apply and transfer the knowledge they have to a new mathematics situation.

Interviews and their performances on working sheets revealed mistakes and difficulties committed were: 1. Aware of task demand, but have misinterpretation or drew illogical conclusion; 2. Lack of metacognitive process; 3. Unable to build meaningful relationship between available facts towards goals; 4. Unable to construct data-based or pattern-based conjecture or unable to justify on conjecture made; and 5. Misunderstanding on deductive and inductive thinking.

In general, similar findings were reported by de Castro (2004) and Harel and Sowder (2007) (Yefdokimov, 2009). De Castro reported the level of MRA of the participants qualified low (73%),



moderate (27%), and high (0%). Yevdokimov predicted students faced difficulty to construct proof caused of lack of understanding on what materials should be applied. Moreover, students failed to give justification on the construction of proof caused of limited understanding on the relationship between objects involved. Partially, the finding no. 5 in the last paragraph is in line with Sumarmo's (1987) and Williams's (Bergeson, 2000) one.

In overall, the AM of both groups classified moderate (neither positive nor negative). The resistance of PBL in the experiment classroom presumed to be the main reason why the difference did not exist either for both groups or within PBL itself. The PBL students were accustomed to learning in conventional environment for long time. They even rarely learned in group or cooperatively. The non-existence of AM difference between PBL and conventional group in this study opposed to Gani's (2007), Juandi's (2006), and Saragih's (2007) findings.

## CONCLUSION

To make thinking and reasoning to be one of the corner-stone on the learning process and activities in school, it needs the commitment of all of mathematics teachers to focus the students' attention and prior knowledge to essential aspects for supporting depth and meaningful learning, enhance students' thinking ability through pose question technique, apply knowledge learned to real situation, take benefit of students' learning experiences, engage students more in the process of teaching and learning, and cultivate students' conceptual understanding for supporting meaningful learning without laying on rote learning only

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