Investigating Secondary School Students’ Difficulties in Modeling Problems
PISA-Model Level 5 And 6

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Abstract
The chart of Indonesian student of mathematical ability development in Program for International Student Assessment (PISA) event during the last 4 periods shows an unstable movement. PISA aims to examine the ability of children aged 15 years in reading literacy, mathematics literacy, and science literacy. The concept of mathematical literacy is closely related to several other concepts discussed in mathematics education. The most important is mathematical modelling and its component processes. Therefore the goal of this research is to investigate secondary school students’ difficulties in modeling problems PISA-model level 5 and 6. Qualitative research was used as an appropriate mean to achieve this research goal. This type of research is a greater emphasizing on holistic description, and phenomenon identified to be studied, students’ difficulties in modelling real world problem in PISA model question level 5 and 6. 26 grade 9 students of SMPN 1 Palembang, 26 grade 9 students of SMPK Frater Xaverius 1 Palembang, and 31 participants of mathematical literacy context event, were involved in this research. The result of investigate showed that student is difficult to; (1) formulating situations mathematically, Such as to representing a situation mathematically, recognizing mathematical structure (including regularities, relationships, and patterns) in problems, (2) evaluating the reasonableness of a mathematical solution in the context of a real-world problem. The students have no problem in solve mathematical problem they have constructed.

Keywords: Mathematical model, Modelling competence, PISA, PISA Questions level 5 and 6, Students’difficulties in solving PISA-model Questions. Mathematics Literacy.

Abstrak
Grafik perkembangan kemampuan matematika siswa Indonesia pada ajang PISA selama 4 periode PISA terakhir menunjukkan pergerakan yang tidak stabil. PISA bertujuan meneliti secara berkala tentang kemampuan anak umur 15 tahun dalam membaca (reading literacy), matematika (mathematics literacy), dan IPA (science literacy). konsep literasi berkaitan erat dengan beberapa konsep-konsep lain yang dibahas dalam pendidikan matematika, tetapi yang paling penting adalah modelling (pemodelan matematika) dan komponen prosesnya. Oleh karena itu tujuan dari penelitian ini adalah untuk mengetahui kesulitan siswa sekolah menengah pertama memodelkan masalah model PISA level 5 dan 6. Penelitian yang digunakan untuk mencapai tujuan penelitian ini adalah penelitian kualitatif. Jenis penelitian ini adalah lebih menekankan pada deskripsi secara keseluruhan, dan

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mengidentifikasi kesulitan siswa dalam membuat model matematika dari soal PISA level 5 dan 6. 26 orang siswa kelas 9 SMPN 1 Palembang, 26 orang siswa kelas 9 SMPK Frater Xaverius dari 1 Palembang, dan 31 peserta kontes literasi matematika di Palembang, terlibat dalam penelitian ini. Hasil investigasi menunjukkan bahwa siswa mengalami kesulitan dalam proses; (1) merumuskan masalah dalam kehidupan sehari-hari ke dalam model matematika, Seperti memginterpretasikan konteks situasi nyata ke dalam model matematika, memahami struktur matematika (termasuk keteraturan, hubungan, dan pola) dalam masalah, (2) mengevaluasi kewajaran dari solusi matematika dalam konteks masalah dunia nyata. Namun siswa tidak memiliki masalah dalam menyelesaikan model matematika yang telah mereka bangun.


Introduction

Program for International Student Assessment (PISA) is conducted by the OECD (Organization for Economic Co-operation & Development) and United Nations Educational, Scientific and Cultural Organization (UNESCO) Institute for Statistics. PISA aims to examine the ability of children aged 15 years in the regular reading (reading literacy), mathematics (mathematics literacy), and Science (science literacy). Indonesia is one of the PISA participating countries that have joined since 2000. The chart of Indonesian student of mathematical ability development in the PISA event during the last 4 periods shows an unstable movement, Indonesian’s students only able to answer questions PISA level 1, 2 and 3, and a few students can solve level 4 questions. Chairman of an international group of mathematicians for PISA 2012, Kaye Stacey (2010), argued that the concept of literacy is closely related to several concepts discussed in mathematics education. But most important is the modeling because the cycle of mathematical modeling is a central aspect of the conceptions of PISA students as an active problem solvers, but students or problem solver often do not need to be involved in every stage of the cycle of modeling, especially in the assessment context, Blum, Galbraith, Henn & Niss, (2007). Therefore, researchers interested in conducting research to investigate middle school students’ difficulties in modeling PISA model problems level 5 and 6.
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Theoretical framework

The structure of the PISA mathematics framework can be characterised by the mathematical representation: ML + 3Cs. ML stands for mathematical literacy, and the three Cs stand for content, contexts and competencies. The PISA mathematical literacy domain is concerned with the capacities of students to analyse, reason, and communicate ideas effectively as they pose, formulate, solve and interpret mathematical problems in a variety of situations. In using the term “literacy”, the PISA focus is on the sum total of mathematical knowledge a 15-year-old is capable of putting into functional use in a variety and some creativity.

1. Mathematical Literacy

The definition of mathematical literacy refers to an individual’s capacity to formulate, employ, and interpret mathematics, and this language provides a useful and meaningful structure for organising the mathematical processes that describe what individuals do to connect the context of a problem with the mathematics and thus solve the problem. The categories to be used for reporting are as follows:

- Formulating situations mathematically

The word formulate in the mathematical literacy definition refers to individuals being able to recognize and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form. In the process of formulating situations mathematically, individuals determine where they can extract the essential mathematics to analyze, set up, and solve the problem. They translate from a real-world setting to the domain of mathematics and provide the real-world problem with mathematical structure, representations, and specificity. They reason about and make sense of constraints and assumptions in the problem.

- Employing mathematical concepts, facts, procedures and reasoning

The word employ in the mathematical literacy definition refers to individuals being able to apply mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems to obtain mathematical conclusions. In the process of employing mathematical concepts, facts, procedures and reasoning to solve problems, individuals perform the mathematical procedures needed to derive results and find a mathematical solution (e.g., performing arithmetic computations, solving equations, making logical deductions from mathematical assumptions, performing symbolic manipulations, extracting mathematical information from tables and graphs,
representing and manipulating shapes in space, and analyzing data). They work on a model of the problem situation, establish regularities, identify connections between mathematical entities, and create mathematical arguments.

- Interpreting, applying and evaluating mathematical outcomes

The word interpreting used in the mathematical literacy definition focuses on the abilities of individuals to reflect upon mathematical solutions, results, or conclusions and interpret them in the context of real-life problems. This involves translating mathematical solutions or reasoning back into the context of a problem and determining whether the results are reasonable and make sense in the context of the problem. This mathematical process category encompasses both the “interpret” and “evaluate” arrows noted in the previously defined model of mathematical literacy in practice. Individuals engaged in this process may be called upon to construct and communicate explanations and arguments in the context of the problem, reflecting on both the modeling process and its results.

2. **Fundamental Mathematical Capabilities Underlying the Mathematical Processes**

A decade of experience in developing PISA items and analyzing the ways in which students respond to items has revealed that there is a set of fundamental mathematical capabilities that underpins each of these reported processes and that also underpins mathematical literacy in practice the seven fundamental mathematical capabilities used in this framework are as follows:

Communication, Mathematising, Representation, Reasoning and argument, Devising strategies for solving problems, Using mathematical tools.

3. **Context**

The reference to “a variety of contexts” in the definition of mathematical literacy is purposeful and intended as a way to link to the specific contexts that are described and exemplified more fully later in this framework. The specific contexts themselves are not so important, but the four categories selected for use here (personal, occupational, societal, and scientific) do reflect a wide range of situations in which individuals may meet mathematical opportunities.

4. **Mathematical Content**

PISA aims to assess students’ capacity to solve real problems, and therefore includes a range of mathematical content that is structured around different phenomena
describing mathematical concepts, structures or ideas. This means describing mathematical content in relation to the phenomena and the kinds of problems for which it was created. In PISA these phenomena are called overarching ideas; (1) **Change and relationships**, PISA recognises the importance of the understanding of change and relationships in mathematical literacy. (2) **Space and shape**, PISA recognises that patterns are encountered not only in processes of change and relationships, but also can be explored in a static situation. (3) **Quantity**, PISA recognises the importance of quantitative literacy. In PISA, the overarching idea of quantity includes: meaning of operations, feel for magnitude of numbers, smart computations, mental arithmetic, estimations. (4) **Uncertainty**, In PISA the overarching idea of uncertainty is used to suggest two related topics: statistics and probability. Both of these are phenomena that are the subject of mathematical study.

5. **Proficiency Levels of modelling competence**

Proficiency levels for modeling, the following development is observed as literacy levels increase.

1. Apply simple given models
2. Recognize, apply and interpret basic given models
3. Make use of different representational model
4. Work with explicit models, and related constraints and assumption
5. Develop and work with complex models; reflect on modelling processes and outcomes
6. Conceptualize and work with models of complex mathematical processes and relationships reflect on, generalize and explain modeling outcomes

The process of modeling constitutes the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures, on the other; as such it is a pervasive aspect of mathematics. Figure 1 schematically represents this mathematical modeling approach by Erik DE CORTE
Method

The research methodology that we use in this study is a qualitative research. Qualitative research is research studies that investigate the quality of relationships, activities, situations, or materials. This type of research is a greater emphasize on holistic description—that is, on describing in detail all of what goes on in a particular activity or situation rather than on comparing the effect of a particular treatment or on describing the attitudes or behaviors of people. There are five steps in qualitative research (Fraenkel & Wallen, 2010): (1) identification of the phenomenon to be studied – Students difficulty in construct mathematical model of real world problems in PISA- model questions; (2) identification of the participants in the study – 26 third grade secondary school students aged 15 years old in SMPN 1 Palembang, 26 third grade secondary school students aged 15 years old in SMPN 1 Palembang SMPK Frater Xaverius 1 Palembang, 31 participants of mathematical literacy context event; (3) generation of hypotheses – students can doing well in formulating real world problem into mathematical problem, employ mathematical problem, and interpret mathematical solution to real world situation; (4) data collection - used students worksheet, video recording and interviewed some students to get deeper information of their thinking process; data analysis – data will be analyzed by holistic descriptive; and (5) interpretation and conclusion – using indicator of modelling competence, mathematical literacy refer to proficiency level of PISA question given as a guideline to interpret and to make a conclusion.
**Result and Analysis**

Students were asked to solve two math PISA model to measure the students’ ability of modeling. Students are also given a paper to write a commentary on the questions provided. After students complete the given problem, they also interviewed to clarify their answers, and asked the cause of their difficulties in solving problems. The following are two models of the PISA math level 5 and 6 are assigned to students.

**Question 1**

![First balance](image1)

First balance

![Second balance](image2)

Second balance

Cuboids has 1 kg in weight

Third balance

How many cube, cuboids, Cylinder that may be added at the right side of third balance such that the weights are in balance? Give 2 the combination of the types of things are possible.

**Question 2.**

Ira and ATI each have candies. Here is a conversation between Ira and Ati

Ira: Ati, will you give me 5 candies of yours so my candies as much with your own candies.

Ati: (Laughter) No Ira. Why do not you give me 5 candies so my candies as much twice of yours candies.

**Question**

Problem 11 How many candy wholly owned by Ira and Ati? Write down how the calculations.

<table>
<thead>
<tr>
<th>Context</th>
<th>Candies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Content</td>
<td>Change and relationships</td>
</tr>
<tr>
<td>Competence</td>
<td>Kompetensi pemodelan</td>
</tr>
<tr>
<td>Level</td>
<td>6</td>
</tr>
</tbody>
</table>

Mathematics literacy:
- Formulating situations mathematically
- Employing mathematical concepts, facts, procedures, and reasoning
- Interpreting, applying and evaluating mathematical outcomes
**Students' answers to First question**

Students answers will be analyze to get the information of student’s difficulty in solving the PISA question.

**Student A**

Students' work shows that students write the weight of cylinder, cube and cuboids directly without any explanation. However, student gave the correct answers and logic thinking to get the number of objects that must be added to the third weights such that the third weight. But the students do not provide an explanation or any kind of work steps that make readers understand his thinking process. Student’s explanation of how he determine the number of object that should be add in the right side of third weights such that the third weights is balance, shows that students understand the problems. He said that he calculated the balance weight of the object on the left side of the third weights based on the weight of each object that he already knew in prior step. Then trying to determine what objects should be added to the third-right weights, and what is the weight of the three objects in order to balance weights. Therefore, the modeling process which appears only process Interpreting, applying and Evaluating mathematical outcomes, while the process formulate problems into mathematical models and solve mathematical models are not visible from the students' responses.
As a follow-up action to clarify the students’ answers, the students concerned were interviewed with the conversation as follows:

**Researcher** : How do you determine the weight of cubes, cylinder and cuboids?
**Student** : By using my logic

**Researcher** : Can you tell me the process to get the answer?
**Student** : By comparing the object is reduced on one side of the scales with the object increases on another side

**Researcher** : So how do the comparison?
**Student** : There are 3 cubes on the left side of weights, whereas on the right side of a cube is reduced while the number the cuboids increasing 1 unit and so does cylinder, so the weight of a cube is equal to 1 cuboids + 1 cylinder. Similarly, the second balance. The cube is reduced 1 unit while cuboids increased 3 unit from the left side to right side of second weights. Thus a cube is equal to 3 cuboids. Therefore a cube weighs 3 kg. Due to heavy weight a cube equal to the weight of a cuboids plus the weight of a cylinder. Cuboids’ weight is 1 kg, so the weight of cylinder is 2 kg.

**Researcher** : Excellent, why do not you write down how you get that answer, so people who check your answer sure that you understand about that?

**Student** : I find it hard to write regularly, I just think in head and count it directly.

In addition, other students also have similar answer with student A.

**Student B**

![Image of Student B's answer](image)

Fig.1. The answer of student B

Almost the same as the previous student, this student also does not provide any explanation. In fact none of the modeling process that emerged from students’ responses. Here is the result of conversations with students in the interview process.

**Researcher** : How did you get answers like this?
**Students** : To get the first combination, the numbers of objects on the right side of the third weights are equated with the number of objects on the left side of the third weights.

**Researcher** : What about the second combination?
**Student** : By comparing objects on one side of the scales decreases with increasing object on the other side (the same thought process to previous student, but students 2 makes mistake in calculating the number of cylinder that grow on the right side of weights ).

On the left side of weights first, there are 3 cube, whereas on the
right side of a cube is reduced while increasing the number of cuboids and cylinder increase 1 and 2 (so he gave the wrong answer)

Researcher : Why do not you write down how you get that answer, so people who check your answer sure that you understand about that?
Student : I do not know how to write it.

Students C

Student C was able to simplify the problem into symbols and mathematical language correctly, and give the correct answer. Modeling process that appears in her answer are formulate the problem into a mathematical model, and interpret the problem, but the process of solving a mathematical model has been formulated not appear. From interviews in mind that students are hard to write how to get results. Students' responses are almost the same as the student answers D.

Student D

Visibly, student C and student D’s way in modeling is almost same. Nevertheless, they have different modeling process. Students C modeling the problem by directly writing the number of objects that exist at weights 1 and 2, while the student D write
the model by compare the number of objects decreases and increases on the sides of the scales first.

**Student E**

![Fig.5. The answer of Student E](image)

Different from the four previous students, Student E can construct mathematical model through the correct process. The teacher of student E said that student E is the student who has high achievement in mathematics.

In contrast, the low achievement student cannot do more than write comment for the questions. Those are their comments:

![Fig.6. students’ comment on first question](image)
Students Answer for The Second Question

Student F

Students simplify the problem and construct a mathematical model. But students are not rigorous thinking and do not fully understand the problem. He did not realize that Ati’s candies reduced when Ira’s Candies increase and Ati’s candies increase when Ira’s Candies reduced. She also solve the model has formulated, but the student gives the wrong answer, because the model he built is wrong. While the process of interpreting and evaluating the modeling does not appear from the answers above.

Student G

Fig.7. The answer of Student F

Fig.8. The answer of Student G
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Students are seen trying to build a mathematical model, but he was wrong in formulate the relationship of the number of Ati and Ira’s Candies. Students are also tried to solve model that has been built which is definitely wrong, because he has built a wrong model. Problems that form the concern here is process of evaluating the suitability of student mathematical solution which he acquired with the fact that it is impossible Ira has 2.5 candy and candy Ati has 7.5.

Students H

The number of Ira’s Candies: 35
Number of Ati’s candies : 25
I calculate it based on conversations and use slightly instinct. If 5 of Ira’s candies given to Ati, so Ira candies is 30 pieces and Ati’s candies is 30 pieces, if 5 of Ati’s candies given to Ira, So Ira’s candies is 40 and Ati’s candies is 20.

Fig.9. The answer of Student H

Student H was able to answer this problem correctly. In the interview he said that he solve the problem by simply using instinct, with the thought process as stated in the answer.

Student I

Therefore, The total number of Ira’s and Ati’s candies is 60 pieces.

Fig.9. The answer of Student I
Based on students' answers on the answer sheet and the results of interviews, it is known that he is not using the process of modeling first. But he tried to choose any number that fulfill to number of Ira and Atis’ candies according to the conversation. Then prove or evaluate whether or not in accordance with the Ari and Ati’s conversation.

**Student E**

![Fig.10. The answer of Student I](image)

Student E is the high achievement students who can solve first question in the correct modeling process. Otherwise, he formulated this real world problem into incorrect mathematical problem and then he cannot solve the problem and interpret the solution to real situation. The student said in the interview process that she hard to construct mathematical model of this second problem because she not accustom to solve this type of problem. Whereas, the first problem is similar with the problem that she had learn in system of linear equations of two variables.

Meanwhile, The comments of students who cannot solve this question are as follows:

![Fig.11. The answer of Student F](image)

Same with student comments on the previous problem, the students see the biggest difficulty in formulating a problem in everyday life into a mathematical model.
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Conclusion

Of the various students' answers, both of Question 1 and Question 2 can be concluded

The result of investigate showed that student is difficult to; (1) formulating situations mathematically, Such as to representing a situation mathematically, recognizing mathematical structure (including regularities, relationships, and patterns) in problems, (2) evaluating the reasonableness of a mathematical solution in the context of a real-world problem. The students have no problem in solve mathematical problem they have constructed. The founded in this research is most of high achievement students cannot solve uncommon problem completely since they cannot formulate the problem mathematically, while students with moderate achievement can solve the problem by using their own way that they called “insting”, “trial and error”, “using own logic”. whereas the low achievement students cannot solve the two problem because they cannot find appropriate formula that they can used to solve the problem, and the problem required high logic.

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