ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 10, No. 1, January 2019, pp. 143-156

EXTERNAL REPRESENTATION FLEXIBILITY OF DOMAIN AND RANGE OF FUNCTION

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Abstract

This study attempts to analyze pre-service secondary mathematics teachers' flexibility of external representations of domain and range of functions. To reach the purpose, a task consisted of thirty question items were designed. Participants of the study were thirty-eight Indonesian pre-service secondary mathematics teachers attending mathematics education department at one private university in Jakarta, Indonesia. Based on the analysis participants written responses, this paper revealed participants' difficulties in providing a proper and consistent definition of the concept of domain and range of functions. We also disclosed the participants' lack of flexibility in doing translation among representations under the concept of domain and range of function. In general, participants written responses to the task did not provide evidence of a solid understanding of domain and range. There are several implications of these findings offered for secondary mathematics teacher education's program.

Keywords: Domain, Flexibility, Function, Pre-service mathematics teachers, Range.

Abstrak

Penelitian ini menganalisis fleksibilitas representasi eksternal domain dan range fungsi mahasiswa calon guru matematika di tingkat sekolah menengah. Untuk itu, sebuah tes yang terdiri dari tiga puluh pertanyaan telah didesain. Penelitian ini melibatkan 38 mahasiswa calon guru matematika yang sedang menempuh studi di jurusan pendidikan matematika di salah satu universitas di Jakarta, Indonesia. Berdasarkan analisis terhadap hasil pekerjaan tulis mereka, artikel ini mengungkap kesulitan mahasiswa dalam mengekspresikan domain dan range fungsi secara benar dan konsisten. Kami juga menemukan kelemahan mahasiswa dalam fleksibilitas antar representasi domain dan range fungsi. Selain itu, mereka juga kesulitan dalam menemukan domain dan range fungsi-fungsi yang diberikan dalam tes. Secara umum, berdasarkan jawaban terhadap soal yang diberikan, mahasiswa calon guru matematika ini tidak memiliki pemahaman yang kuat tentang domain dan range fungsi. Terdapat implikasi dari hasil penelitian ini untuk program pendidikan guru matematika tingkat menengah.

Kata kunci: Calon guru matematika, Domain, Fleksibilitas, Fungsi, Range.

How to Cite: Aziz, T. A., & Kurniasih, M. D. (2019). External representation flexibility of domain and range of functions. *Journal on Mathematics Education, 10*(1), 143-156.

A great majority of researchers in mathematics education has agreed that the concept of functions is the most powerful notion, the basic concepts, the heart of mathematics (Clement, 2001), and plays a substantial role in all level of mathematics curriculum. At the beginning of instruction associated with the function, teacher usually presents the concept of domain and range of the function. The domain is defined as the set of meaningful inputs x, whereas the range is described as the set of corresponding outputs y (Rockswold, 2012). These concepts become critically important in learning function or further mathematical concepts as it leads students to generalize ideas. Students' accurate understanding of domain and range function could assist them in comprehending linear transformation (Dorko & Weber, 2014) and inverse function (Arnold, 2004).

The importance of the concept of domain and range function, unfortunately, is not converged with students' comprehension about the concept. There are several previous studies devoted to exploring students' understanding of domain and range function. The studies reported that a large majority of students have difficulty in determining the domain of multivariable functions (Martínez-Planell & Gaisman, 2012), the domain of a composite function (Neger & Frame, 2005; Özkan & Ünal, 2009), and the domain of square root function (Drlik, 2015). Also, even though students are taught the way how to determine the domain and range of a function, they encounter difficulty when dealing with various types of problems. The reason might lay in the fact that understanding associated with domain and range of functions is likely to be overlooked by most instructors in university (Dorko & Weber, 2014) or high school level (Arnold, 2004). In other words, there is a lack of awareness of teachers in presenting the topic of domain and range function. Instead of having students catch on this topic comprehensively, most teachers are likely to present it briefly and focus on the operation of the function.

Another possible reason is that within the context of the topic of functions, the presentation tends to emphasize on single representation instead of considering various representations. The salient aspects of function concepts are the diversity regarding representations and interpretations (Sajka, 2003). Students focus heavily on algebraic symbol impedes them to possess a comprehensive understanding of multi-representation. Martínez-Planell & Gaisman (2009) found that when students were not exposed to distinct representations, they demonstrated a weak understanding of domain and range of functions. Elia & Spyrou (2006) revealed three factors might contribute to students' acquisition in determining domain and range of a function, one of which is the ability to employ various modes of representations.

The function could be expressed in various ways such as a table, ordered pairs, algebraic symbol, and graphics. Generally, students are likely to have a narrow view about function in which function deals solely with algebraic formula (Clement, 2001). Many researchers put emphasizes on the importance of various representations to help students grasp the concept of function. Therefore, for students to gain a comprehensive understanding of domain and range function, teachers are necessary to present various representations of function.

Goldin & Steingold (2001) distinguished two facets of representations, namely internal and external representations. Internal representation refers to the images a person generate in his/her brain for mathematical objects and operations (Cuoco & Curcio, 2001) or cognitive processes to mathematical ideas (Yilmaz, Durmus, & Yaman, 2018). In this respect, internal representations of an individual could not be observed directly or abstract. Meanwhile, external representation could be observed physically as the forms of it are an algebraic expression, real number line, Cartesian coordinate, diagrams, and so forth (Goldin & Steingold, 2001). Sierpinska (1992) added that making the connection among different representations of functions is another challenge for students.

The ability of students to do translation among representations is characterized as flexibility or translation (Bannister, 2014). Dufour-Janvier, Bednarz, & Belanger (1987) argue that the psychological processes involved in the translation process. Comprehending the concept of multiple representations and moving from one mode of representation to another are important aspects as it demonstrates students' understanding of function (Moschkovich, Schoenfeld, & Arcavi, 1993). Gagatsis & Shiakalli (2004) added that this ability could enhance students' success in problem-solving particularly and mathematics education generally.

A sheer number of studies and attention are devoted to understanding the concept of function and its teaching strategies as well, yet specifically, domain and range function receive little to no attention in the research.

As understanding of domain and range function might contribute to a comprehensive understanding of function, thus perhaps this time should be spent talking about domain and range function. Through a review of the literature, there are several studies conducted to investigate students understanding of the concept of domain and range function. Most of the previous studies in the same field concentrated solely on single external representation, such as graphical representation (Cho & Moore-Russo, 2014; Cho, 2013; Martínez-Planell, Gaisman, & McGee, 2015) or symbolic representation (Dorko & Weber, 2014; Özkan & Ünal, 2009). However, a study concerning the flexibility of external representations of domain and range of functions is not yet investigated. Even though both internal and external representations interact and important to effective mathematics teaching and learning, in this study we focus on the external one. Besides, what is new in this study is that it involved pre-service secondary mathematics teachers. Understanding their flexibility of external representation of domain and range of the function is of value as it might help teacher educators to make an effort to refine pre-service secondary mathematics teachers' mistakes and misconception about the concept of domain and range of function.

METHOD

As the present study set out to investigate pre-service secondary mathematics teachers' flexibility of external representations of domain and range function, we collected data quantitatively using test administration. The participants of the study consisted of the thirty-eight pre-service secondary mathematics education department in one private university in Jakarta, Indonesia. They were selected conveniently for the study as they have taken a course on the concept under study, that is, differential calculus. As the course is offered during the first semester, hence they were those who were in the second, fourth, and sixth semester of their four-year secondary mathematics teacher education program. Twenty-three of them were females, and seven were males.

We developed a test of domain and range of functions understanding using multiple representations. The test consisted of thirty items, i.e. five items were statements' analysis, nine items were multiple-choice questions, and the rest were essay questions. Most of the items were developed specifically for this study by the researchers, and several of them were taken from items used in the previous study (Cho, 2013). Content and face validity of the instruments was confirmed by expert opinions. Two experts in mathematics education took part as validators.

It measured three aspects related to flexibility external representations of domain and range functions. The first aspect is composed of seven items assessed students' understanding of the concept of domain and range function. The first two items requested participants to explain the definition of domain and range of function using their own words. The other five items asked students to analyze and respond to presented statements regarding the concept of domain and range function. Participants could choose among 'I don't know', 'Incorrect', 'I doubt', and 'Correct' options. The second aspect consists of two items measured students' knowledge about interval. It asked students to translate information from line number into interval notation and vice versa. The reason behind involving knowledge of interval in the test was related to the fact that this knowledge plays a significant role in facilitating students in determining domain and range of function. The third aspects consisted of twenty-one items assessing students' translation ability across various representations of domain and range of function. One item is requested participants to translate information from the graph into interval notation. Five items are requested the participant to translate information from graph to set notation. One item asked participants to translate information from the graph into a two-set arrow diagram. Six items are about translating information from algebraic expressions into set notation. One item requires participants to select presented graphs of functions whose domain and range are the same. Two items are about the restriction of domain and range of function. Three items focus on determining possible values of domain and range of function. One item is devoted to having students translate information from algebraic expression into the ordered set. Finally, one item asked students to determine domain and range of function from two-set arrow diagrams. All participants were instructed to complete the task in 100 minutes.

RESULT AND DISCUSSION

The main purpose of the paper is to draw attention to investigate pre-service secondary mathematics teachers' flexibility of external representations in understanding domain and range of functions. This study highlighted conception held and difficulties encountered by pre-service secondary mathematics teachers as they attempted to address tasks related to domain and range of functions. The findings of this study were mainly based on the analysis of participants written responses gathered from test administration. Subsequently, it is categorized according to themes. There are four main themes observed and classified as described in Table 1.

Table 1. Participants' responses to the definition of domain and range of a function

The domain	The range
Referring to the definition of the function	Referring to the definition of the function
Origin Set	Result set
The input of a function or the value of x	The output value
Left set in a two-set arrow diagram	Right set in a two-set arrow diagram

Students' understanding of the definition of domain and range function

Various participants' responses to the definition of domain and range were categorized into the following classifications as shown in Table 1. The first typical response is that they attempted to describe it by referring to the definition of the function. The function is defined as a rule that relates to every member of one set with a member of another set (Downing, 2009). Based on this, they argued that domain is a member of one set that will be associated with a member of another set. Conceptually it is, of course, an acceptable definition. Nevertheless, using this explanation might be hard to comprehend immediate practical concerns or cases. A question such as, "What is the domain of $f(x) = 2x$?" could not be addressed with such explanation. Besides, the definition of function was also taken into account as an attempt to delineate the meaning of range.

Second, several participants claimed that the domain is defined as origin set, whereas the range is described as the result set. Unfortunately, no further explanation was given to describe their answers in detail. Such responses are also acceptable to some extent as it is likely to be failed when attempting to understand immediate practical cases. Also, it is prevalent in most Indonesian classrooms as the teachers tend to use the terms of domain and origin set as well as range and result set interchangeably.

Third, some participants argued that the domain of a function is illustrated as the input of a function or the value of x and range of the function is the output or the value of y. This response is almost close to the desired response. However, an explanation is of which values of x or values of y satisfy all the requirements for meaningful $f(x)$ is not addressed. Therefore, the statement is unsatisfactory. Meaningful $f(x)$ seems to be unnoticed aspects in teaching domain and range of function.

Fourth, several participants refer to the position of domain and range at a two-set of diagram arrow. At the beginning of function lesson, mathematics teachers in Indonesia tend to present a two-set of diagram arrow as a way to illustrate the concept of function. It is followed by a comprehensive explanation about domain, codomain, and range. Teachers, generally speaking, set the domain at the left side, whereas they set the co-domain and range at the right side. The way of how to determine the domain, codomain, and range of a function is by looking at where a member is located.

Based on the above responses, it appears that the majority of participants were not able to express the meaning of domain and range of function properly. Set of possible values of x to make meaningful $f(x)$ is an ignored notion. The finding of this study is in line with a study conducted by Elia, Panaoura, Eracleous, & Gagatsis (2007) who uncovered students' inconsistencies in constructing the definition of function. Their understanding seems to be influenced by teachers' articulation of such a concept. Besides, teaching processes which focus heavily on algebraic manipulation might contribute to this obstacle. Presenting the formula or the rule at the beginning of mathematics lesson is likely to be prevalent in most Indonesian classrooms rather than articulating definition. Aziz, Pramudiani, & Purnomo (2017) supported this finding in which they found that mathematics teachers do not seem to have the intention to guide students to express definition of algebra concept correctly.

Furthermore, the students' responses towards statements we provided about the domain and range of the function are presented in Table 2.

Table 2. Participants' responses to the statements

Item No	Statements	DK	DA	DB	AG	NR
4	The domain and range of a function corresponding to the y and x-axis of the Cartesian coordinate system.	1	28	4	3	2
5	Any real numbers divided by zero is zero.	0	29	0	7	2
6	Among real numbers set, the square root of any negative numbers does not exist.	1	2	3	30	2
7	$\sqrt{0} = 0$	0	1	1	34	2
8	The range of a function is determined by the function as well the domain.	0	0	4	30	4

Note: DK = I don't know; IN = I disagree; DB = I doubt; AG = I agree; NR = No Response

Students' knowledge about interval

In this study participants' knowledge of interval were evaluated. Albeit relatively neglected, comprehension of intervals is prominent as it might contribute to students' success in coming to grips with determining domain and range of functions. The students' response to determining the domain and range of the function is presented in Table 3.

Table 3. Participants' Responses to Determining Domain and Range of Function

Item No	Sub-items	Correct	Incorrect	No response
3	Domain	34	0	4
	Range	28	6	4
9	A	27	7	4
	B	21	13	4
	C	24	9	5
10	A	25	8	5
	B	26	7	5
	C	22	10	6
11	Domain	26	5	7
	Range	17	13	8
12	Domain	1	14	23
	Range	1	14	23
13	Domain	1	16	21
	Range	1	16	21
14	Domain	0	22	16
	Range	3	19	16
15	Domain	16	11	11
	Range	12	14	12
16	Domain	3	13	22
	Range	2	12	24
17	-	5	26	7
18	Domain	13	17	8
	Range	4	23	11
19	Domain	13	16	9
	Range	8	19	11
20	Domain	17	11	10
	Range	14	12	12
21	Domain	2	14	12
	Range	1	24	13
22	Domain	2	24	12
	Range	3	19	16
23	Domain	14	12	12
	Range	14	12	12
24	-	3	23	12
25	-	31	2	6
26	-	30	3	5
27	-	29	3	6
28	-	10	18	10

29	-	25	6	7
30	-	18	8	12

In the task, there are two sub-categories namely; translating information from real number line to interval notation (Item 9) and translating information from interval notation to real number lines (Item 10). Table 3 shows that most of them were able to deal with the task. They appear to have a robust knowledge of interval and recognize how to represent it in another form. However, several participants made a mistake by disregarding the order of the numbers in interval notation in which it should be written from the smallest to the largest.

Students' algebraic manipulation ability

Having capability of manipulating algebraic expression is necessary for determining domain and range of a function. Therefore, in the task we provided three items consisted of knowledge about division by zero, taking the square root of negative numbers, and taking the square root of zero. As shown in Table 2, it appears that most of them do not have a serious misconception. Nevertheless, when they are presented with a task, they could not capitalize on their algebraic knowledge. In other words, their knowledge is not by their performance when solving problems. It was reported in the literature that students might possess a high level of conceptual knowledge but lack procedural skill (Keating & Crane, 1990). Therefore, the interplay between conceptual and procedural knowledge or between knowledge and practice is important to address mathematical problems successfully.

Students' understanding of domain and range of functions through various representations

Understanding of the concept of a function includes the capability of moving from one representation to another representation, flexibility of using effective representation in solving a problem, and capability of discerning multiple representations when working on a function (Eisenberg & Dreyfus, 1994). Therefore, this study discloses participants' flexibility of external representation in an attempt to determine domain and range of functions. External representations used in this study are set of ordered pairs, two-set arrow diagram, graphical representation, symbolic representation, set notations, and interval notations.

Based on participants' responses, their difficulties are observed when attempting to translate information from symbolic and graphical representation to set notation or interval notation. The observed sources of these difficulties are (1) dominance on the integers; (2) special angles ;(3) poor algebraic manipulation; (4) expressing range in x; and (5) focusing on restriction on the Cartesian coordinate. Dominance on the integers tends to be a major cause of their inability to determine domain and range of function presented symbolically or graphically. On the contrary, when they are presented with multiple-choice items in the task having them select possible values substituted to the value of x as well as f(x), they do not get in trouble as the given choices are in the form of the integers. This dominance might lead students to put real numbers aside and discern continuous curve as discrete things. It seems that it is important for them to have a robust understanding of the properties of real numbers. Several researchers also found that students' mistakes when working with algebra are in consequence of the dominance of the integers (Almog & Ilany, 2012).

Besides, most participants do not have adversities in translating information from a two-set arrow diagram and ordered set. The reason might be that it capitalizes on the integers and they are in favour of working with it instead of real numbers. In the didactic process, most mathematics teachers also tend to use the integers frequently as examples. Therefore, in this task, they do not face a big challenge in locating the domain and range.

In this study, we also find that they consider that the limit set on the Cartesian coordinate is the end of restriction to the function. The main reason might be due to their ignorance about the meaning of arrow at the end of curved lines. Even though there is considerable research indicating the importance of graphical representation in teaching and learning process as it could aid students in looking at the concept or problems in distinct ways as, students' comprehension of the use of Cartesian coordinate needs to be improved. These results are in good agreement with another study which has shown that students often seem to concentrate on observed aspects of a graph instead of seeing the graph in its entirely (P. Cho & Moore-Russo, 2014). Abdullah (2010) also revealed students adversities using Cartesian graph. Working excessively on integer numbers might also contribute to this ignorance in which they only read integer coordinates.

Among functions presented, most of the participants had trouble determining domain and range of sine function presented symbolically and graphically. Works of literature have shown that trigonometry is perceived as one of mathematics topic in which most students undergo crucial adversities in learning (Gür, 2009; Kamber & Takaci, 2018; Orhun, 2001). The reason is due to that the topic of trigonometry lacks coherence in mathematics education. Focusing on how to present the topics so that students comprehend it meaningfully becomes a challenge for mathematics teachers.

Besides, item asked to translate information from graphical representation to a two-set arrow diagram was not able to be addressed by most participants. Participants' inaccurate understanding of the graphical representation of a function in Cartesian coordinate might impede them to catch the necessary information provided and then translate it to another representation such as two-set arrow diagram. Even though working with a two-set arrow diagram seems straightforward, it becomes more challenging when necessary information is not presented directly.

The findings of the data analysis assert that generally speaking participants indicates a lack of flexibility among external representations in an attempt to determine domain and range functions. The finding of this study converges with prior research showing (Bannister, 2014) that teachers could exhibit flexible, disconnected, or constrained conception. In this study, pre-service teachers tend to possess constrained conception in which they demonstrated the construct from one perspective and did not deal with various external representations.

Determining domain and range of function from two-set arrow diagrams

There is one item that asked students to determine domain and range of function represented by two-set arrow diagram. It seems that the majority of them did not encounter difficulty in coping with this item. As elements of each set were illustrated clearly, they assign the element into domain and range easily. This success could be supported by at least two possible reasons. Firstly, the elements of both sets are integer numbers which students tend to favor. Secondly, a two-set arrow diagram is the most familiar representation of function and relation concepts. At the beginning of the function lesson, most mathematics teachers present this diagram as well as showing its domain, co-domain, and range. On the contrary, few participants showed their inability to determine the range of function. The reason may lay in the fact that they were not able to make a clear distinction between co-domain and range of function.

Translating information from graph to set notation or interval notation

Participants' ability to translate information from graphical representation into interval notation or set notation was examined by six items. The items asked participants to determine the domain and range of function presented graphically and writes it in the form of set notation or interval notation. The graph of functions used in the task, to wit, linear function (Item 11), rational function (Item 12), the Sine function (Item 13), quadratics function (Item 14), square root function (Item 15), and arbitrary function (Item 16).

Based on Table 3, it seems that most participants tend to be able to cope with determining domain and range of linear function. However, most of them provided incorrect answers when working with a rational function, the sine function, quadratics function, as well as square root function. Besides, determining the range of the function is not as easy as determining the domain of the function. It is evident that most of them failed in determining the range of function.

There are four typical mistakes made compiled. The first is dominance on the integers. Several participants do not seem to get accustomed to working with real numbers. It is obviously clear when they attempted to translate information from graphical representations to set notation or interval notation. Therefore, when determining domain and range of the function given, they solely consider the integers and ignore other numbers. It seems that the participants do not have an accurate comprehension of the properties of real numbers.

The second is expressing range in x. Several participants express a range of function in terms of x instead of y or $f(x)$. It seems that it might be due to their carelessness or ignorance. Based on their responses to item 4 evaluated their understanding concerning the relation between domain and range of function and Cartesian coordinate system, most participants are likely to have an accurate understanding.

The third is focusing on special angles. When attempting to determine the domain of Sine function, most of the participants only consider special angles such as 30^0, 45^0, 60^0, 90^0, and so forth. Therefore, the obtained ranges from these angles are limited. The curve of Sine function is not discrete. The curve is smooth or continuous as it is defined for each real values of x. The reason may lie in the fact that mathematics teachers tend to overemphasize the use these special angles on various mathematical activities. Accordingly, non-special angles are put aside.

The fourth is focusing on restriction on the Cartesian coordinate. When analyzing participants written responses to several items associated with this category, it appears that they restrict their domain or range based on graphical limitation. They are not likely to grasp the meaning of arrow at the end of the curved line. Therefore, when determining domain or range of a function, several participants restricted it by considering where the Cartesian coordinate comes to an end.

Translating information from the graph into a two-set arrow diagram

There is one item asked participant to translate information from graphical representation to two-set arrow diagram. The type of question is multiple choices, and it seems rather straightforward. Nevertheless, most participants are not able to address the task. The presented function does not continue in which it consists of several open intervals. Most participants did not pay attention to this and considered it as a continuous function. Knowledge of interval is likely to contribute to participants' success in addressing the task.

Translating information from algebraic expressions into set notation or interval notation

There are six items requested participants to determine domain and range of functions represented by algebraic expression and express it in the form of set notation or interval notation. Each item in this category has its unique characteristics. There are four different sort of functions taken into consideration, to wit, rational function (Item 18), square root function (Item 19 and Item 22), a linear function (Item 20 and Item 23), and the Sine function (Item 21) (See Table 3).

According to the table, it seems that participants have puzzlement in dealing with these items. Compared with other tasks, tasks involving linear function (Item 20 and Item 23) possessed a high tendency to answer correctly although no more than half of them succeeded in dealing with it. It appears that linear function is the most straightforward facet of function. Besides, working with trigonometric functions is still a challenge for them as only two out of them managed to address it correctly. Also, determining range seems to be more difficult than determining the domain of the function.

There are similar four typical mistakes made by them when translating information from algebraic expressions into set notation or interval notation, to wit: dominance on the integers, focusing on special angles, error in algebraic manipulation, and writing range in terms of x.

Selecting graphs of functions whose domain and range are the same

Item 24 asked participants to select three out of six graphs of functions whose domain and range are the same. Only a few of them could deal with it. Inability to catch information from the graph tends to be a major cause of their difficulty.

Restriction on the domain and range of function

There are two items included in this category. The first item (Item 28) requested participants to determine the range of function whose domain was restricted at first. The second item (Item 30) asked the participant to select one out of four graphs of functions represented the function whose domain which was restricted. Table 3 indicates participants' responses to these two items. No more than half of the participants were able to provide correct responses to these items. It seems that restriction on domain and range is an unfamiliar topic for them as it is not introduced widely in the high school mathematics curriculum. Conceptually, participants' knowledge of restriction on domain and range has been evaluated on item 8 and most of them were able to give a favorable response.

Determining possible values of domain and range of function

Three multiple-choice items were designed to ask participants determining presented values of domain and range of functions (Item 25, Item 26, and Item 27). The functions are square root functions, rational functions, and combination between rational and square root function. Based on Table 1, the majority of the participants were able to select correct choices. As the options were in the form of integer numbers, thus they do not face any adversities.

Translating information from algebraic expression into an ordered set

The last category in the task is to have participants translate information from algebraic expression into the ordered set. In other words, participants were asked to determine domain and range of function represented as algebraic expression and express it in the form of the ordered set. As the item is multiple choices, most of them can deal with it.

CONCLUSION

The objective of this study is to investigate pre-service secondary mathematics teachers' flexibility of external representations of domain and range of functions. From the study that has been carried out, it is possible to conclude that participants exhibit inconsistencies in constructing a definition of domain and range of function, lack of flexibility among external representations of approaching domain and range of function, and inability to determine domain and range of functions.

The findings of the present study have several notable implications for classroom instructions. The topic of domain and range of function should not be disregarded by teachers. As a topic of domain and range are introductory in the topic of function, review of interval algebraic manipulation including the use of multiple representations seems to be appropriate and prominent before continuing to subsequent topics. Besides, teaching and learning approach to introducing the concept of domain and range functions needs to be improved. This research was concerned with secondary pre-service mathematics teachers attending courses at university; however, the results should be applicable also to teacher educators, in-service mathematics teachers, and high school students as well. To sum up, the implications of the study's findings could be considered with the purpose to promote pre-service secondary mathematics teachers' understanding of the concept of domain and range of function.

The present study adds to the paucity of studies on pre-service secondary mathematics teachers' understanding of the domain and range in multiple representations, providing insights that correspond with the previous study on the topic. Further study of the issue is still required. In consideration of this study, it appears interesting to conduct subsequent research using an in-depth interview to gain a deep comprehension of how pre-service secondary mathematics teachers' difficulties in understanding and determining domain and range. It could enrich our knowledge about the concept examined in the present study. Besides, further research will be required to observe the way how mathematics teachers or teacher educators introduce the concept of domain and range.

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