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EXAMINING OF MODEL ELICITING ACTIVITIES DEVELOPED BY MATHEMATICS STUDENT TEACHERS

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Abstract

The purpose of this study is to examine the model eliciting activities developed by the mathematics student teachers in the context of the principles of the model eliciting activities. The participants of the study conducted as a case study design were twenty one mathematics student teachers working on seven groups. The data collection tools were the developed model eliciting activities and semi-structured interviews. The model eliciting activities and the transcriptions of the interviews were deeply analyzed based on the principles. The results showed that while one group's activity was not a model eliciting activity, the ones of two groups were appropriate to the all principles. Other model eliciting activities were completely or partly appropriate to the principles. It was seen that the reality and model construction principles were binding role in developing these activities. The self-assessment and construct documentation principles were directly related to each other. The construct share ability and reusability, and effective prototype principles were the principles which were associated with the others and could be elicited effectively by tracking future implementations.

Keywords: Mathematical Modeling, Model Eliciting Activities, Model Eliciting Activities Principles, Mathematics Student Teachers

Abstrak

Tujuan dari penelitian ini adalah untuk menguji model memunculkan kegiatan yang dikembangkan oleh guru mahasiswa matematika dalam konteks prinsip-prinsip model memunculkan kegiatan. Para peserta studi yang dilakukan sebagai desain studi kasus dua puluh guru satu siswa matematika bekerja pada tujuh kelompok. Alat pengumpulan data yang model maju memunculkan kegiatan dan wawancara semi-terstruktur. Model memunculkan kegiatan dan transkripsi dari wawancara mendalam dianalisis berdasarkan prinsip-prinsip. Hasil penelitian menunjukkan bahwa sementara aktivitas satu kelompok itu bukan kegiatan model yang memunculkan, yang dari dua kelompok yang sesuai dengan semua prinsip. Kegiatan Model memunculkan lainnya benar-benar atau sebagian sesuai dengan prinsip-prinsip. Hal itu terlihat bahwa realitas dan konstruksi model prinsip-prinsip yang mengikat peran dalam mengembangkan kegiatan ini. Penilaian diri dan membangun prinsip-prinsip dokumentasi yang langsung berhubungan satu sama lain. Konstruk Mudah dibagikan dan usabilitas, dan prinsip-prinsip prototipe yang efektif adalah prinsip-prinsip yang berhubungan dengan orang lain dan bisa menimbulkan efektif dengan melacak implementasi masa depan.

Kata kunci: Pemodelan Matematika, Model yang Memunculkan Kegiatan, Model yang Memunculkan Prinsip Kegiatan, Guru Matematika

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Today, emphasis is placed on the development of individuals who can use mathematics in real life, and these skills are measured by international comparative exams such as PISA and TIMSS. In addition, students working only on traditional problems can not exactly relate mathematics with the real life. Mathematical modelling serves to this relation by recognizing the opportunity to solve real life problems with mathematical methods. Therefore, mathematical modeling is seen as an important concept for PISA students (Edo, Hartono & Putri, 2013). The problems which include a series of modeling cycles in which present ways of thinking are cyclically revealed, tested and renewed should be introduced to students (Lesh, 2005; Prahmana & Suwasti, 2014). Teachers should carefully choose the modelling activities including the series of modelling cycles. It is important for teachers to use “Model Eliciting Activity (MEA)” which is one way of implementation of modelling. Thus the real life situations are made sense of, and teachers help their students to explore, extend and correct their mathematical thinking (Kaiser & Sriraman, 2006). The MEA is different from traditional word problems because it requires solvers to evaluate the validity of solutions, and to improve, share and if necessary revise their solutions (Lesh & Harel, 2003). These problems are open ended so that students suggest various solutions to these problems (Chamberlin & Moon, 2008; Lesh & Harel, 2003; Lesh, Hoover, Hole, Kelly & Post, 2000; Lesh & Zawojewski, 2007; Mousoulides, 2007). The MEA is also the simulations of real life experiences aiming at revealing the thinking and understanding of the students or the teachers (Lesh & Harel, 2003; Mousoulides, Christou & Sriraman, 2008) and also named as “thought revealing activities” (Lesh et al., 2000). The MEA reveals the students’ thoughts about a real life situation to be modelled and helps them to understand the existence of more than one ideal solution appropriate to the situation (Mousoulides, Christou & Sriraman, 2008).

Lesh, et al. (2000) realized multi-tiered teaching experiments by working together with students, teachers and researchers and as a result of them, the six principles emerged to develop the MEA (Chamberlin & Moon, 2005). The principles are reality, model construction, self-assessment, construct documentation, construct share ability and reusability, and effective prototype principles (Lesh, et al., 2000). The reality principle advocates the situations that students may come across in their real lives (Chamberlin & Moon, 2005) and helps students understand the abstract mathematical concepts more easily and increase their motivation (Chamberlin, 2002). The model construction principle states the necessity of developing models for solving the problem successfully (Chamberlin & Moon, 2005). The MEA should make students, givens, goals and possible solution methods (Lesh, et al., 2000). The MEA should make students need developing models for analyzing the real life situations, focus on goals, relations, actions and patterns, identify strengths and weaknesses of the alternative ways of thinking and eliminate the inappropriate and less functional models (Lesh & Caylor, 2007; Lesh, et al., 2000). The self-assessment principle is the students’ ability to find solutions without getting help or approval from their teachers, to assess the usefulness of the solutions (Chamberlin & Moon, 2005; Lesh & Caylor, 2007; Lesh, et al., 2000). The construct documentation, which is considered to be the reason why the MEA is named as “Thought Revealing Activity” (Lesh, et al., 2000), require students to reveal their own thoughts and to document their thinking process (Chamberlin & Moon, 2005) and this principle is related to self-assessment (Chamberlin & Moon, 2005). The construct share ability and reusability principle is about whether the solutions are usable in a similar situation; in other words whether the developed model can be generalized to other situations and so it can be understood whether the solution is successful or not (Chamberlin & Moon, 2005).

The effective prototype principle deals with whether the solution provides a useful prototype or metaphor to interpret other situations and whether the students remind former problem situations when they come across with a similar problem structure (Lesh, et al., 2000). If students remember the solution method of MEA even after a few months or years, it means this MEA fulfils this principle (Lesh & Caylor, 2007).

The six principles should be considered during developing and applying the MEA (Bukova Güzel, Tekin Dede, Hıdıroğlu, Kula Ünver & Özaltun Çelik, 2016). To benefit from the MEA, the mathematics teachers or the student teachers should develop the MEAs that are appropriate to their students by considering these six principles. There appears in the international field a study (Yu & Chang, 2011) including original MEAs developed by the participants and the researchers examined the appropriateness of these MEAs in the context of the principles. In Turkey, because the modeling implementations are not very common, the selection or creation of modelling activities is especially difficult. Existing MEAs can be reviewed and used (Eraslan & Kant, 2015; Şahin & Eraslan, 2016; 2017) however teachers may not use them because there are cultural differences, the use of technology is not widespread in our schools yet, students can have difficulty in understanding problems including lots of data, and some problems cannot be suitable for student level. Surely the existing MEAs will give teachers an idea. However, they will need to develop original MEAs that match the learning, cultures and levels of students, and the curriculum. When the development of the MEA is provided in this way, it is possible to create an MEA repertoire for the more common use in schools. With this aim, Deniz and Akgün (2016) gave place different MEAs developed by mathematics teachers in their study.

For teachers and student teachers to be able to develop MEAs complement with the principles, how these MEAs reflect the development process by gaining the awareness of the principles becomes important. In this context, a 14-week lesson on modeling was conducted with the student teachers. In this course, the theory of Vygotsky's social constructivism (Fosnot & Perry, 2005), which is based on the idea that the learning of the individual takes place through the interaction with his colleagues and teachers in a broader sense, was adopted. In this direction, the student teachers structured their modelling knowledge by discussing through group work. In this process, they made preliminary studies in each week and presented reports to their classmates on the topics of modelling, modelling processes, modelling implementations and modelling tasks. The presentations were followed by class discussions and they restructured their knowledge about themes. They were given various modelling tasks and worked on the solution of these tasks. They likewise presented their solution approaches and evaluated other solutions as a group. After each modelling task solution, the task was reached consensus by discussing the appropriate and non-appropriate solutions. At the end of the course, the student teachers had awareness, knowledge and experience on modeling through active discussions. At the same time, discussions were held on the use of modelling in mathematics lessons in our country. At the end of these discussions, it was concluded that the modelling tasks were needed to be developed for students to meaningfully understand mathematics by using their experiences. Thus, the student teachers designed the MEAs that they could use in their future lessons. In the context of the study, the MEAs developed by the mathematics student teachers were examined according to the MEA's principles framework (Lesh et al., 2000) in a detail way.

METHOD

This study was based on a qualitative case study design (Yin, 1987). Because the appropriateness of the developed MEAs were examined in the context of the principles framework in a detail way, the case study design was conducted. The study was carried out within the context of the Mathematical Modelling Course, one of the selective courses in the department of secondary mathematics education in a state university in Turkey. While one of the researchers was also the course's instructor, the other researchers followed the lesson as participant observers.

Participants

The participants were twelve female and nine male enrolled in the Mathematical Modelling Course. These twenty one student teachers voluntarily formed seven groups including a two-person group, five three-person groups and a four-person group (named as G1, G2, etc.). The participants' pseudonym names and their groups were presented in the Table 1.

Table 1. The Participants' Pseudonyms and Genders

Groups	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
Pseudonyms	Ayla Merve	Ozan Mehmet Caner	Serdar Akın Metin	Deniz Selma Candan	Defne Duygu Erdem	Melis Yasemin Derya	Ege Alp Esin Ebru

Data Collection Tools

The data was obtained from the participants' MEAs (see Appendix A), their solutions and the semi-structured interviews. The MEAs and their solutions were examined by the researchers and then the questions of the interviews for each group were determined. While some questions were asked all the groups regarding the principles of the MEA, some of them were peculiar to the specific group's MEAs and its solution. The common questions and to which principle they serve were as follows:

- Explain whether the problem situation requires constructing mathematical model or not. (Model Construction Principle)
- Explain your ideas about whether the problem situation is meaningful for the students and their lives and experiences. (Reality Principle)
- What do you think about whether the students can evaluate the validity of the alternative solutions or not? (Self-Assessment Principle)

- What do you think about whether the students can express their ideas clearly or not? (Construct Documentation Principle)
- What do you think about whether the constructed model can be sharable and usable or not? (Construct Share ability and Reusability Principle)
- Explain your ideas about to what extent the constructed model is meaningful for others and whether the problem situation will provide a useful prototype to interpret similar situations. (Effective Prototype Principle)

Data Analysis

The researchers examined the MEAs and their solutions independently in terms of their suitability to the principles. While this content analysis conducted, the researchers examined both the MEAs as documents and the transcriptions of the interviews to identify the participants' statements regarding the existence of the principles. As a result of the content analysis, they discussed their own analysis and noticed that some MEAs were appropriate to certain principles but some of them were troublesome. However, by examining the documents, they reached the conclusion that the existence of the effective prototype principle might not be observed explicitly. Because the students who would needed a while to be able to remind and use the problem statement and solution of the MEA in the future implementations. But the content of the MEA, its possible solution developed by the participants and the data obtained from the interviews enabled a prediction about the existence of the effective prototype principle. In some situations, the participants explained that the appropriateness of any principle could not be identified because of the instruction not including modelling in our country in the interviews. The researchers determined the appropriateness of the MEA by considering its structure. While these analyses were conducted, some categories were constructed through the MEA principles and the coding process based on the theoretical framework (Strauss & Corbin, 1990) was realized. In this context, we identified the categories for each principle as "completely appropriate", "partially appropriate" and "inappropriate" and defined them (see Table 2).

Table 2. Definitions of the Categories in the Evaluation about the Principles

	Completely appropriate	*Partially appropriate*	*Inappropriate*
Reality Principle	Including realistic aspects such as the context, the figures, the data, etc	Including some aspects which were not completely realistic	Including unrealistic aspects
Model Constructio n Principle	Involving model/s construction peculiar to the real context	Involving model/s construction to some extent	Not involving model/s construction or including existing model/s
Self-Assessment Principle	Including statements about the necessity to enable self-assessment	Including deficit statements about the necessity to enable self-	Not including statements about the necessity to enable self-assessment

		assessment	
Construct Documentation Principle	Including statements enabling to document students' thought processes explicitly	Including deficit statements enabling to document students' thought processes explicitly	Not including statements enabling to document students' thought processes explicitly
Construct Share ability and Reusability Principle	Enabling to construct mathematical model/s which can be used in similar situations and generalized to different situation	Enabling to construct mathematical model/s which can be used in similar situations and generalized to different situation to some extent	Not enabling to construct mathematical model/s which can be used in similar situations and generalized to different situation
Effective Prototype Principle	Including statements about the students' remembering of the problem statement and constructed models	Including statements about the students' remembering of the problem statement and constructed models to some extent	Not including statements about the students' remembering of the problem statement and constructed models

To enable the study's validity and reliability, the triangulation for data analysis was carried out by explaining the categories with verbatim transcript interviews. After the examination of the MEAs according the categories, the researchers compared their analyses and the agreement percentage between two of the researchers (Miles & Huberman, 1994) was over 70%. In addition, the data collection and analysis process were tried to be identified in detail and the results were supported by the MEA excerpts and the participant statements from the interviews.

RESULTS AND DISCUSSION

The appropriateness of the MEAs to the principles were addressed in this section and the extracts from the MEAs and their solutions, and interviews conducted with the groups were presented. The categories for each MEA were seen in Table 3.

Table 3. The Analyses of the MEAs related to the Principles

	Principles																	
	Reality			Model Construction			Self-Assessment			Construct Documentation			Construct Share ability & Reusability			Effective Prototype		
	CA	PA	In	CA	PA	In	CA	PA	In	CA	PA	In	CA	PA	In	CA	PA	In
G_1: Fuel Tank Problem	✓			✓			✓			✓			✓			✓		
G_2: Photosynthesis Problem	✓					✓	✓			✓					✓	✓		

G_3: Measuring Temperature Problem	✓	✓	✓	✓	✓	✓
G_4: Pisa Tower Problem	✓	✓	✓	✓	✓	✓
G_5: Biathlon Problem	✓	✓	✓	✓	✓	✓
G_6: Block Problem	✓	✓	✓	✓	✓	✓
G_7: Construction Problem	✓	✓	✓	✓	✓	✓

CA: Completely Appropriate, PA: Partially Appropriate, In: Inappropriate

The Reality Principle

The contexts of the MEAs were on determining the amount of the fuel inside a tank (G1), identifying the amount of the oxygen produced by a leaf in one hour (G2), revealing the way of a machine sent from the earth to other planets to measure the temperature (G3), calculating the leaning angle of Pisa Tower in a year (G4), ranking the runners to compete in biathlon (G5), calculating the height of blocks by the help of sun light (G6), and identifying the profit function for construction of the houses (G7). It was presented to what extent those MEAs provided the reality principle in Table 3.

The MEAs except the G3's were appropriate to the reality principle. Because the contexts and presentations of the MEAs except one were likely to be meaningful by high school students. The reasons why the G3's MEA was inappropriate to the reality resulted from the figure used in the problem (see Figure 1). The representation of the planets, their orders from the Sun and their arrangements were inaccurately presented. In addition, some statements from the problem situation such as "reaching the Sun" showed the inappropriateness.

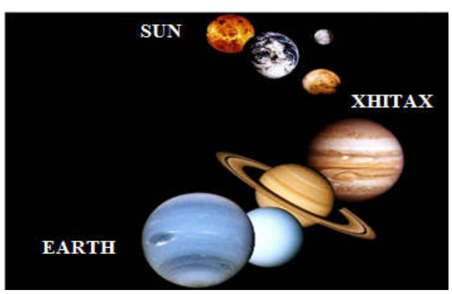

Figure 1.The Picture in the Measuring Temperature Problem

During the interview, the participants realized that they should reorganize the data for the problem after the question about the existence of the reality principle. Although they felt this unrealistic situation, they insisted on the fact that students would not be able to distinguish whether the represented picture was real or not in case the MEA would be implemented.

Serdar : Yes, we could have been more realistic.
Res : That means you could have taken real distances?
Serdar : Yes.
Metin : The real distance should have been found by the help of the picture.[...] I think a student may not know that this shape is like that, I mean. I guess student will not even think the earth must be here or there.

When the G4's MEA was examined, it was assessed as partially appropriate to the reality principle because the assumptions such as the fact that Pisa Tower would not be affected by environmental factors to calculate the slope angle in 1993 and the amount of slope would remain constant were not expressed. Because those assumptions were not expressed, the Pisa Tower was likely to collapse, and this would in turn affect the MEA's reality in the negative. The groups whose MEAs were appropriate to the real life emphasized the reality very often in their interviews. For example, since the G5's MEA for the Erzurum Winter Olympic Games included both an up-to-date organization and was aimed at one of the major ski resorts for our country, it was seen that the place could be recognized by all students. The statements of Erdem were as follows:

Erdem: Firstly, we wanted it to be a daily situation. We thought we had to choose a daily situation and at that time there was Erzurum Winter Olympic Games and we did not already know the game before. While doing the project, the assignment, we searched ourselves and were interested in many things. As seen here, we determined which runners got the gold medals and which ones joined the games most.

Since the Block Problem of the G6 included realistic numerical values and a situation where any person could encounter (e.g. determining the solar front facades, taking into account the sun's position when parking the car), and students were asked to produce solutions by using actual data, it was accepted as appropriate to real life. The statements of the participants from the G6 were as follows:

Derya : Actually we were thinking something different. But in the end, we decided something to be found around us such as the sun, block, etc.
Res : Ok. Is it a situation that students can come across in the daily life?
Derya : Certainly. Say, he will park his car to this part of the block [the shadowed part of the Ege Block] and he will go to work at noon. Let the sunlight not come down on his car. What happens then? Let it not subject to the sun. [...] You will buy a house and it matters if it will have the sunlight or not. Does it have the morning light? This can happen in our daily life.

The Model Construction Principle

The MEAs and their solutions were completely appropriate to the model construction principle except two groups' MEAs (see Table 3). The G1 stated that constructing a logarithmic function was needed to calculate the amount of the fuel used in the tank and in interview explained their models. Since the G1's MEA had a structure that encourages the construction of model and required to construct models to reach the results, the MEA was assessed as being completely appropriate.

Merve : We tried to develop an MEA based upon a function. This was our starting point. Apart from this, while engaging in the problem, we wanted for student to feel the necessity of constructing a function.

Res : Well, you will be a secondary math teacher. Do you think a high school student needs to construct a model in your designed MEA? Why?

Merve : Well, they need, because this problem cannot be solved without constructing a model or a function.

G_1 took the tank in cylinder form and reached the dimensions of the tank by taking advantage of the dimensions of the wheel in the picture. They calculated the volume of the tank by moving from the dimensions they obtained and determined how much weight of the fuel was in by using their physics knowledge. They constructed the logarithmic function showing the amount of the consumed fuel by adding to the account the numerical values in the table.

> How do I find the weight of the fuel oil inside the tank? How do I find out the cost of the fuel by using these data that I have gained? For this, I have to construct the function between the fuel oil weight and the cost of the fuel.
> We estimate the base diameter and the height of the tank by taking the length of the wheel diameter is 100 cm. Then, we can solve the problem by finding the volume of the tank, weight and other data. If the diameter of the tank base is 120 cm and the height is 500 cm, we find the volume by using the formula $V=\Pi.r^2.h$. When d is density, r is diameter, h is height and g is the acceleration of gravity; the weight is found by using the G=V.d.g formula and then the spent fuel oil is determined. For this, the data in the table should be associated and the cost of the spent fuel is constructed as$(1+\log_4 x)/3$ in case the weight of the fuel oil is x.

The G_2 solved the problem without constructing a model and used the ready model, Fibonacci sequence. The G_2 considered that using existed models realized model construction principle. However, when students working on this MEA did not remember the Fibonacci sequence, they could solve the problem by making assumptions about leaf count and using the given chemical reaction model. Since remembering and using existing models were at the forefront instead of constructing models, the MEA were evaluated as inappropriate in model construction principle.

Caner : In that number of brunches we need to guess how many leaves they have. Also we have to know the amount of the oxygen that a leave produces by using a photosynthesis equation. We think it is difficult to solve without constructing a model.

Ozan : Doing without constructing a model... It is hard to guess how many brunches and leaves there are in a tree. I think, model gives us an idea to make guesses. So, we made use of the sequences of Fibonacci. I mean we tried to find out the number of the brunches in this way, it gave us an idea.

The G_4 stated that a function should be developed to measure the slope angle of Pisa Tower in 1993. They considered a right triangle whose length of the hypotenuse was x. Based on the knowledge that the Pisa Tower deflects 7 cm to the south every 10 years they constructed their model as follows.

1993-1372=621
621x7/10=4,347
In 621 years, the tower deflects 4,347 m. Thus the slope angle is α=arccos(4,347/x).

The G5 constructed a function of total time (T) consisted of the time of the skiing, the numbers of target which they could not shoot. When the model was constructed, the number of target that could not be hit in the first shoot was assigned as n_1 and the second one was assigned as n_2. The model was as follows:

$$T = \left[4.\left(\frac{time\ to\ ski\ 100\ km}{100\ km}.7{,}5\ km\right)\right] + (n_1 + n_2).30\ sn$$

The Self-Assessment Principle

While two of the MEAs were inappropriate to the self-assessment principle, the others were completely appropriate (see Table 3). When the G_1's MEA was analyzed, it was thought that there was no contradictory statement regarding the self-assessment principle and they also stated that the problem statement was prepared according to the self-assessment principle. However, they emphasized students should need to get the teacher's approval because of their habits and not being experienced in modelling. Although they expressed this view, their MEA were categorized as appropriate because it was such as to enable self-assessment for the students. The participants reflecting their opinions were given below:

Ayla: Students may ask some questions while working on the MEA such as "Will we find an exact result or will we come closer to something?", "Is our own result true or not?"
Res: What may be the reason of this?
Ayla: What is modelling? That sort of thing. … If he didn't see anything like that [modelling problem] before. There are also habits of students: Teacher asks questions and they answer and then teachers approves that their results are correct or incorrect. Their expectations may be in this direction because they are used to it.

In the Photosynthesis problem, there was no statement thought to be inappropriate to the principle but the unrealistic structure of the MEA would prevent to make self-assessment. So, the MEA was evaluated as inappropriate. In contrast, the G_4 stated that they could achieve the convenience to this principle in statement that the teachers support the students develop their thinking. So the MEA was seen appropriate to the principle. The MEA of the G_5 had clear definition of the biathlon game and the information given in the text was adequate to construct mathematical models. Because of the clearness of the statements in the text and the participants' thoughts were in that direction, the MEA was evaluated as appropriate to the self-assessment principle. Since the G_7 stated that lots of values were given, students would be confused and made some mistakes. So they would need help by their teacher. Because of this, their MEA was seen inappropriate. The participant statements are as follows:

Alp: I mean he cannot think all variables we gave together.
Esin: There are lots of numerical values and they include great numbers. They needed to be

calculated in a careful way.
Alp: I guess students can be confused.

The Construct Documentation Principle

When the MEAs and their solutions were examined, it was seen that the MEAs except the G_3's and G_7's correlated with the construct documentation principle (see Table 3). There was not any target statement related to the students expressing their ideas clearly in the MEAs by G_1, G_2, G_4, G_5, and G_6. The solutions and interviews of these group indicated that these MEAs were completely appropriate to the construct documentation principle.

The G_1 stated that students should document what they thought while solving the problem, otherwise they needed to be encouraged by their teachers. Similarly their solution also indicated the existence of the principle. Since the group took the π as 3, this might cause mathematically inadequate results. However, it did not affect the evaluation of the principle because they included the examination of MEA's structure and the existence of solution stages in detail.

We take the π value as 3.
The volume is 60.60.3.500=5400000 cm3. When the density of fuel is approximately 0,8 g/cm3, the mass is found as 5400000.0,8=4320000 g. If we consider the mass as kilogram we find it as 4320 kg and the weight as G=mg=4320.10=43200 newton. Our function was f(x)= (1+ $\log_4 x$)/3 and we can substitute the weight in the place of x: 43200=$2^a$$\log_2 \quad 43200 = a$. a equals approximately to 15,3.
If $4^b=2^{15,3}$, the cost of the spent fuel in 1 km is found approximately as b=7,6.(1+7,6):3=2,86 TL.
The passenger bus takes this route in 20 minutes and 20 minutes is 1/3 hour. If we make assumption as it has a speed of 90 km, the distance between Manisa and Izmir is 90:3=30 km.
The cost is 30.2,86=85,8 when it is full. While it is returning, it costs 0,65.30=19,5 TL. The total cost is 19,5+85,8=105,3 TL.
So the cost of this tanker going and returning from Izmir to Manisa is approximately 110 TL.

The G_2 thought that the solution should be cascaded by the teachers to make their students express their thoughts. While cascading the solutions and writing all what was thought, they claimed that students could reveal their thoughts.

Res : Does the MEA require the student to express their thoughts in details? For example, you gather the papers after students solved the problem, you only have numeric values. Students do not write their thoughts and you are not able to understand how they solve. What would you do in such a situation? Would you take precautions ahead?
Mehmet : I can give some steps of the modeling process, ask students to solve the problem and explain their solutions according to these steps. They will already have to write down what they think when we want the step-by-step answers.

The G_4 stated students could express their thoughts mathematically in case they would be supported by their teachers. Since their solution included detailed explanations, the Pisa Tower Problem was evaluated as completely appropriate to the construct documentation principle.

Let's compare the person in the photo to the tower. We calculated the first column of the tower as 6 times the person with the red t-shirt in the photo, the six columns after the first as 4 times the person and the last column as 2 times the person. Thus, the height of the tower is equal to 32 times the person. If we take the height of the man 1.70 m, the approximate length of the tower is 1.70x32=54,4. We can get about 55 m. The slope angle is α = arccos (4.347/55) = arccos0.079.

Since there did not appear any statement about expressing the thoughts clearly in the text of the Construction Problem, it was accepted as a result of the pre-examination that it was not appropriate to the construct documentation principle. The G_7 stated that students could not be able to think many variables together and express clearly what they thought. Since the MEA included lots of numerical values, students would have difficulty in explaining their thoughts about them.

The Construct Share Ability and Reusability Principle

The MEAs except the G2, G3 and G4's were evaluated as completely appropriate to the construct share ability and reusability principle (see Table 3). The G_1 emphasized that their MEA and solution could be generalizable to certain situations about fuel and fuel transportation. G_1 explained their solution was useful in different situations with these words: "This is almost useful for every petrol tanker. If a man working on transportation of petrol wants to carry much more petrol and decrease the petrol cost, he will absolutely prefer the vehicles whose fuel tank will greater than the one in the problem". In addition they stated in the interview that because the constructed model was suitable for most of the transporter vehicles, it can be generalized for similar situations.

Merve : Because, the purpose of similar situations is a relationship between weight and fuel oil. I mean a bus firm can also calculate and use this. Ummm. A shipper can also use. Every kind of thing I mean can use it. Cargo for example.

The G_2 thought that the Photosynthesis Problem could only be used in chemical reactions in a limited way and so it did not include any generable model. Since an existing model was presented to students in the MEA and students were not required to construct models, it was evaluated as inappropriate to this principle. The constructed model to solve the Pisa Tower Problem was usable to measure the height of the tower for any year. The problem situation in the MEA was not sufficient when considered in real life context because some precautions are being taken to prevent the tower from tilting too much in real life. Since students would not take this precautions into account, the fact the models could be used in different situations or adapted to another situations would cause erroneous results. So the G_4's MEA was partly appropriate to this principle. When examined the Biathlon Problem, the participants stated that their constructed model was generalizable to the similar situation by giving an example from the situation about a porter's visiting every flat and distributing orders. Because of this, their MEA was evaluated as completely appropriate to the principle. Their expressions were as follows:

Defne : It can be generalized to the similar situations.
Res : Any example?
Erdem : Think of a porter for example, he is supposed to visit every flat, and think of the way he had to take as the stairs, the period of going up this amount of stairs, or there must be floors, and as for this if it was 4 floors for example we can multiply this period with four. For example and I know it will be much detailed, but he may lose time while giving the bread. For example it can be similar to our problem.

The Effective Prototype Principle

While the MEAs of the G_1, G_2, G_5, G_6 and G_7 were completely appropriate and the G_4's MEA were partially appropriate to the effective prototype principle, the G_3's MEA was inappropriate to this principle (see Table 3). The G_1 thought that every student could solve the MEA in case they had prior knowledge. Because the MEA required the construction of a model including the relationship between the weight and the cost, the participants indicated that students used the similar reasoning in correlating two variables in any other similar problem. In addition, they stated that students could recall what their thought processes were in this MEA when they would encounter multiple data belong to the given two variables in future problems. Because of this, it was evaluated as completely appropriate to the effective prototype principle. Merve's expressions were as follows:

Merve: If they will face with any problem including so many data about two different variables, they can benefit from their methods used in this MEA.

When the Photosynthesis Problem was examined, it was seen as a prototype for other problem situations and the G_2 thought that students could solve the problem in case they knew the Fibonacci raw. The G_2 also stated that students could use the models in that MEA while solving problems about reaction equations in chemistry lessons. So this MEA was completely appropriate to the principle. However the G_4 explained that their model was effective and functional because of including the approximate value of the height of the tower, they ignored the realistic assumptions regarding the model. So the Pisa Tower Problem's solution and the constructed model were evaluated as partially appropriate to the effective prototype principle. The G_6 stated that their MEA was useable for different future conditions such as determining the shadow area in different time zone, places and in south hemisphere. They expanded their views about effective prototype by emphasizing the selection of the picnic places. So, the MEA was completely appropriate.

Derya: Well, for example in January, it is 1:30 p.m. and North hemisphere. In conditions including different months, time zones and South hemisphere, students can use this problem's solution.
Melis: Yes, when we changed the problem, they can use. Supposing that they will go picnic and search for the best place to have a picnic. Then they can benefit from this to identify the places where trees overshadow until the nightfall.

CONCLUSIONS

In this study which aims to examine the MEAs developed by the student teachers according to the principles in Lesh et al.'s (2000) framework, all the MEAs except one were appropriate to some or all of the six principles. Six activities were considered to be MEAs although they were not completely appropriate to all of the principles, and the incomplete principles were thought to be developed. However, it was impossible to see the Measuring Temperature Problem as an MEA. The most important causes of this were that the problem did not include a real context, did not require constructing model because of unrealistic context and involve a situation against a scientific reality. Even though the participants tried to provide the real life connection by using their physics and astronomy knowledge and phenomena, their deficiencies reflected on the MEA and its solution. This situation makes it impossible to question the existence of other principles in MEAs where the reality principle is not provided. If there is no reality, the constructed model is wrong. Also, a model to be sharable and usable, and a prototype for future situations is seen impossible. However, even if the participants constructed a model in the solution process, interpretation and validation of this model would not be realized in the real life context. In addition, students would not be able to make self-assessment as their real life knowledge and the situation given in the problem would be contradicted. The importance of establishing interdisciplinary relationships in MEAs is emphasized (English, 2009; Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). However, as in the Measuring Temperature Problem, the inaccurate physics and astronomical knowledge of the students prevented establishing the proper interdisciplinary relationships. On the other hand, the Fuel Tank, Photosynthesis and Block Problems provided this interdisciplinary feature because the participants correlated the necessary disciplines in these problems. This situation also led to the fact that their MEAs provided the reality principle. The Biathlon and the Fuel Tank Problems were found to be completely suitable for all principles. While one of these problems used an interdisciplinary association (Fuel Tank Problem), the other dealt with an international up-to-date event (Biathlon Problem). Both MEAs represented complex real-life situations, contained meaningful contexts for students, and had a generalizable structure for different situations.

The reality principle plays a binding role when evaluated in terms of all the principles. Because the models created in MEAs with incorrect real-life contexts will not be able to be generalizable and adaptable to different situations, to allow students to evaluate themselves and to enable them to document their thinking processes (e.g. Measuring Temperature Problem). In addition, although incomplete information in the context of real life are given in MEAs, the existence of other principles can be explored in the light of established assumptions. Mathematical models can be constructed in the direction of determined assumptions, students can evaluate themselves and explain their thoughts. However, the fact that the constructed models can be sharable and reusable, and the problem context can enable prototypes for future situations cause troubles and the models can be required to be revised.

The model construction principle is also a need for the MEA development process based on reality. When examining the most basic definition of MEAs, Lesh et al. (2000) defined MEAs were activities required constructing models. Chamberlin and Moon (2008) stated that students should develop their own mathematical models instead of using an existing formula or model. Therefore, the existence of the model construction principle in the MEAs in which the existing models are used cannot be questioned. For example, in the Photosynthesis Problem, students needed to use an existing model. Since the model appropriate to the problem was not constructed, the model share ability and reusability principle was not provided.

The self-assessment and construct documentation principles are two principles that are directly related each other. For an MEA to have these two principles requires firstly to be appropriate for real life, personal meaningful, appropriate for the readiness of the students, consistent with their pre-knowledge and their modeling experience. In addition expressing the task clearly and directing students to express their thoughts are also important aspects. For example, the Biathlon Problem, which was consistent with students' pre-knowledge, appropriate to the students' readiness and included the expression of the task clearly, was related to a sports that was not familiar to students. Through being given detailed information about biathlon sports, the students' familiarity was provided and the personal meaningfulness was provided. Therefore, it was decided that students could make self-assessment. On the other hand, the Construction Problem would create complexity for students and would not be suitable for their readiness because it involved a lot of data and required the association of lots of variables despite it included a real context for students. Therefore, this MEA were not considered appropriate to these two principles. It may also be important to consider classroom practices for identifying the existence of these two principles. In this context, classroom norms, the tasks that these norms impose on students and teachers, and in-class and in-group interactions can facilitate the examination of these principles. In addition, the use of tools by the teacher, such as materials, photos and videos, will ensure that the students have knowledge of the context. In addition, by providing a solution plan with the problem, and by enabling student to write their thoughts step-by-step, they can make the students evaluate themselves and express their thoughts. Some statements such as "You can ask things you do not understand in the problem. Remember that you will only discuss your questions regarding the solution within your group.", "If you feel that you have missing information when solving the problem, try to complete the missing information by exchanging ideas within your group." or "You should discuss what you have done within your group and make joint decisions." (Bukova Güzel, et al., 2016) can be given written with the MEA and so students can be encouraged to make decisions in their groups without asking for help from teacher.

Although the participants had not been informed about the principles in the study, two MEAs were approved for all principles. While an MEA was not considered appropriate to principles, other MEAs were generally found partly or completely appropriate. This result is in contradiction with the result of certain studies. Chamberlin and Moon (2005) indicated that the participants could not be successful in developing MEAs unless anyone was informed or got education about how to develop MEAs. Deniz and Akgün (2016) stated that the participants had difficulty in developing MEAs fully appropriate to the principles although they were instructed on mathematical modelling and the MEA principles. Additionally, Yu and Chang (2009) explained that developed MEAs were unsuccessful in the last four principles except for reality and model construction principles.

The construct share ability and reusability, and effective prototype principles were the principles which were associated with the others and could be elicited effectively by tracking future implementations. The fact that the MEAs did not fit the last four principles in their study was not arisen in this study. It can be said that although the participants had not been informed about the MEA principles in this case, having knowledge and experience on modeling and modeling applications reflected on their MEAs. What makes the difference in this study is the detailed interviews conducted with the participants as well as the examination of the MEAs. Because, more information regarding principles which could not be seen in the MEAs as written documents were obtained from the interviews. It is suggested in future studies to add classroom practices and thus to examine the principles that can be evaluated over a long period such as effective prototype principle.

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APPENDIX A. THE MODEL ELICITING ACTIVITIES DEVELOPED BY THE MATHEMATICS STUDENT TEACHERS

G_1: Fuel Tank Problem

The fule weight in tank (Newton)	The cost of fuel per km (TL)
4^1 newton	0,67 TL
4^2 newton	1,12 TL
4^3 newton	1,34 TL
4^4 newton	1,68 TL
4^5 newton	2,06 TL
4^6 newton	2,34 TL
4^7 newton	2,67 TL

In the figure, a fuel tank was given and the cost of the fuel consumes according to its weight per kilometer were given in the table. This tank will go to Manisa from Izmir as full and later it will come back to Izmir after unloading the fuel. The tank consumes the fuel worth 0.65 liras per kilometer when it travels unloaded. The radius of the wheel of the tank is 50 cm. How much fuel does the tank consumes in terms of liras, when goes and turns back from Manisa?

G_2: Photosynthesis Problem

One hectare trees absorbs 6 tons carbondioxite in a year. A tree with an average height absorbs 12 kg carbondioxite in average in a year and emits the amount of oxygen to the atmosphere which a family needs in a year**.** As $6CO_2+6H_2O \rightarrow C_6H_{12}O_6+6O_2$ is a photosynthesis equation, what is the amount of the oxygen a leave produces in an hour? Please find it (C:12, H:1, O:16).

G_3: Measuring Temperature Problem

We have a special machine. We want to throw this machine to space in order to measure the temperature of planets. For the measurement the machine needs to visit every planet for a short while. It is predicted that our machine will arrive the Sun in 6 days. In order for the machine to reach the sun as soon as possible and explode there, we need to write the software of the position function which is dependent on time of the route on the machine. Will you help us to find this function?

G_4: Pisa Tower Problem

The Pisa Tower seen beside was completed in 1372. Because of the tower's construction, it is known that the tower deflects 7 cm to the south every 10 years. Find the angle of deflection in the year 1993.

G₅: Biathlon Problem

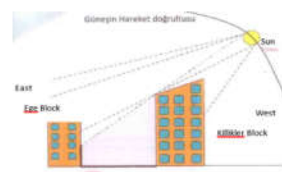
This year, in the branch Biathlon, one of the games to be played in Erzurum Winter Olympic Games, there are three racers who got golden medals in 2008 Biathlon World Championship and their performances in the biathlon's relay race are given in the table.

Racers	The period of skiing 100 kilometers	The number of right shots	Total period
Ivan Tcherezov	300 sec	3-5	
Nikolay Kruglov	250 sec	3-3	
DmitryYaroshenko	350 sec	5-3	

In Biathlon game, the main activity which is 20 km for men is an individual event and includes four shooting stages. The 10 km sprint is made up of the first two shootings. In the in-turn races, every biathlon racers ski for 7.5 km and shoot two times. In the following parts, in a track of 12.5 km, racers stop for 4 shooting stages. Every racer carries a rifle of 3.5 kg in their backs. These rifles have clips for 5 bullets. In Biathlon races the racer having the least time in total wins the race. In sprint and relay race every missing shoot, racers ski for 150 meters punishment tour and this means an addition of 30 seconds. According to the given information, if these racers joined Erzurum Winter Olympic Games, calculate these racers' duration finishing the 4x7.5 relay race. Try to interpret their ranks.

G₆: Block Problem

The picture was taken when the shadow length was equal to the real length (January, 1:30 p.m. North Hemisphere). According to this, find the height of Killikler Block according to the given data as closer as to the reality. And find the time when the Ege Block starts not getting the sun light.



G₇: Construction Problem

The place of the plot	The cost of the plot: TL	The size of the plot: m^2	The cost of the flat: TL	The size of the flat: m^2	The maximum number of the flats that can be built	The selling price per flat: TL
Sea side	1.500.000	750	65.000	190	2	450.000
Urban	800.000	600	60.000	180	6	150.000
Suburban	400.000	800	55.000	170	7	100.000

There is a building contractor who has got the enough capital to buy these plots and build houses on them. Where should he build the houses to get the maximum profit? Find the profit function.