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IMPROVING THE REASONING ABILITY OF ELEMENTARY SCHOOL STUDENT THROUGH THE INDONESIAN REALISTIC MATHEMATICS EDUCATION

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Abstract

By taking the role as a mentor and a facilitator, a teacher in the 4th grade of elementary school needs to look at the condition of the students in the concrete thinking stage. Learning process needs to be adjusted such that the abstract objects in mathematics can be represented through concrete objects as a bridge to enter the knowledge that the students already had, especially for the material of fraction. This research aims to analyze the achievement and the improvement of students' mathematics reasoning ability through the implementation of Indonesian realistic mathematics education (PMRI) approach. The research subject consisted of 51 students in the experiment group and 45 students in the control group which categorized into three levels (low, intermediate, and high). The result suggests that the achievement and the improvement of students' reasoning ability in the mathematics learning using PMRI approach are better than the conventional learning.

Keywords: Realistic, Mathematics Education, Fraction, Reasoning, Elementary School

Abstrak

Sebagai pembimbing sekaligus fasilitator, seorang guru di kelas 4 sekolah dasar perlu memperhatikan kondisi para siswanya yang berada pada taraf berfikir konkret. Proses belajar perlu disesuaikan sehingga objek abstrak pada matematika dapat "direpresentasikan" melalui benda konkret sebagai jembatan memasuki pengetahuan yang telah dimiliki siswa, khususnya terhadap materi pecahan. Penelitian ini bertujuan untuk mengkaji pencapaian dan peningkatan kemampuan penalaran matematika (KPM) siswa melalui penerapan pendekatan pembelajaran pendidikan matematika realistik Indonesia (PMRI). Subjek penelitian terdiri dari 51orang kelas eksperimen dan 45 orang kelas kontrol yang terbagi menjadi tiga level (rendah, sedang dan tinggi). Hasil penelitian menunjukkan pencapaian dan peningkatan KPM siswa pada pembelajaran matematika dengan pendekatan PMRI lebih baik dari pembelajaran konvensional.

Kata kunci: Realistik, Pendidikan Matematika, Pecahan, Penalaran, Sekolah Dasar

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It is hard to imagine how the learning process runs and what the learning result is towards the people who do not possess the listening ability and aims to know the beautiful chirp of birds, even more, towards the people who have the listening ability but never hear the birds chirp (Syamsuri, Purwanto, Subanji, & Irawati, 2017). They might give a written answer, but they cannot understand it and it is not the knowledge they have. This phenomenon happens since their condition does not support to know kinds of voice. It is similar to a teacher as an instructor and a facilitator needs to look carefully at the condition or the ability of their students involved in the learning process.

Fraction is a topic which is first introduced to the students of elementary school when they are in the 4th grade. Therefore, it is important for the students to engage in a proper and joyful learning of fraction such that they can avoid misconception in understanding it (Wahyu, Amin, & Lukito, 2017). The learning process has to be started by understanding the concept of fraction through activities utilizing concrete object, either the discrete one or the continuous one. Once they understand the meaning of fraction, then it is given a symbol corresponding to the representation they have seen and understood through the concrete object.

Some reports suggest that many students experience difficulty in understanding and solving fraction problems. It even develops an anxiety among students to learn. Morge (2011) reports that the topic of fractions can be intimidating and difficult for children, even into the middle grades. Similar to this, Naiser, Wright, and Capraro (2003) suggests that fractions are often difficult for students to fully comprehend. Teachers must find a variety of strategies to use in the classroom for teaching fractions. The same statement was reported that "the teacher said that one of the topics in grade five that is difficult to be understood by fifth grade students is fractions (Julie, Suwarsono, & Juniati, 2013).

Furthermore, Li & Smith (2007) reports that a design of learning fraction for teacher to implement the learning is needed. The issue of difficulty related to fraction is not only experienced by students but also the teacher or pre-service teacher in choosing strategy to implement the learning of fraction. When the students do not understand the concept of fraction, they give answer "nothing" or "zero" referring to the result of division when the divisor is greater than the dividend. Fraction becomes important to be understood by students to support them to learn the other material. Laursen (as cited in Brown & Quinn, 2007) stated that the inability to perform basic operations on common fractions has led to error patterns that emerge in learning algebra. Problems can arise when students attempt to apply misunderstood shortcuts, learned with fractions, to situations involving algebra.

When students consciously engaged in the process of finding mathematical concept using a concrete object, then it will give a strong trace in recalling the finding. The experienced things will leave stronger trace compared to the one gained from reading or listening. The finding they got from the learning process based on their own thinking pathways will be logically accepted and create a more meaningful learning (Treffers, 1987). Dahar (1988) stated that from the cognitive development theory of Piaget, we know that the younger children are still at the stage of learning concrete concepts, while the more difficult or more abstract concepts are more suitable to be learned by the elder. In the concrete level, students are regarded to master the concept if they have recognized the object they deal with.

Based on the constructivist paradigm, learning is an active activity where the students construct their knowledge by themselves. Suparno (1997) stated that constructivist thinks that knowledge is the result of human construction. A human constructs knowledge through the interaction with objects, phenomena, experience, and environment.

Through the realistic learning approach, students will understand the concept and operation of fraction through mental activities. Regarding the realistic learning, the PMRI Team (2010:18) states that Realistic Mathematics Education (RME) is viewed to be a potential approach to increase students' understanding of mathematics. Inspired by the philosophy of RME, they developed an approach to improve mathematics learning in Indonesian schools. It is known as PMRI which stands for Pendidikan Matematika Realistik Indonesia or Indonesian version of RME. Based on this argument, mathematics is not to be learned only, but also to be implemented in daily life activities (Sumirattana, Makanong, & Thipkong, 2017). In this paper, the operational definition of PMRI is a learning which utilizes contextual problem and concrete object to understand concept and problem and to represent fraction based on the chosen context.

In fact, there are many didactical phenomena related to the mathematics learning. However, fewer people pay attention to them and do not use them though they are full of mathematics principles. Efforts to raise the potential are needed such that the learning could be meaningful, the students' understanding towards mathematics can be well constructed, and the scientific institutionalization could happen within them.

The learning model emphasizing algorithms, without giving enough time and chance to find the knowledge, it will be meaningless for students. Bruner (as cited in Dahar, 1988) states that discovery learning is suitable with the idea of knowledge active searching by human which leads to a good result. Although it is not easy for students to find the concept as teacher expects, context needs to be addressed such that it can stimulate students to do the learning activities based on the learning purposes (Wang, Zhang, & Zhou, 2013).

In the level of concrete, students can be regarded as they have mastered a concept if they recognize the object they have ever known. Dahar (1988) states that somebody has reached the understanding towards a concept in the concrete level if he recognizes the object they have ever met before. A child who has ever played a toy and gives the same response when the child finds the toy, then it is said that the child has mastered the concept at the concrete level. In order to master the concept in the concrete level, students need to look carefully at the object, and then distinguish it from the other stimuli in his environment.

Following the students' thinking pathways in the learning process, based on the learning trajectories principle quoted by Clements & Clara (as cited by Westenskow, 2012), learning trajectories are a mapping of the progression of learning of mathematical concepts and skills. Following the students' thinking pathways in the mathematics understanding process, it can be useful to give chance for the students to actualize their potential. Therefore, the students' potential can be explored optimally such that it can be applied in the daily life and in the other problem solving as an external representation.

Mathematics material is dominated by abstract objects. The abstract of mathematics should be modified to be more concrete such that the students can imagine through a contextual problem (Swanson & Williams, 2014). The learning process which is started by utilizing concrete objects will give a deep trace because it suits their cognitive development stage which moves from concrete to formal. Learning activities involving concrete object will create action and involve several senses of the students.

It is possible for the students to give response and solution algorithm formally and informally when they are engaged in the contextual problem. The learning sequence done by a student to reach the goal may vary from one student to the others. However, a learning designer has to consider between the development stage of students' thinking process and the stage based on the mathematics structure. Therefore, students have to understand mathematics by not only memorizing it but also by given wider chance to think of various problem understanding.

According to Koenig (2007), reasoning and proving activities in the instructional program from pre-kindergarten to senior high school level should consist of the following: (1) Recognize reasoning and proof as the basic aspect of mathematics; (2) Make and investigate conjecture in mathematics; (3) Develop and evaluate argument in mathematics; (4) Choose and use various kinds of reasoning and proving methods.

Reasoning, according to Shurter & Pierce (as cited in Sumarmo, 2014) can be defined as a process to reach a logical conclusion based on relevant fact and source. Human often finds problems requiring them to find the solution and draw a conclusion. A conclusion can be drawn through reasoning by looking at every relation of the available arguments or information. Therefore, reasoning ability needs to be developed since early childhood through designed learning process at school. Through exercises of reasoning development, students can see the problem and the adequacy of information to draw a conclusion.

When the students see that the given information is inadequate, then they can draw an initial conclusion suggesting the need for additional information. Further, they can look for additional information by utilizing their existing knowledge as long as the information can be derived from the given problem. Therefore, the teacher needs to know how far the students understand the given information.

METHOD

Research Location and Sample

The experiment was done at two elementary schools, the SD Negeri 2 Banda Aceh and SD IT Nurul Ishlah Banda Aceh. Each school involved two groups of the 4^{th}-grade students. The selection of the two schools as the research location was done through several considerations such as the aspect of the number of students within the group and the aspect of mathematics teacher.

This research was done by setting two treatments towards the subject. One group consisted of 51 students was treated by PMRI approach learning as the experiment group, while the other group consisted of 45 students became the control group and was treated by conventional learning. The conventional learning means that the learning is implemented without any intervention, addition, or assistance from others.

Research Instrument

The data collection was done through observation, test, and unstructured interview. The observation was done in a structured way and unstructured way. The instrument used was test and nontest. The tests consisted of a test of initial mathematics ability and test of mathematics reasoning ability. The instruments were also equipped with lesson plan and concrete objects. The process of data collection used instruments which have been validated upon the readability, have been revised according to the feedback from content expert and language expert, have been tried to the nonsubjects both teachers and students.

Indicators for the mathematics reasoning ability are: (1) using or interpreting mathematics model such as formula, graph, table, scheme, and drawing conclusions from them; (2) solving problems using the appropriate mathematics method such as arithmetical, geometrical, or analytical; (3) communication mathematics information effectively using symbols, visual, numerical, or oral representation; and (4) assessing the accuracy level of the conclusion based on the quantity information.

Mathematics Reasoning Ability

Mathematic reasoning ability is defined as the students' ability to check the adequacy or the need of data and all the relationship among available arguments and information to draw conclusions. Shurter & Pierce (as cited in Sumarmo, 2014) states that the problem of mathematics reasoning ability consists of nonroutine problem designed to know the students' ability in mathematics reasoning. The problem posed consists of problem with illustration and problem without illustration equipped with data to find the solution. However, it was not a ready-made data to enable students to create and investigate conjectures as well as develop and evaluate the argument towards the problem-solving. In the problem construction, it is expected that a reasoning process happened to lead to drawing a logical conclusion based on the relevant facts and sources.

Initial Observation

Initial observation was done to search the possibility to conduct research based on the requirements and to observe the implementation of the learning process. One of the requirements is the number of learning group (classroom) in the 4th grade is minimum 2 groups. The observation focused on the qualification of the mathematics teachers who taught in the targeted elementary school and their experience in joining PMRI workshop or training. There were seven elementary schools in Banda Aceh which we visited before setting the research location, namely two private schools and five public schools.

Concrete Object

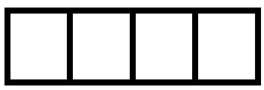
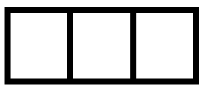
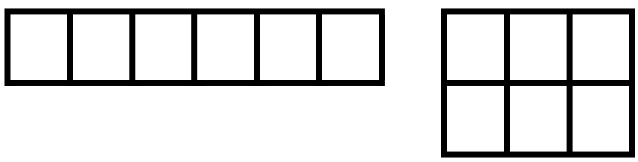
The concrete object used as the learning media was a rectangular board and divided into 24 equal parts such that it resembled a chocolate shape as illustrated in Figure 1.

Figure 1. Chocolate

The rectangular fraction board model was divided into 24 parts (6×4 unit) but not separated so that it can be used to represent fraction with the denominator of 2, 3, 4, 6, 8, 12, and 24 and the numerator of 1 to 24. The fraction board was equipped with paperboards in various sizes such that they can cover part of the board to show the certain value of fraction. The selection this material was regarded to be useful for students to understand the concept of fraction and to do the operation of addition and subtraction of fraction with certain numerator and denominator. The board was made of wood or plywood in 132×88 cm size. Through this size selection, a "chocolate chip" can be made to form 24 small chips in 22×22 cm size.

Eight sizes represented different values of fraction: $\frac{1}{2}$ with size 2×6 or 3×4, $\frac{1}{3}$ with size 2×4, $\frac{1}{4}$ with size 1×8 or 2×4, $\frac{1}{6}$ with size 1×4, $\frac{1}{8}$ with size 1×3, $\frac{1}{12}$ with size 1×2, $\frac{1}{24}$ with size 1×1, as well as one size of 4×6. The example of paper size representing each value of fraction can be seen in Table 1.

Table 1. Form and Representation of Fraction

Value of Fraction	Paper Size as Representation	Paper Form
$\mathbf{\frac{1}{6}}$	1×4	
$\frac{1}{8}$	1×3	
$\frac{1}{4}$	1×6 and 2×3	

The fraction rectangle was equipped with paperboards with a rectangle shape and certain size such that it represents the value of the desired fraction. The example of board of fraction representing $\frac{1}{2}$ can be seen in Figure 2.

Figure 2. Board of Fraction

Paperboard was used to cover part of fraction rectangle to represent the corresponding fraction. Each group of students was given two fraction boards to give wider chance for the students to try representing fraction. The two fraction boards also enabled students to do the addition of fraction.

"Cobek"

Cobek, or mortar, is a kitchen utensil made of clay which is usually used to traditionally grind chili, onion, and another spice in Aceh, as shown in Figure 3.

Figure 3. Cobek or mortar

A mortar was given to each group to be broken into pieces to represent the meaning of fraction, especially the concept of fraction as part of a whole.

Figure 4**.** Making real object representing the fraction value

Research Limitation

There are five representations of fraction, namely: part of a whole, division, ratio, measurement, and operator. This research focused on the definition of fraction as part of a whole. This concept can be shown by using discrete and continuous objects.

Data Analysis

The quantitative data was gained through the test of initial mathematics ability and the test of mathematics reasoning ability. The data of initial mathematics ability was gained before the fraction learning process and was used to classify the level of students (low, intermediate, and high), while the test of mathematics reasoning ability was given before and after the learning from the two elementary schools. There are two variables involved in this research, the realistic mathematics education was the independent variable and mathematics reasoning ability was the reasoning ability.

The test items were made in a form of essay to consider the aspect of reasoning. The essay could be used to see the improvement of the reasoning ability through pretest and posttest to be analyzed the improvement score. The data analysis used *Software* Statistical Package for Social Science (SPSS), the 18^{th} version. In the initial stage, all data of the test result was presented in a table of two major parts, namely the experiment group and the control group.

RESULT AND DISCUSSION

The addition process between two fractions with different denominators is started by the understanding towards addition of two fractions with similar denominators. There were several ways the students did to add $\frac{1}{2}$ and $\frac{1}{3}$ by using the fraction board. The steps were selecting the paperboard corresponding to the represented fraction.

(1) Step 1: Selecting a chocolate chip representing $\frac{1}{2}$ and $\frac{1}{3}$.

Figure 5. Chocolate chips representing $\frac{1}{2}$ and $\frac{1}{3}$

(2) Step 2: Two different chips were substituted using the similar chips.

Figure 6. Chocolate chips as the fractions having similar value

(3) Step 3: Checking that the substitution has been correctly done (if necessary).

(4) Step 4: If the substitution has been done correctly, then the students calculated the addition of $\frac{1}{2}+\frac{1}{3}$ from the illustration, namely $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1+1+1+1+1}{6}=\frac{5}{6}$. The steps of addition are illustrated at Figure 5 and Figure 6, while Figure 7 suggesting the result.

Figure 7. Chocolate chips as the result of addition

The learning process on fraction using fraction board as a model of chocolate chip can be done through the following steps.

(1) Reinforcing the understanding towards the meaning of fraction through a concrete object without symbol of fraction. This step will create a strong trace in the students' mind since they directly observe and experience to understand the fraction.

(2) Through the fraction model using concrete object, teacher shows various different value of fraction to the students referring to different size of fraction models. Teacher also shows the relative similar size of fraction models to the students referring to the equivalent fractions.

(3) Further, students learn about the division of concrete object, the continuous and the discrete one, to introduce the equivalent fraction through the process of concrete division as well as to create understanding of fraction interpretation as a division.

(4) Teacher introduces the symbol of fraction.

(5) Teacher introduces the operation on fraction through concrete object.

The illustration in Figure 8 presents the example of using paperboard on the fraction board to represent fraction of $\frac{1}{3}$.

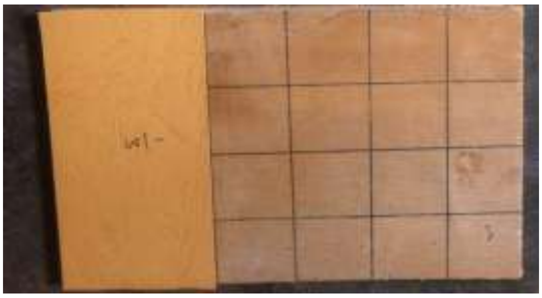


Figure 8. Representation of fraction $\frac{1}{3}$

Figure 8 represents the fraction of $\frac{1}{3}$. It is then replaced by two paperboards representing value of $\frac{1}{6}$, then the two of them together make value of $\frac{2}{6}$ or equivalent to $\frac{1}{3}$ as shown in Figure 9.

Figure 9. Chocolate chip representation, each paperboard represent fraction of $\frac{1}{6}$

The use of concrete objects which are fraction board equipped with paper board have been able to improve the students' reasoning ability. It can be seen from the process and result of their learning especially in problem solving and mathematics reasoning.

A problem was given before and after the learning process. The students gave different answer and it shows the quality improvement of the answer provided. The example is given below.

A picture was given as illustrated in Figure 10. Students were asked to give explanation whether the shaded region AKD is $\frac{1}{4}$ of the rectangle ABCD.

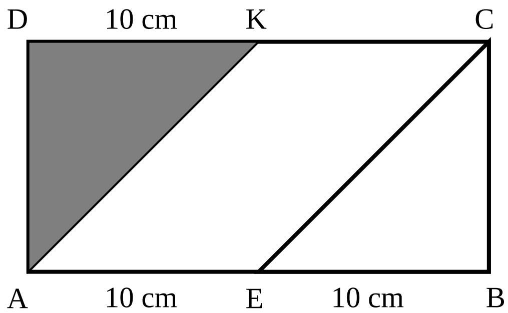


Figure 10. Problem illustration

Before the learning process, a student provided answer that the AKD is $\frac{1}{3}$ and is not $\frac{1}{4}$ of the ABCD. However, after learning process, the students provided different answer as seen in Figure 11.

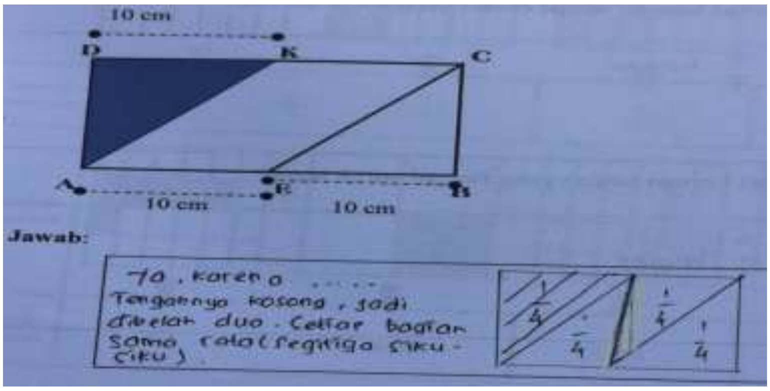


Figure 11. Student's answer towards problem

The result of the independent t-test of the N-Gain of mathematics reasoning ability gave the t value of 2.977 and Significant (P) value of 0.004 for all categories. Since the value of $P = 0.004 < 0.05$, then the H_0 was rejected. Thus, there was a significant difference of the improvement of mathematics reasoning ability between the students taught by using PMRI approach learning and the students taught by using conventional learning.

In general, the achievement and the improvement of the students' mathematical reasoning ability taught by using PMRI approach are better than the students' mathematical reasoning ability taught by using conventional learning. The students were active in utilizing concrete object in solving problems both independently and within group, such as finding the equivalent fractions.

Figure 12. Finding the equivalent fraction

However, the achievement of students in the category of intermediate initial mathematics ability gave no significant difference. The same phenomenon was also shown by the students in the category of low initial mathematics ability. There was no significant difference in terms of achievement and improvement of the mathematics reasoning ability between the students taught by using PMRI approach and the students taught by using conventional learning. Comprehensive information is illustrated in Table 2.

Table 2**.** Achievement and improvement of mathematics reasoning ability

Ability	KAM	**Achievement**		**Improvement**	
		Mean		Mean of NGain	
		PMRI	PKV	PMRI	PKV
Mathematics Reasoning	T	28,17	20,08	0,4719	0,1840
	S	19,23	13,50	0,2858	0,1373
	R	13,15	12,40	0,1358	0,1505
	Total	20,00	14,91	0,2927	0,1566
	KAM	Conclusion		Conclusion	
	T	PMRI>.PKV		PMRI>PKV	
	S	PMRI=PKV		PMRI>PKV	
	R	PMRI=PKV		PMRI=PKV	
	Total	PMRI> PKV		PMRI>PKV	

CONCLUSION

The achievement and the improvement of the students' mathematical reasoning ability taught by using PMRI approach are better than the students' mathematical reasoning ability taught by using conventional learning. However, the achievement of students in the category of intermediate initial mathematics ability gave no significant difference. The same phenomenon was also shown by the students in the category of low initial mathematics ability.

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