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MATHEMATICAL PROOF: THE LEARNING OBSTACLES OF PRE-SERVICE MATHEMATICS TEACHERS ON TRANSFORMATION GEOMETRY

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Abstract

Several studies related to mathematical proof have been done by many researchers on high-level materials, but not yet examined on the material of transformation geometry. The aim of this research is identification learning obstacles pre-service teachers on transformation geometry. This study is qualitative research; data were collected from interview sheets and test. There were four problems given to 9 pre-service mathematics teachers. The results of this research were as follows: learning obstacles related to the difficulty in applying the concept; related to visualize the geometry object; related to obstacles in determining principle; related to understanding the problem and related obstacles in mathematical proofs such as not understanding and unable to express a definition, not knowing to use the definition to construct the proof, not understanding the use of language and mathematical notation, not knowing how to start the proof.

Keywords: Mathematical proof, Pre-service mathematics teachers, Geometry.

Abstrak

Beberapa penelitian terkait pembuktian matematis telah dilakukan oleh banyak peneliti pada materi-materi tingkat tinggi, namun belum ada yang meneliti pada materi geometri transformasi. Tujuan dari penelitian ini adalah mengidentifikasi hambatan belajar calon guru pada transformasi geometri. Penelitian ini merupakan penelitian kualitatif, data dikumpulkan dari lembar wawancara dan tes. Ada 4 masalah yang diberikan kepada 9 calon guru matematika. Hasil penelitian adalah sebagai berikut: hambatan belajar terkait kesulitan dalam mengaplikasikan konsep; terkait dengan memvisualisasikan objek geometri; terkait dengan kesulitan dalam memahami prinsip-prinsip; terkait dengan memahami masalah dan terkait dengan kesulitan dalam pembuktian matematis seperti tidak memahami dan tidak dapat mengungkapkan definisi, tidak tahu untuk menggunakan definisi dalam mengkonstruksi bukti, tidak memahami penggunaan bahasa dan notasi matematika, tidak tahu bagaimana memulai pembuktian dan tidak tahu bagaimana memvisualisasikan masalah yang diberikan.

Kata Kunci: Pembuktian matematis, Calon guru matematika, Geometri.

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Geometry is an integral part of the learning of mathematics (Fachrudin, Putri, & Darmawijoyo, 2014; Sukirwan, Darhim, Herman, & Prahmana, 2018; Ahamad, Li, Shahrill, & Prahmana, 2018). However, the development of learning geometry at this time is less developed. One reason is the difficulty in forming a real construction student carefully and accurately, the notion that to paint geometry requires precision in the measurement and requires a long time, and not infrequently students experiencing obstacles in the process of evidence (Rizkianto, Zulkardi, & Darmawijaya, 2013; Novita, Prahmana, Fajri, & Putra, 2018). Meanwhile, the painting plays an essential role in teaching geometry at school for painting geometric connection between physical space and theory. If further investigation of the link between the objects with the abstract geometry student obstacles in learning geometry, it will be alleged that in fact there is a problem in teaching geometry at school relates to the formation of abstract concepts. Learn abstract concepts cannot be done only with the transfer of information, but it takes a process of formation of concepts through a series of activities experienced directly by students (Nurhasanah, Kusumah, & Sabandar, 2017). The series of abstract concept formation activity of these are referred to the process of abstraction.

Studying mathematics meant to be also studied branches of mathematics is the science of geometry. Everything in this universe is geometry so that the branches of mathematics through geometry learn about the concepts embodied in the objects that exist in nature through geometric concepts. Thus, assessment of learning geometry must continue to be developed so that each learner can analyze the geometry of objects into a concept of geometry and can construct geometry knowledge with formal proofs. Mata-pereira & Ponte (2017) say that a proof is a connected sequence of assertions that includes a set of accepted statements, forms of reasoning and modes of representing arguments. Stefanowicz & Kyle (2014) say that a proof is a sequence of logical statements, one implying another, which explains why a given statement is true.

However, mathematical proofs in geometry material lately become an obstacle that they seem poorly developed. Difficulties analyzing geometric properties are realized in the form of theorems to create a concept widely experienced by the students. Proof becomes a severe matter in determining the school curriculum in every different country. This is what makes reasoning and proof NCTM enters into one of the standard processes. This means that in every lesson a teacher must enter the elements in each classroom. Maya & Sumarmo (2011) state that possessing mathematical proving ability was certainty ability because it is an essential ability that should be possessed by all students who learn mathematics. Komatsu (2017) state also that proof and proving to play a crucial role in the discipline of mathematics and should be an essential component of mathematical learning.

Several studies have been conducted regarding evidence learning in secondary school (Harel & Sowder, 1998; Mariotti, 2006). Several research methods have also been conducted in proof-learning. Duval's research (1991) has identified the arguments and evidence that explain the difficulties students experience in understanding and making proof. Other researchers such as Balacheff, 1988; Harel & Sowder, 1998; Marrades & Gutiérrez, 2000 focuses more on identifying the types of empirical and deductive evidence generated by students that enable student progress in learning to prove.

Some researchers identify and explain the reasons why students are unwilling or unable to complete deductive evidence from their allegations (Arzarello, Micheletti, Olivero, & Robutti, 1998). Researchers also focus on analyzing learning especially on deductive evidence (Antonini, 2003; Stylianides, Stylianides, & Philippou, 2007; Antonini & Mariotti, 2008). More details can be found in Mariotti's research (2006), Reid and Knipping (2010), and Hanna and De Villiers (2012).

In traditional learning, mathematical proof is only used as a means to eliminate the doubts of students on the concepts taught. However, the evidence is not used as a means of increasing the higher mathematical ability. Just as revealed by Hana (Christou, et al. 2004) that the function of evidence and proof are: verification, explanatory, systematization, invention, communication, construction, exploration, and incorporation. Verification of proving and the proof is regarded as the most fundamental functions in the proof because both are products of the process of the development of mathematical thinking very mature. Verification refers to the truth of the statement while explanations provide insight into why this is true.

As for the role played proof in mathematics, namely: 1) to verify that a statement is true, 2) to explain why a statement can be said to be true, 3) to establish communication mathematics, 4) to find or create new math and 5) To make systematic statement in an axiomatic system (Knuth, 2002). Hanna (Stylianides, Stylianides, and Philippou, 2007) say that there are three main reasons why the ability to prove the need to be improved. First, the proof is crucial to learn to explore mathematics. Second, the ability of students in the proof can improve their math skills more broadly, because the evidence "involved in all situations where the conclusion must be reached in the making of decisions to be made", and the third, the difficulties experienced by high school students and college students will affect their ability to perform mathematical proofs on a broader level again, so it is crucial for students to learn mathematical proof on the level of previous education.

METHOD

This study was conducted to analyze student learning obstacles, especially regarding the difficulty students epistemology regarding materials, both presented in the form of materials or materials in lectures. This research method is descriptive qualitative research that aims to describe obstacles regarding epistemology student learning mathematical proofs related to the subject of transformation geometry. Subjects were nine student teachers Unswagati contracting mathematics courses transformation geometry consisting of 3 students with high prior knowledge mathematically, three students with medium prior knowledge of the mathematically and three students with low prior knowledge of mathematically. The beginning of knowledge is based on the acquisition of student achievement index in the previous semester. For students learning obstacles regarding epistemology student at transformation material using five indicators: concept, visualization, principles, understand the problem, and mathematical proofs.

RESULTS AND DISCUSSION

This research resulted in qualitative data. Learning obstacles students understand the concepts in terms of epistemology transformation geometry in working on the geometry transformation is divided into 5 types, it is in terms of indicators in assessing learning obstacles, namely: a) learning obstacles related to the difficulty in applying the concept; b) learning obstacles related to visualize the geometry object; c) learning obstacles related to obstacles in determining principle; d) learning obstacles related to understanding the problem and e) related obstacles in mathematical proofs.

Related understanding and applying the concept of learning difficulty is the difficulty experienced by students in understanding and applying the concept by the command matter. Examples of these obstacles one student does not understand the concept, students cannot mention the definition of transformation. It happened at the beginning of the mathematical knowledge of students with high, medium or low. Here is one example of the questions and responses of students experiencing barriers to learning. For example, on the following question: write the definition of a transformation in the field of V.

Learning obstacle students with high prior knowledge is described as follows. S1 had trouble with the concept, to define the transformation; these students do not write domain/codomain of a function called transformation. S5 can write correctly and complete the definition of transformation. S8 experienced obstacle of the concept, to define the transformation, these students do not write domain/ codomain of a function called transformation.

Learning obstacle students with prior knowledge is being described as follows. S3 having trouble against the concept, it cannot define the transformation correctly; these students mention that the transformation is a bijective function, but do not write domain /codomain of these functions. S4 in defining transformation by merely mentioning that a transformation is injective functions only. S9 can write the definition of transformation correctly and completely. Obstacles students with low initial knowledge are described as follows. S2 no difficulty is in writing the definition of transformation. S6 and S7 to write the definition of transformation are not complete. Both of these students do not write domain/codomain of a function is the transformation in the field of S6 V. It also found one in writing notation.

Related Learning Obstacles Visualize learning obstacles related visualize the geometry object. The point is that students have obstacles regarding describing the line of the transformation result. Examples of these obstacles include the inability of students in painting properly and appropriately. Here is one example of the questions and responses of students who experience learning obstacles. For example, in Question 2 is as follows: Draw the line $g' = M_h(g)$ if h = {(x,y) |y = x + 1} and $g = \{(x,y)|y = -x\}$.

Learning obstacle students with high prior knowledge is described as follows. S1 is having trouble visualize right lines g and h so that images reflection created false images. S5 can visualize by what is known and questioned, but the images are still made without a ruler. S8 can paint lines mirroring the results appropriately. Obstacles students with prior knowledge are being described as follows.S3 had difficulty in visualizing the line h so that the reflection is illustrated one. S4 in the paint did not use a ruler, but the results are correct reflection depicted. S9 cannot describe all that is known. Obstacles students with low initial knowledge are described as follows. S2 can paint reflection results correctly. S6 and S7 cannot describe all that is known so that the reflection nothing, other than that, the two students were not drawing Cartesian coordinates as a first step in painting a line mirroring results.

Learning obstacles related to the principle of. Learning obstacles is the difficulty experienced by students in terms solves the problem by defining the principles to be used in solving the problem of transformation. Examples of this difficulty are the inability of students in the mentioned properties of isometry, so it cannot be a member of reasons of the questions in the matter. Here is one example of the questions and responses of students who have difficulty learning. For example, the following problem: Given: T and S isometry. Determine the statements below True or False? Give your reason.

a. If *g* is a line, then *g*'= (TS)*(g)* is also a line.
b. If g // h and g'= (TS)(g), h'= (TS)(h) then g'// h'.
c. If S is a reflection of the S is involutory.

Learning obstacle students with high prior knowledge is described as follows. S1 can answer correctly, but the reasons expressed by one. S5 can be answered correctly and the reasons for appropriately. S8 can mention the definition of isometry correctly, but cannot answer questions related to the principle of isometry, so that reason used improperly. Obstacles students with prior knowledge are being described as follows. S3 was having difficulty writing down the definition of isometric and members wrong reasons related to statements given. S4 can write for the right reasons, but they are notational wrong. S9 is important to specify the definition of isometry, giving the wrong reasons. Obstacles students with low initial knowledge are described as follows. S2 is important to specify the definition, and the reason given was also incorrect. S6 and S7 wrong in writing down the definition and does not include the reason (no answer).

Learning obstacles related to understanding the problem. Is the difficulty of learning obstacles experienced by students regarding understanding the problem to solve the problem by using the steps in the completion of the write down what is known and asked about the matter. Examples of this difficulty are the inability of students to solve problems by the steps to completion. Here is one example of the questions and responses of students who have difficulty learning. For example, on the following question: define a line equation $g' = M_h(g)$ if h = {(x,y) |y = x + 1} and $g = \{(x, y) | y = -x\}$.

Learning obstacle students with high prior knowledge is described as follows. S1 was having trouble determining what is known and troubleshooting procedures are still wrong. S5 can understand the problem, find out what is known and asked, can solve the problem by the settlement procedures. S8 can understand the problem and solve it according to the procedure. Obstacles students with prior knowledge are being described as follows. S3 can understand the problem and solve the problem with proper procedures, but there are errors in arithmetic operations. S4 can understand the problem and solve the problem according to the procedure. S9 cannot understand the problem and cannot solve it. Obstacles students with low initial knowledge are described as follows. S2 can understand the problem but cannot finish the correct procedure. S6 and S7 are not able to understand the problems and did not finish.

The difficulty is in proving mathematical. Learning difficulty is the difficulty experienced by students in constructing the proof of the matter. Here is one example of the questions and responses of students who have difficulty learning. For example, the following problem: to prove that the reflection on the line g is an isometry.

Learning obstacle students with high prior knowledge is described as follows. S1 can construct evidence correctly, but there is still incorrect notation. S5 can construct evidence properly. S8 can construct proofs in part, at the end of the part that is wrong in giving reasons. Obstacles students with prior knowledge are being described as follows. S3 and S4 obstacles for constructing proofs can't use the existing definition. S9 trouble is to begin constructing proofs. Obstacles students with low initial knowledge are described as follows. S2 cannot use a definition for constructing proofs. S6 and S7 begin constructing the evidence about be proved, difficulty in starting the construction of the evidence and not be able to use the definition for constructing proofs.

Some mathematical proofs related research shows some of the things that are essential mathematical proofs. Activities considered difficult by students to learn and teachers to teach include justification or proof (Suryadi, 2007). Research studies conducted Dryfus (Jones and Rood, 2001) showed that students always fail to look at the adequacy of the evidence because they are too often asked to prove things that are obvious to them. Students also fail to distinguish between the different forms of mathematical reasoning such as heuristic or argument, explanation or proof. A significant gap in the research literature is still at least main set students "because it looks right" instead of "because he worked on issues" for the argument that believed.

The research result Knuth (2002) showed that teachers recognize the many roles of the play proof in mathematics, in learning the role of evidence should not be abandoned, and the evidence as a tool for learning mathematics. The results also show that many teachers still have limitations in determining the nature of the evidence used in the study of mathematics.

Heinze and Reiss (2003) research results showed that some students found with some of the answers wrong though ideas about the solution are proving correct. This occurs in the empirical argument. Many errors occur experienced by students is related to aspects of the structure of evidence. Most of the students interviewed were mostly already know that the argument does not form empirical evidence.

Study interviews in this study also showed that the three aspects of methodological knowledge of relevant proof when assessing the evidence. It seems that on this aspect conclusion chain is not problematic, because the proof is right mostly depicted as true. However, some cases it was not clear whether the students understand every step of the evidence compiled. The problem with this aspect of the scheme of evidence, in particular, an inductive argument, often trained using inductive argument in elementary school. Students have difficulty bridging the gap between empirical arguments to formal arguments. This is confirmed by a study by Lin (Heinze and Reiss, 2003) showed that Taiwanese students a different problem that is wrong or improper argument transformation in improving formal arguments were observed.

Mariotti study (2001) revealed the geometry construction is an essential part of the experience of students who should be organized. Results of the study Mariotti showed that if the geometry is just a pencil and paper geometry theory perspective, it is difficult to understand. When students draw on paper students can only focus on the images being constructed and can't manipulate it.

CONCLUSION

Average-ability students, especially in mathematical constructing proofs of 11.33 (maximum score 28) with details as follows: for students with high mathematical prior knowledge gained an average of 19.67; students with early mathematical knowledge have gained an average of 9.33, and with low prior knowledge mathematical earned an average of 5.00. There are five kinds of difficulties related to the students regarding epistemology on geometry transformations, namely a) learning difficulties related to the difficulty in applying the concept; b) associated learning difficulties visualizing the geometry object; c) learning difficulties related to difficulties in determining principle; d. Related learning difficulties to understand the problem and e. Related difficulties in mathematical proofs. Specialized in mathematical proofs, students have difficulty, among others: do not know how to start the construction of the evidence, can't use the definition (concept) and the principle already known, and are likely to begin construction of the evidence with what must be proved.

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