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TRIGONOMETRIC CONCEPTS: PRE-SERVICE TEACHERS' PERCEPTIONS AND KNOWLEDGE

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Abstract

This paper explored pre-service teachers' perceptions and knowledge of trigonometric concepts. Convenience sampling technique was used to select a sample of 119 (25 females, 94 males) second year science/mathematics pre-service teachers from two Colleges of Education in the Northern Region of Ghana. Trigonometry Perception Questionnaire (TPQ) and Trigonometry Assessment Test (TAT) were used to collect data on pre-service teachers' perceptions and conceptual knowledge of trigonometry. Data were coded and keyed into Statistical Package for Service Solutions (SPSS version 20) and analysed using descriptive statistics. The results suggested that pre-service teachers perceived trigonometry as abstract, difficult and boring to learn; and had limited conceptual knowledge of basic trigonometric concepts. Consequently, more that 50% of them were unable to construct and reconstruct the appropriate mental structures for meaningful understanding to enable them respond to important basic trigonometry tasks. To achieve quality mathematics education, teacher educators must change their instructional practice and teach for understanding. Since understanding is the key to teacher's instructional actions, processes and knowledge, we recommend teaching trigonometry for understanding during teacher preparation.

Keywords: Trigonometric concepts, Pre-service Teachers, Teachers perceptions, Teacher knowledge

Abstrak

Penelitian ini mengeksplorasi persepsi dan pemahaman calon guru terhadap konsep trigonometri. Teknik *convenience samping* digunakan untuk memilih sampel sebanyak 119 calon guru sains/matematika tahun kedua (25 perempuan, 94 laki-laki) dari dua Sekolah Tinggi Pendidikan di Wilayah Utara Ghana. *Trigonometry Perception Questionnaire* (TPQ) dan *Trigonometry Assessment Test* (TAT) digunakan untuk mengumpulkan data tentang persepsi dan pengetahuan konseptual calon guru tentang trigonometri. Data dikodekan dan dimasukkan ke dalam Statistical Package for Service Solutions (SPSS versi 20) dan dianalisis menggunakan statistik deskriptif. Hasilnya menunjukkan bahwa calon guru menganggap trigonometri itu abstrak, sulit, dan membosankan untuk dipelajari; serta memiliki pengetahuan konseptual yang terbatas tentang konsep trigonometri dasar. Akibatnya, lebih dari 50% dari mereka tidak dapat membangun dan merekonstruksi struktur mental yang tepat untuk pemahaman yang bermakna sehingga memungkinkan mereka memberikan respon terhadap tugas trigonometri dasar yang penting. Untuk mencapai pendidikan matematika yang berkualitas, guru/pendidik harus mengubah kegiatan pembelajaran dan mengajar mereka agar mudah dipahami. Karena pemahaman adalah kunci untuk kegiatan pembelajaran, proses, dan pengetahuan guru, peneliti menyarankan untuk mengajar trigonometri dengan pemahaman konsep yang kuat selama program persiapan calon guru berlangsung.

Kara Kunci: Konsep trigonometri, Calon guru, Persepsi guru, Pengetahuan guru

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Trigonometry is a branch of physical mathematics that deals with the understanding of concepts and their applications. Its content areas include angles, measurement of angles, triangles and their relationships (Rizkianto, Zulkardi, & Darmawijaya, 2013; Ahamad, et al. 2018). It blends geometric, graphical, and algebraic reasoning that provides a space for making sense when solving problems involving triangles, trigonometric expressions and graphs. Trigonometry has application in

Geography and Astronomy and is widely used in other fields of study such as Electricity, Cartography, Geometry, Maritime, Optics and Physics (Weber, 2005; Tuna, 2013). In Ghana, Trigonometry is one of the six content domains delineated in the Senior High School (SHS) mathematics curriculum. It covers trigonometric concepts, processes and their applications to problem solving (Ministry of Education, 2010). Conceptual understanding of trigonometric ideas at the SHS level provides the foundation for meaningful learning of mathematics at the Colleges of Education.

The content of trigonometry in the College of Education (CoE) curriculum includes trigonometric ratios of angles, inverse trigonometric functions, bearings, maxima and minima of trigonometric functions and their graphs, solutions to simple trigonometric equations, trigonometric identities, compound angles; $sin(A \pm B)$, $cos(A \pm B)$, $tan(A \pm B)$; and their applications (Ministry of Education, 2007). Cognisant of the fact that the student must understand, make sense of, and benefit from the numerous advantages trigonometry offers, the curriculum placed emphasis on constructivist's strategies such as group work, project work, discussion, and discovery learning. It further stressed on the development of concepts through practical activities that rely on concrete materials in developing the relevant cognitive skills for meaningful problem solving. This instructional paradigm is grounded in the action-process-object-schema (APOS) theory.

According to the APOS theory (Asiala, et al. 1996), a learner must have sound mental structures to make sense of a given mathematical situation. These mental structures are the likely actions, processes, objects and schemas required to learn a concept. For a given concept, the theory requires that the likely mental structures be detected, and the appropriate learning activities designed to support the construction and reconstruction of these mental structures. Dubinsky (2010) asserts that the APOS theory and its application to classroom practice are based on two key assumptions: (1) the mathematical knowledge of an individual is the ability to respond to perceived mathematical problem situations and their solutions by either constructing or reconstructing mental structures to deal with the situations; and (2) an individual does not learn mathematical concepts directly but rather applies the mental structures to make sense of the concepts or situations. Consequently, learning trigonometry is facilitated if the individual possesses the required mental structures for the concepts. In the absence of the mental structures, responding to conceptual problem situations and understanding trigonometric concepts and their application may be almost impossible.

Understanding trigonometric concepts is a requirement for advanced studies in survey, physics, architecture, and other branches of engineering. Understanding trigonometry serves as a tool for enhancing students' cognitive skills and provides a framework for coordinating concepts (angles, measurement of angles and lengths, shapes and similarity, vectors, polar coordinates and parametric curves). Trigonometry in the curriculum is, therefore, a fitting ground for exploring, connecting and relating mathematical ideas, and for meaningful combination of different scientific disciplines.

Research on Trigonometry

Weber (2005) compared two groups of students taught in two different ways- the lecture-based and experimental-based instruction paradigms. The results indicated that students taught in the lecture-based classroom developed limited understanding of trigonometric functions while those who received the experimental-based instruction developed a deep understanding of the concepts. Students who received lecture-based instruction were generally unable to justify why trigonometric functions had properties as they are. They were also unable to form reasonable estimates for trigonometric functions. Interviews with participants affirmed that students who received lecture-based instructions did not understand trigonometric expressions as procepts (the ability to think of mathematical operations and objects) (Gür, 2009), which covers both concept and process. Based on the meaningful learning theory and the conceptual field theory, Klein (2015) studied the knowledge students possessed about trigonometry that enabled them to comprehend related concepts. The findings showed that identifying students' previous knowledge and making explicit the knowledge-in-action resulted in attitudinal change. Similarly, Gür (2009) explored the types of errors, misconceptions, and the obstacles that occur in trigonometry among grade 8 students. The study indicated that students made serious conceptual errors, had varied misconceptions and experience numerous obstacles in learning trigonometry. The errors and misconceptions were based on underlying obstacles. One fundamental obstacle was that trigonometry and its related concepts were abstract and non-intuitive.

Although students may have difficulties with angles in degrees, Akkoç and Akbaş-Gül (2010) indicated that students' conceptions about trigonometry is mostly based on angles in radians. Students are unable to develop strong knowledge of radians to be able to relate them to real numbers as the domain of trigonometric functions. To address student difficulties with radians, research (Akkoç & Akbaş-Gül, 2010; Moore, 2012) suggests teaching the concept by explicating their connections to arcs for easy understanding. Defining trigonometric functions primarily in relation to arc lengths could help students overcome their learning difficulties with the radian concept.

The studies reviewed suggest that students have problems with the concept, process, and procept in learning trigonometry. These problems stem from students' personal epistemologies of mathematics - perceptions of/beliefs about mathematics learned and experienced in school. Apart from informing the problems associated in learning, the results also suggest possibilities of teaching trigonometry in ways that can result in perceptual change.

Perception and Performance in Mathematics

Cherry (2017) defines perception as the sensory experience of a person which involves recognizing environmental stimuli and responding to it. It includes the physical senses and the cognitive processes for interpreting the senses. Perception does not only create a person's experience but also allows the person to act within the environment. The individual constructs meanings by sensing objects as they engage in daily activities and the sensations lead to percepts. These percepts are conditional, in the same sense as mathematical hypotheses are provisional. As we acquire new information, our percepts change. Beliefs, which are used as they appear, represent the things we do in life without question. Beliefs comprise the values that are perceived but related to a method or mode of thinking (Mutodi & Ngirande, 2014). Perception is a component of what we do based on our beliefs. Thus, students' perception of trigonometry depends on what they believe and think about trigonometry. Due to limitations in what students are able to perceive, they are highly selective in choosing what to perceive based on its relevance to them. In the process of filtering, different people will respond in different ways and manners even when they are subject to the same conditions. Different people will always have different experiences and abilities of perceiving objects and hence their perceptions. Perceptions therefore relate to the way we act or react. The way and manner we think (perceive) determines our actions and such actions are a function of our perceptions (Kakraba, Morkle, & Adu, 2011).

One of the significant assumptions of the constructivist view of perception is that perception is not predicted exclusively by external stimuli. As a result, it is presumed that motivational state, emotional state, culture as well as one's expectation have influence on one's perceptual perception (Adegboye, 2000). There is always an inclination to perceive certain features of a stimulus. The inclination to perceive is influenced by various factors. The set of factors that influence perception is termed the perceptual set. A student's previous experience with trigonometry and the meaning that is made with new concepts can make the student to think that trigonometry is rigid to understand.

Student's difficulties with angle and angle measure, which are the starting point for learning trigonometry in our curriculum, are the most basic problems inhibiting the depth of understanding of trigonometric concepts (Cetin, 2015). Research (Gür, 2009; Cetin, 2015) indicate that although some students have perceived knowledge of angle and arc, they could not use their perceptual knowledge in the construction of angles. Cetin (2015) then concluded that although students had visual image of the concept of an angle, they had not truly studied the basic concepts of trigonometry. These findings suggest that knowledge of conceptual structures is different from conceptual understanding. Considering the importance of trigonometric concepts in everyday life and the challenges students face in understanding them, it is essential to explore pre-service teachers' perceptions of the basic concepts they are prepared to teach.

Statement of the Problem

Research studies (Weber, 2005; Moore, 2012; Gür, 2009; Cetin, 2015) revealed that students have difficulties in trigonometry. These difficulties emanate from a number of factors including; lack of motivation, abstractness of trigonometric concepts, lack of understanding of fundamental concepts, and students' inability to connect concepts in trigonometry. Several studies have focused on students' perceptions and understanding of trigonometry. For instance, Weber (2005) explored students' understanding of trigonometry, whilst Wongapiwatkul, Laosinchai and Panijpan (2011) investigated factors that enhance students' conceptual understanding of trigonometry using earth geometry and the great circle. Tuna (2013) also investigated prospective mathematics teachers' conceptual knowledge of degree and radian. Despite the numerous studies in trigonometry available, there is limited research on pre-service teachers' perceptions of trigonometric concepts at the Colleges of Education in Ghana. Hence, the study was designed to investigate pre-service teachers' perceptions and knowledge of trigonometric concepts in Ghana to bridge the existing research gap.

Research Questions

As perception is central to a student's acquisition of knowledge in teacher preparation, the study was guided by the following questions:

1. What are pre-service teachers' perceptions of trigonometric concepts?
2. How knowledgeable are pre-service teachers in basic trigonometric concepts?
3. What are pre-service teachers' conceptual difficulties in trigonometry?

Purpose of the Study

Pre-service teacher perceptions lie at the heart of teaching (Fajet, Bello, Leftwich, Mesler, & Shaver, 2005). This study was undertaken to explore pre-service teachers' perceptions and knowledge of trigonometric concepts. Examining and documenting pre-service teachers' perceptions, and knowledge of trigonometric concepts can contribute to a better understanding of how they view teaching geometry concepts, what they have for classroom practice as well as an assessment of what they need to acquire to become competent mathematics educators. The study sought to add to the body of knowledge on pre-service teacher's perceptions, knowledge, and understanding of trigonometric concepts. Findings can provide clear guidance and direction for future researchers in their study and to policy makers in making curriculum and teacher development policy decisions. As perceptions have the tendency of shaping the way people learn, results of the study can have significant influence in determining how individuals organise and define their tasks. Although Ghana operates a national pre-university mathematics curriculum, pre-service teachers experience trigonometric concepts in the different contexts. Consequently, a study of their perceptions and knowledge is an assessment of how they interpret trigonometric concepts in their teacher education through the lenses of their prior experience – how they were taught trigonometry, and how they learned trigonometry – to make sense of it.

METHOD

Research Design

The study was a descriptive research designed to explore pre-service teachers' perceptions, conceptual knowledge, understandings and difficulties with trigonometric concepts. Consequently, descriptive tools were used to explore second year science/mathematics pre-service teachers thinking, basic conceptual knowledge, understandings and challenges in solving problems on trigonometry.

Participants

Convenience sampling technique was applied to select 119 second-year pre-service teachers (25 females and 94 males) from two colleges of education in the Northern Region of Ghana. The choice of second year pre-service teachers was based on the fact that the year group had studied enough content and pedagogy and ready to put what they have learnt into practice. This group of pre-service teachers was scheduled to start teaching practice on various subjects including mathematics at the basic level. This made it imperative to explore their perceptions, knowledge, understandings, and difficulties in trigonometric concepts they were expected to teach.

Research Instruments

The Trigonometry Perception Questionnaire (TPQ) and the Trigonometry Assessment Test (TAT served as the data collection tools. The TPQ had 12 items put into two sections. Section 1consisted of four items seeking the biographic information on each respondent. Section 2 was made up of eight Likert scale type items on a five-point scale that sought participants' perceptions on trigonometry. The Likert scale type items offered pre-service teachers the opportunity to rate their thinking or opinions about trigonometric concepts. The TAT had 10 open-ended questions that provided participants a window of opportunity to express their personal understanding of trigonometric concepts. Respondents constructed their own responses to show their knowledge and application of basic trigonometric concepts. Participants completed the test within an hour.

Data Analysis

The Trigonometry Assessment Test (TAT) yielded both quantitative and qualitative data. Data from the TAT were analyzed thematically to reflect the research questions. Themes were identified, coded and fed into the Statistical Package for Service Solutions (SPSS) so that the frequency count and the corresponding percentage on every task on the questionnaire were examined. Based on pre-service teachers' ability to work different problem types, their level of knowledge according to the action-process-object-schema (APOS) framework was determined (Asiala et al., 1996). APOS was most appropriate to analyze student perception and understanding of the concept of trigonometry because of the theory's ability to describe student understanding on different levels of conceptual understanding. By describing and coding the data based on the action, process, and object levels of understanding, a very specific methodology was devised to interpret student performance. Document analysis was also applied to participant constructed responses to identify the nature of their difficulties. Finally, data from the five (5) point Likert scale type perceptions questionnaire were analyzed using SPSS.

RESULTS AND DISCUSSION

The results of the study were organized under themes based on the research questions. The questions were on pre-service teachers' perceptions, knowledge, and conceptual difficulties of trigonometric concepts.

Pre-service Teacher's Perceptions of Trigonometric Concepts

To explore pre-service teachers' perceptions of trigonometric concepts, eight Likert Scale type statements were provided for participants to indicate their level of agreement to the statements. Frequency counts of respondents' level of agreement to each statement were determined and converted into percentages as shown in Table 1.

The pre-service teachers in the present study had varying positive and negative perceptions of trigonometric concepts. Positively, trigonometric concepts were perceived to increase their reasoning and analytic capacity and their academic success is a function of understanding them. On the negative side, they saw the concepts to be abstract, rigid and boring to learn. These attributes affected their performance in mathematics. From Table 1, equal proportions of the participants (83.9%) indicated that trigonometry was too rigid and abstract, and boring while 77.3% agreed that trigonometry was a difficult topic. A majority of the respondents (88.3%) were of the view that trigonometry increased their reasoning and analytical power. Similarly, 82.4% indicated that their perceptions of the trigonometry affected their performance but less than half the number of participants (34.5%) agreed that it should be removed from the mathematics curriculum.

Table 1: Pre-service Teacher's Perceptions of Trigonometry (N =119)

Statement	SD f (%)	D f (%)	U f (%)	A f (%)	SA f (%)
Trigonometry is a difficult topic in mathematics.	6 (5.0%)	20 (16.8%)	1 (0.9%)	55 (46.2%)	37 (31.1%)
Trigonometry is a boring topic.	5(4.2%)	14 (11.8%)	1 (0.8%)	29 (24.4%)	70 (58.8%)
Trigonometry is too rigid and abstract.	3 (2.5%)	7 (5.9%)	10 (8.4%)	58 (48.7%)	41 (34.5%)
My academic success does not depend on understanding trigonometry.	17(14.3%)	30 (25.2%)	6 (5.0%)	26 (21.9%)	40 (33.6%)
Trigonometry should be removed from the curriculum	30 (25.2%)	48 (40.3%)	0 (0.0%)	26 (21.9%)	15 (12.6%)
Studying trigonometry increases my reasoning and analytical power.	2 (1.7%)	7 (5.8%)	5 (4.2%)	44 (37.0%)	61 (51.3%)
My perception about trigonometry depends on its usefulness in my daily activities.	20 (16.8%)	22 (18.5%)	3 (2.5%)	24 (20.2%)	50 (42.0%)
My perception about trigonometry has affected my mathematics performance.	16(13.4%)	5(4.2%)	0(0.0%)	48(40.4%)	50(42.0%)

Note: SD—Strongly Disagree, D—Disagree, U—Unsure, A—Agree, SA—Strongly Agree.

Pre-service Teachers' Knowledge of Trigonometric Concepts

To assess pre-service teachers' knowledge of trigonometric concepts, a 10-item Trigonometry Achievement Test was administered. Twenty-five (25) participants were not available to take the test as they were either sick, at a workshop, or engaged by authorities. Constructed responses of 94 out of the 119 participants who took the test were analysed and presented in Table 2.

Question 1, which explored respondents' basic knowledge in graphing trigonometric functions, required respondents to sketch the graph of a sine function indicating the period, domain, range, y-intercept, symmetry, maximum and minimum points, and increasing and decreasing intervals. Table 2 indicates almost all the participants were unable to solve all the sub items of the questions. Generally, the tasks seemed difficult for most pre-service teachers. Specifically, only 10% of the teachers could draw the sine function and indicate the domain, range and period. Less than 10% of participants could draw the graph of the sine function and indicate the other basic features (namely the maximum and minimum points, the line of symmetry, y-intercept and the ranges at which it is increasing or decreasing). This suggests that the pre-service teachers in the present study had action knowledge on the graph of trigonometric functions but lacked a process, object and schema knowledge on the graph of the sine function.

Questions 2 and 3 which required pre-service teachers to briefly explain why $\sin^2 x + \cos^2 x = 1$ and $tanx \times cotx = 1$, tested understanding of the basic identities. Table 2 indicates that none (0%) of the respondents were able to explain why the fundamental identity $\sin^2 x + \cos^2 x = 1$ holds while 50% were able to correctly establish the relation between the two trigonometric objects, $\cot x$ and $\tan x$. They could define $\cot x$ as a ratio of 1 and $\tan x$ to explain the relation.

Question 4 asked pre-service teachers to calculate the value of $\tan 90^{\circ}$ without using tables or calculator while question five examined the pre-service teachers' knowledge on simplifying trigonometry ratios. About half the number of the respondents correctly initiated the process by defining $\tan 90^o$ in terms of $sine$ and $cosine$ but only 19.1% were able to arrive at the correct solution. Similarly, 28.7% correctly initiated the process of defining cos x and sin x in item 5 and ended up with 21.3% arriving at the correct response. This suggests that 42.6 % of the respondents had an action knowledge of the item since they could express $\tan 90^o$ as a ratio of $\sin 90^o$ and $\cos 90^o$. However, 19.1% of the respondents had process, object and schema knowledge of the question. In the same vein, 28.7% teachers had action knowledge of task 5a but 21.3% had the overall process, object and schema knowledge on item 5.

Question 6 required pre-service teachers' knowledge of relationship between the sides of right-angled triangle and the acute angles in the triangle to solve problems on trigonometric ratios. Table 2 indicates that 61.7% drew a right-angled triangle to illustrate the solution process but only 36.2% correctly arrived at the solutions. The results for this item suggested that pre-service teachers had better knowledge in solving trigonometric ratios using the right-angled triangle. This demonstrates that they had very good action, process, object, and schema knowledge of trigonometric ratios.

Question 7 required respondents' knowledge and understanding of the equivalence of $\cos(-x)$ whiles question eight required their understanding of why tangent and cotangent functions are positive in the third quadrant with explanations. The results indicated that although 43.6% of pre-service teachers provided the correct answer to Question 7, only 35.1% provided the correct explanation in task 7a. Similarly, only 19.1% could successfully explain the sign conventions as required in item 8a. The results that only 35.1% and 19.1% could tackle tasks 7a and 8a respectively suggest few pre-service teachers knew the concepts and fewer understood what they know.

Question 9 tested participants' knowledge in expanding the compound angles and their understanding of how to apply their knowledge of the expansion to compute angles without using mathematical Tables or calculators. Question 10 also examined pre-service teachers' ability to derive the sine and cosine rules. Table 2 indicates that 60.6% and 35.1% successfully expanded $cos\ (A+B)$ and $sin\ (A+B)$ respectively. However, 30.9% could apply their expansion to initiate the computation of $cos\ (120^o)$ with 30.9% arriving at the correct answer. Similarly, 41.5% could apply their expansion to initiate the computation of $sin(75^o)$ with 29 (30.9%) arriving at the answer. As shown in Table 2, 45.7% and 37.2% were able to derive the *cosine* and *sine* rules respectively. The results suggested that about two-fifths of the pre-service teachers had good action knowledge on expansion and derivation of the sine rule. However, only about one-third of the respondents had the process, object, and schema knowledge on the expansion to compute the values of $cos(120^o)$, and $sin(75^o)$. Table 2 further shows that 37.2% of the participants were able to derive the *sine* rule while 45.7% could manage with the *cosine* rule. This suggests that less than 50% of the participants had action, process, object, and schema knowledge to derive the rules.

Table 2. Pre-service Teachers' Responses to Trigonometry Assessment Test (TAT) (n = 94)

	Questions	Incorrect f (%)	Correct f (%)
1.	Sketch the graph of a sine function	84 (89.4%)	10 (10.6%)
	a. Indicate the domain	84 (89.4%)	10 (10.6%)
	b. Indicate the range	84 (89.4%)	10 (10.6%)
	c. Indicate the period	86 (91.5%)	8 (8.5%)
	d.Indicate the y – intercept	86 (91.5%)	8 (8.5%)
	e. Indicate the maximum points	86 (91.5%)	8 (8.5%)
	f. Indicate the minimum points	86 (91.5%)	8 (8.5%)
	g. Indicate the symmetry	86 (91.5%)	8 (8.5%)
	h. Indicate the increasing interval	86(91.5%)	8(8.5%)
	i. Indicate the decreasing interval	86(91.5%)	8(8.5%)
2.	Please briefly explain in details why $\sin^2 x + \cos^2 x = 1$	94(100.0%)	0(0.00%)
3.	Show that $\tan x\ \times \cot x = 1$. And briefly explain.		
	a. Define $cot\ (x) = \frac{1}{tan\ (x)}$	80(85.1%)	14(14.9%)
	b. Substitute and compute the answer	36(38.3%)	58(61.7%)
	c. State that a number multiplied by its inverse is equal to 1	47(50.0%)	47(50.0%)
4.	Without using a table or a calculator what is the value of $\tan 90 =$?. Please explain.		
	a. Define $\tan 90° = \frac{\sin 90°}{cos 90°}$	54(57.40%)	40 (42.6%)
	b. Show the value for $\sin 90°$ dan $\cos 90°$	55(58.5%)	39 (41.5%)
	c. Substitute and state that the result is indeterminate	76(80.9%)	18 (19.1%)
5.	Simplify $(\sin x)\ (\cos x)$	74(78.7%)	20(21.3%)
	a. Define $\sin x\ a$ and $\cos x$	67(71.3%)	27(28.7%)

	Questions	Incorrect f (%)	Correct f (%)
	b. Expand these functions	74 (78.7%)	20 (21.3%)
	c. Simplify and state the result	74 (78.7%)	20 (21.3%)
6.	Given $\tan x = \frac{4}{5}$ establish a relation for cosec (csc) using a right-angled triangle?		
	a. Draw a right angle triangle to illustrate question and label it	36 (38.3%)	58 (61.7%)
	b. Using Pythagoras formula compute the value of the third side	36 (38.3%)	58 (61.7%)
	c. From the right angle triangle compute $sin\ (x)$	37 (39.4%)	57 (60.6%)
	d. Define $cosec(x) = \frac{1}{sin(x)}$	60 (63.8%)	34 (36.2%)
7.	$\cos(-x) = ?$ Please explain your answer.		
	a. Statement of answer.	53 (56.4%)	41 (43.6%)
	b. Explanation of answer.	61 (64.9%)	33 (35.1%)
8.	Why are the tangent and cotangents functions positive in Region III of the unit circle? Please explain.		
	Ability to explain results	76 (80.9%)	18 (19.1%)
9.	a. Expand $\cos(A + B)$	37 (39.4%)	57 (60.6%)
	b. Expand $sin(A + B)$	61 (64.9%)	33 (35.1%)
	a.i. Express $cos\ (120°)$ as $cos\ (30° + 90°)$	68 (72.3%)	26 (27.7%)
	a.ii. Substitute and compute for $cos\ (120°)$ into the above expansion	65 (69.1%)	29 (30.9%)
	b.i. Express $sin(75°)$ as $sin(30° + 45°)$	55 (58.5%)	39 (41.5%)
	b.ii. Substitute and compute $sin(75°)$	65 (69.1%)	29 (30.9%)
10.	Derive the $sine$ and $cosine$ rules respectively		
	a. Ability to derive the sine rule.	59 (62.8%)	35 (37.2%)
	b. Ability to derive the cosine rule	51 (54.3%)	43 (45.7%)

Pre-Service Teachers' Conceptual Difficulties in Trigonometric Concepts

Teachers' conceptual difficulties influence the way they teach in the classroom. To identify the pre-service teachers' conceptual difficulties in trigonometric concepts, their self-constructed responses to the test items were analysed. Analysis of the responses indicated that pre-service teachers generally had difficulties in almost all the basic trigonometric concepts they are being prepared to teach. Their difficulties were reflected in their performance in Table 2.

Table 2 further shows that apart from sub questions 3b, and 6a, b, and c, more than 50% of the pre-service teachers could not solve the tasks tested. Their greatest difficulty was in explaining why the trigonometric identity $\sin^2 x + \cos^2 x = 1$ holds as none could explain this identity. The second most difficult concept for the pre-service teachers was in graphing the sine function. Table 2 indicates that more than 89% of the respondents could not state the domain and range, while over 91% could not find the intercepts, line of symmetry, the maximum and minimum points, and determine the intervals for which the function is increasing and decreasing to enable them graph the sine function. Similarly, the respondents had difficulty explaining the sign conventions of trigonometric ratios in the various quadrants as almost 81% could not state the signs of the tangent and cotangent functions in the third quadrant. Analysis of respondents' explanations of why $\sin^2 x + \cos^2 x = 1$, $tanx \times cotx = 1$, and procedures in graphing the sine function indicated that many pre-service teachers had conceptual difficulties with trigonometry relations. Hence, they could neither explain the identity nor apply the relationships to prove the identity.

Summary Results

This study explored pre-service teacher's perceptions and knowledge of trigonometric concepts. The study identified four main findings. First, even though the pre-service teachers viewed trigonometric concepts to be abstract and boring, they perceived it to increase their reasoning and analytic capacities. Second, most pre-service teachers lacked the basic knowledge of trigonometric concepts. The few that knew the concepts did not understand them. Third, most pre-service teachers could not draw the graph of the basic sine function and indicate their features. Finally, most pre-service teachers had conceptual difficulties with trigonometric relationships a situation that limits their ability to explain basic trigonometric identities or apply the relationships in proving basic trigonometric identities.

Discussion

Generally, pre-service teachers' perceptions of trigonometric concepts varied from task to task. Although their perceptions varied by task, most of them had negative perceptions of trigonometric concepts. Trigonometry is perceived as rigid, abstract, difficult and boring to learn. This negative perception is attributed to teachers' not taking time to teach for understanding. Rather they rush through the curriculum to cover the content. The press to cover the content often results in superficial or rote understanding. Students appear to learn the trigonometric concepts in isolation which affects their ability to establish connections between them. This finding is consistent with Wasike, Ndrurumo and Kisili's (2013) conclusion that secondary school students had positive and negative perceptions about mathematics.

Knowledge of trigonometric concepts and their relationships are the basis for understanding trigonometry (Tuna, 2013). Students' mathematical understanding is demonstrated by their performance and application of their knowledge in solving problems. Solving mathematical problems involves the deployment of knowledge into organizing one's mental structures that include actions, processes, objects and schema. Generally, the results show that most pre-service teachers are deficient in their action-process-object-schema knowledge (Asiala, Brown, DeVries, Dubinsky, Mathews, & Thomas, 1996) which limits their ability to actively deal with problems involving trigonometry. In effect, they experience difficulty in simplifying, expanding, and applying basic trigonometric concepts and identities in solving problems. Although the sine and cosine rules are fundamental concepts in trigonometry, less than 38% and 44% were able to derive the sine and cosine rules respectively. Similarly, apart from the test item on expanding $cos(A+B)$ that 57% of the teachers were able to handle, over 50% could not solve any of the items correctly. Specifically, 89.4% of the respondents could not sketch the graph of the sine function; 78.7% could not simplify $sinxcosx$ and none could explain why $sin^2x+cos^2x=1$. This suggests that most pre-service teachers had little conceptual knowledge and understanding of trigonometric functions and identities.

Although majority of the participants in the present study could expand $cos\ (A+B)$, it is interesting to observe that over 72% of pre-service teachers could not apply their knowledge on the expansion to compute $cos\ (120)^o$. They could neither see the connection between $cos\ (A+B)$ and $cos\ (30^o+90^o)$ nor between sin (A+B) and $sin(30+45)^o$. These findings suggest that the pre-service teachers lacked conceptual knowledge to simplify, apply and explain basic trigonometric concepts. Essentially, the pre-service teachers had little process, object and schema knowledge on the expansion since their ability to compute $cos\ (120)^o$ using their own expansions were problematic. If the knowledge of the basic concepts and their relations are fundamental to understanding trigonometry, then the present findings suggest pre-service teachers' conceptual understanding is weak. The weak conceptual understanding of the pre-service teachers in trigonometry might be due to their negative perceptions since perceptions positively relate to performance in mathematics (Wasike, Ndurumo, & Kisilu, 2013)**.**

Another finding was that most pre-service teachers seemed to have a clear conceptual understanding of the relationship between cotangent and tangent as demonstrated in their action-process-object-schema on the task. However, majority could neither explain why the tangent and cotangent functions are positive in the third quadrant nor relate trigonometric ratios in a right-angled triangle to explain why $sin^2x+cos^2x=1$. The relational difficulties in most of the tasks suggest that the participants did not understand conceptual relationships in trigonometry. In effect, most pre-service teachers in the present study had instrumental understanding of the trigonometric concepts in the curriculum.

CONCLUSION

Overall, the findings in this study suggest that most of the pre-service had limited conceptual knowledge of basic trigonometric concepts. Although participants valued trigonometric concepts, they perceived trigonometry to be too abstract and difficult. Consequently, most of them were unable to construct and reconstruct the appropriate mental structures for meaningful understanding to enable them respond to important basic trigonometry tasks. The pre-service teachers' difficulties suggest knowledge gap and limited understanding of conceptual relations of trigonometry. We therefore conclude that to achieve the goal of quality mathematics education, tutors in the colleges of education in Ghana need to change their instructional practice to develop pre-service teachers' understanding of the trigonometric concepts. This study was limited to only two selected colleges of education. However, the findings have significant implications for mathematics teachers in Colleges of Education, policy makers in Teacher Education of the Ministry of Education. For policy makers, the findings of the study reveal that rushing to cover more curriculum content would lead to superficial understanding. Understanding is the key to the pre-service teachers' application of action, process, object, and schema knowledge. Therefore, we recommend that mathematics teacher educators implement teaching strategies that would enable pre-service teachers to clearly understand trigonometric concepts. This will erase their negative perceptions, enhance their knowledge and improve upon their performance in trigonometry.

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