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DEVELOPMENT OF A THREE DIMENSIONAL GEOMETRIC THINKING TEST FOR EARLY GRADERS

Zeynep Akkurt Denizli[1], Abdulkadir Erdoğan[2]

[1]Department of Elementary Education, Faculty of Educational Sciences, Ankara University, Ankara, Turkey
[2]Department of Mathematics Education, Faculty of Education, Anadolu University, Eskişehir, Turkey
E-mail: zeynep0akkurt@gmail.com

Abstract

This study aimed to develop a three-dimensional geometric thinking test to determine the geometric thinking of early graders in the paper-pencil environment. First, we determined the components of three-dimensional geometric thinking and prepared questions for each component. Then, we conducted the pilot studies of the test at three stages in six state schools located in the city of Ankara in Turkey. The first draft test consisting of 54 items was administered to 384 students; the second draft test consisting of 53 items was administered to 120 students and the third draft test consisting of 45 items was administered to 268 students. In order to establish the content validity of the test, prior to all the administrations, it was subsumed to the review of 13 experts and by considering the content validity rates, the items to be included in the test were determined. As the items in the test are scored as correct/false (1/0), KR-20 reliability coefficient was used in the calculation of the reliability of the test. In the last administration, KR-20 reliability coefficient was found to be 0.87; thus, it was concluded that the test is reliable.

Keywords: Geometric thinking, Three-dimensional geometric thinking, Three-dimensional geometric thinking test, Early graders.

Abstrak

Penelitian ini bertujuan untuk mengembangkan tes berpikir geometris pada ruang dimensi tiga bagi siswa kelas rendah berjenis *paper and pencil*. Pertama, komponen-komponen tes berpikir geometris pada ruang berdimensi tiga ditentukan terlebih dahulu dan pertanyaan-pertanyaan di setiap komponen disiapkan. Selanjutnya, penelitian rintisan uji coba tes tersebut dilaksanakan dalam tiga tahapan di enam sekolah negeri yang berlokasi di kota Ankara, Turki. Draf pertama berisi 54 butir soal diujicobakan kepada 384 siswa. Draf kedua terdiri dari 53 butir soal diujicobakan kepada 120 siswa, dan draf ketiga terdiri dari 45 butir soal diujicobakan kepada 268 siswa. Untuk memperoleh tes dengan validitas isi yang baik, tes tersebut diberikan kepada 13 ahli untuk diulas sebelum diujicobakan dan selanjutnya ditentukan butir-butir mana yang dimasukkan ke dalam tes. Karena butir-butir soal diberi skor benar/salah (1/0), koefisien reliabilitas butir soal menggunakan KR-20. Di uji coba terakhir, diperoleh koefisen reliabilitas KR-20 sebesar 0,87; maka disimpulkan bahwa tes yang dikembangkan reliabel.

Kata kunci: Berpikir geometris, Berpikir geometris pada ruang dimensi tiga, Tes berpikir geometris pada ruang dimensi tiga, Siswa kelas rendah.

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Geometric thinking includes the definition and classification of geometric objects, explanation of the relationships between them, drawings, reasoning, visualizations and proofs produced in relation to these objects and relationships (NCTM, 2000). Knowledge and skills encompassed by geometric thinking related to daily life and the real world have enforced its inclusion in any curriculum of education starting from elementary school for a long time (Prahmana, Zulkardi, & Hartono, 2012). One of the dimensions of geometric thinking is three-dimensional thinking which captures many objects first experienced and perceived by the child and their properties are one of the important issues that should be explored for early graders to develop their geometric thinking.

Yeh & Nason (2004) noted that three-dimensional geometric thinking has three components, which are the external material world, internal spatial ability, and communication. The external material world captures all the geometric objects (the natural objects such as tree and seashell, ideal objects such as triangle and cube), their motions (e.g. growth of a tree, rotation of a cube), and their properties (e.g. angle and edge). The internal spatial ability refers to an individual's capacity of perceiving and knowing the external material world. Perception of external objects by means of the internal spatial ability occurs through the third component; that is, communication/signs. Communication is possible through spoken and written language, mathematical symbols, pictures, diagrams and geometric objects (Yeh & Nason, 2004). Seen from this perspective, it can be said that the development of three-dimensional geometric thinking can occur by the individual's perceiving geometric objects and making sense of them by using his/her spatial ability through communication.

Thinking that van Hiele's study is inadequate in terms of elucidating other fields of geometry such as the geometry of three-dimension, Gutierrez (1992) explored the relationship between the van Hiele's geometric thinking levels and the three-dimensional geometric thinking and thus determined the levels of three-dimensional geometric thinking. Level 1 is recognition. It refers to the level where three-dimensional objects can be compared considering the general properties of three-dimensional objects (corner, edge, and face). The students at this level can have some ideas about the properties of objects such as angle size, edge length and parallelity from the appearance. These students cannot visualize a three-dimensional object they do not see, and they can only predict the results of the movement of this object.

Level 2 is analysis. It refers to the level where comparison of three-dimensional objects can be made considering their properties such as angle size, edge length, and parallelity. The students at this level can recognize the properties of geometric objects by observing them or considering their names and can determine the results of their movements by looking at their positions before and after the movement.

Level 3 is informal deduction. It refers to a level which is built on the isolated properties of three-dimensional objects and where informal realities can be presented. The students at this level can decide whether two and three-dimensional objects are identical without performing any physical and mental movement by doing mathematical analyses and can explain the properties of three-dimensional objects on the basis of their representations or mathematical structures. At this level, students who can visualize the movements of objects they have not seen can establish the connections between the elements (face, edge, and corner) of three-dimensional objects in their first and last positions.

Level 4 is deduction. It refers to a level where the objects and their movements can be analyzed on the basis of their formal definitions. The students at this level have the highest level of visualization ability. Each of these four levels defined by Gutierrez (1992) requires a more mental operation and spatial visualization than the preceding level; with increasing level, more detailed explanations are expected about the properties and movements of three-dimensional objects.

NCTM (2000) defines the skills to be possessed by students related to the geometry of three-dimension from pre-school period to the twelveth-grade. These standards, emphasizing the importance of spatial visualization and construction of two and three-dimensional objects for geometric thinking, covers skills such as recognition of three-dimensional objects, their construction, drawing, comparison, determination of their properties to be imparted to students starting from the pre-school period up to the third grade. As for the third to the fifth grades, skills such as classification of three-dimensional objects according to their properties, performing disintegration and integration operations, recognizing identical and similar objects, producing assumptions and deductions on the basis of the properties of three-dimensional objects, constructing three-dimensional objects, comparing objects with their drawings and calculating the areas and volumes of three-dimensional objects by using standard and non-standard units are captured within these standards.

Research has reported different findings related to early graders' three-dimensional thinking. While there is some research reporting that a five-year-old child can produce drawings considering the difference between a two-dimensional object and a three-dimensional object (Wolf, 1988), there is some other research arguing that only eight-year-old children can produce drawings in line with the rules of perspective without looking at the picture (Murphy & Wood, 1981). In addition to these, there is some other research claiming that the third graders drew three-dimensional objects as if they were two-dimensional and even the seventh and the ninth graders cannot accurately draw a three-dimensional object from the perspective (Mitchelmore, 1980). Thus, it may not be expected that early graders have higher levels of three-dimensional geometric thinking; however, research results have shown that important steps have been taken to promote geometric thinking at elementary education level.

This view is supported by various findings. While the third graders are counting the structures made up of identical cubes, they are able to use their own strategies (Battista & Clements, 1998). Furthermore, six-year-old children can distinguish between a sphere or a circle (Wolf, 1988), while children aged seven to nine start to visualize the plane representations of three-dimensional objects in their minds (Piaget & Inhelder, 1967). In addition, research has also pointed out that three-dimensional geometric thinking has not been addressed by taking all of its aspects into consideration. It can be maintained that the existing research having been conducted at different levels of schooling and generally focusing on only one component of three-dimensional thinking (drawing of three-dimensional objects, understanding of the structures made up of identical cubes, plane representations of three-dimensional objects) seems to be not enough to explore students' three-dimensional thinking.

Pittalis & Christou (2010) identified the three-dimensional thinking skills to be possessed by the fifth to the ninth graders considering the standards set by NCTM as the ability to manipulate different representational modes of 3D objects, the ability to recognize and construct nets, the ability to structure 3D arrays of cubes, the ability to recognize 3D shapes' properties and compare 3D shapes and the ability to calculate the volume and the area of solids. Though the study conducted by Pittalis & Christou (2010) on the fifth to the ninth graders filled an important void by including many components of three-dimensional geometric thinking simultaneously, there is no study encountered in the literature investigating early graders' three-dimensional geometric thinking with all the components of it. Therefore, it is thought to be important to develop tools allowing a comprehensive exploration of early graders' three-dimensional geometric thinking. In the current study, it was intended to develop a pen and paper test to evaluate early graders' three-dimensional geometric thinking.

Proposed Level of Three-Dimensional Geometric Thinking

In the current study, the skills identified by Pittalis & Christou (2010), the skills defined by Gutierrez (1992) for each of the three levels of geometric thinking, and the NCTM (2000) standards were revised considering the cognitive level of the target population. In the following components, two of which are newly added and five of which are similar to those proposed by Pittalis & Christou. They are: (1) recognizing three-dimensional geometric objects, (2) determining the locations of three-dimensional geometric objects relative to each other, (3) using different representations of three-dimensional objects, (4) recognizing the properties of three-dimensional objects and comparing objects according to their properties, (5) establishing the relationship of the two-dimensional and three-dimensional, (6) recognizing three-dimensional structures made up of identical objects, and (7) calculating areas and volumes of three-dimensional objects.

Recognizing the Three-Dimensional Geometric Objects

Recognizing three-dimensional objects encompasses the skills of distinguishing two-dimensional objects from three-dimensional objects and recognizing the similar three-dimensional objects among a group of three-dimensional objects. Students who are in the period between the pre-school education and second grade of elementary school are expected to name three-dimensional objects (NCTM, 2000). Though they do not know all the properties of objects at elementary stages, they can recognize them as including square, triangle etc.; however, recognition of an object with its all properties regardless of its shape, color, and location requires the use of more developed spatial relationships (Gutierrez, 1992).

Determining the Locations of Three-Dimensional Geometric Objects Relative to Each Other

Determination of the locations of objects relative to each other is related to spatial ability (Yeh, 2013). NCTM (2000) states that first and second graders should be able to explain the locations of objects relative to each other by using simple words such as "next to". The students with lack of experience have been observed not to be able to conceive that objects can have different appearances in different locations and to experience difficulties in the determination of locations that can be described with words such as right-left, front-back (Kol, 2010). When they are 3 to 4 years old, children can point at the objects that are shown as targets, located close to them or whose locations are marked; yet, the exact coding of the location of an object can only be accomplished in later ages (middle childhood period) (Piaget *et al.*, 1960).

Using Different Representations of Three-Dimensional Objects

Three-dimensional geometric objects are usually represented by real physical objects, objects in the computer environment and objects drawn on paper (Gutierrez, 1992). Plane representations are the most widely used representations at schools; yet, the use of these representations requires a large amount of mental operation (Gutierrez, 1992). Understanding the drawn representation of a three-dimensional object entails not only the recognition of the depth involved in the drawing but also elements of the object but also visualization of this objects as a whole in the mind (Deregowski & Bentley, 1987). Thus, it can be argued that early graders' understanding of three-dimensional objects on the basis of these representations are highly difficult but important (Gutierrez, 1992).

Recognizing the Properties of Three-Dimensional Objects and Comparing Them

Complementary elements of a three-dimensional object (corner, edge, face etc.) in fact make it possible to recognize and interpret this object. For instance, geometric figures making up each face and the number of these figures help the recognition and naming of a prism (Pittalis & Christou, 2010). Thus, the recognition of three-dimensional objects requires the ability to distinguish which properties of the parts making up them do not change. To do so, it is necessary to compare the elements having similar or different structures and relate the elements of the object to each other (Markopoulos, 2003). Moreover, comparison of different three-dimensional objects according to their properties relies on the analysis of the properties of these objects (Gutierrez, 1992).

Establishing the Relationship of the Two-Dimensional and Three-Dimensional

Establishing the relationship of two-dimensional and three-dimensional is a mental operation requiring the analysis of parts and reconstruction of them in such a way as to create a new figure in the mind and the transition from two dimensions to three dimensions by focusing on complementary parts (Brown & Wheatley, 1997). The transition between two dimensions and three dimensions includes the acts of unfolding/folding. Piaget & Inhelder (1967) argue that the operations of rotation and construction of three-dimensional objects depend on the occurrence of these acts. As these acts entail the transformation of an object to another object by means of visualization in the mind, the skill of establishing the relationship between the two-dimensional and three-dimensional is related to visualization of a three-dimensional object rather than recognition of it (Bishop, 1980; Cohen, 2003).

Recognizing the Three-Dimensional Structures Made up of Identical Objects

Recognition of structures made up of identical objects includes their analysis, visualization of them from different perspectives and construction of their abstract components in the mind. Counting the identical cubes constituting a structure requires cognitive operations such as coordination, integration, and structuring (Battista & Clements, 1998). These operations requiring the identification and organization of the components in a structure (Battista, 2004) help elementary school students to form the volume formula (Battista & Clements, 1998). For instance, in order to count the cubes in a structure made up of identical cubes, the columns and lines constituted by 16 cubes can be determined and used. While counting the identical objects presented to them in the form of drawing (Ben-Haim *et al.*, 1985; Olkun, 1999) and drawing these objects (Yolcu & Kurtuluş, 2010), students experience difficulties.

Calculating the Area and Volume of Three-Dimensional Objects

Measurement of the areas and volumes of three-dimensional objects by means of units plays an important role in the construction of the numerical operations involved in these measurements and association of the formulas with the structure. This association is necessary to understand and visualize the internal dynamic of the structure (Battista & Clements, 1998). The size and shape of the area to be measured and the shape of the units to be used in the measurement (triangle/square) are the variables determining the difficulty of the measurement of the area (Owens & Outhred, 2006). The measurement of volume that is more difficult than the measurement of the area for students requires more spatial visualization and structuring of the object whose volume will be measured. As the determination of the number of identical objects to fill in a box allows the use of different strategies, it is one of the acts that facilitate the construction of the volume formula (Battista, 2004).

METHOD

Study Groups

The three-dimensional geometric thinking test (3DGT) was planned to be developed for the first grade to the fourth grade of elementary school students (6-10 years old). The test development process was completed as a result of three applications conducted in six state schools in the city of Ankara. The first application was initiated in May 2015 and the second and third applications were conducted in the months of October and November of the same year. As the second and third applications conducted at the beginning of the term, the second-grade to the fifth-grade students were selected as they were thought to better reflect the actual state of the first grade to the fourth-grade students (See Table 1).

Table 1. The distribution of the participating students across the grades

The First Application (May 2015)		The Second Application (October 2015)		The Third Application (November 2015)	
Grade	The number of students	Grade	The number of students	Grade	The number of students
1st grade	92	2nd grade	26	2nd grade	74
2nd grade	106	3rd grade	37	3rd grade	76
3rd grade	94	4th grade	31	4th grade	61
4th grade	92	5th grade	26	5th grade	57
Total	384	Total	120	Total	268

Development Process of the 3DGT

In order to construct the 3DGT, first, the above-mentioned components of geometric thinking in three dimensions were determined. In the 3DGT, which is a pen-paper test, drawing representation was used; accordingly, the component of using different representations of three-dimensional objects was not included in the test.

After the components had been determined, items were developed for each component. One item in the test was directly quoted from the study of Pittalis & Christou (2010) and ten items were taken from the same study and then adapted. The other items were originally developed for the study. After the expert review, final forms of the items were given and in this way, a total of 54 items were included in the first draft test. The application of this test lasted nearly 50 minutes in the first and second-grade students and 40 minutes in the third and fourth-grade students. After the first application, the items in the draft test were reorganized considering the findings obtained in the application process and the notes taken by the researcher during the application process. After the expert review, the final form of the second draft test was given to include a total of 53 items. Its application in the first and second grades lasted nearly 45 minutes and in the third and fourth grades, nearly 35 minutes. In light of the item analyses and expert review conducted after this application, the final form of the test was given to include a total of 45 items. In this third draft test as different from the other draft tests, there is not the component of *determining the locations of three-dimensional objects relative to each other*. The third application conducted with this last test lasted nearly 50 minutes in the first and second-grade students and 40 minutes in the third and fourth-grade students. In this application, all the tests were collected when 50 minutes were over. As a result of the reliability analysis and item analysis results conducted after the third application, no change was made in the test and thus this test became the final form of the 3DGT.

In the development process of the 3DGT, while conducting item analyses, item discrimination and item difficulty indices were calculated. In these calculations, while determining the top and bottom groups, 27% of the group at the top and 27% of the group at the bottom were taken (Reynolds *et al.*, 2006). The items having an item difficulty index lower than 0.40 were determined to be difficult, the items having an item difficulty index between 0.40 and 0.60 were determined to be moderately difficult and the items having an item difficulty index higher than 0.60 were determined to be easy. The items with an item discrimination index of 0.19 or lower were the considered to be the items to be excluded from the test; the items with an item discrimination index between 0.19 and 0.30 were considered to be the items to be corrected and the items with an item discrimination index of 0.30 or higher were considered to be items discriminating the individuals at the top group from the individuals at the bottom group (Arık *et al.*, 2014; Büyüköztürk, 2007).

Validity and Reliability of 3DGT

In order to establish the content validity of the developed test, expert opinions were sought (Lawshe, 1975; Shepard, 1993). In the test development process, the opinions of 8 math education researchers, two middle school mathematics teachers, a primary school teacher, and two measurement and evaluation experts; a total of 13 experts, were sought. In the expert opinion form, the experts were asked to evaluate the suitability of the test items for the determined components by marking one of the following response options: suitable/changeable/unsuitable. Moreover, they were asked to write their opinions about the correction of items, their suitability for grade levels, comprehensibility and suitability of their drawings.

Prior to each application, the content validity ratios of the items were calculated for each item by calculating the ratio of the number of the experts marking the response option "suitable" to the 1 minus of the half of the number of all the experts (Lawshe, 1975). In order to test the statistical significance of the obtained content validity ratios, the minimum values at the significance level of .05 were taken into consideration. As the minimum value determined for 13 experts is 0.54, the items with the content validity ratio higher than 0.54 were used in the test (Lawshe, 1975). As the items in the 3DGT developed in the current study are scored as true/false (1/0), in the calculation of the reliability of the test, KR-20 reliability coefficient was used (Büyüköztürk, 2007; Fraenkel & Wallen, 1993). In order to evaluate the reliability of the measurement outcomes, the standard error of the measurement was calculated (Büyüköztürk *et al.*, 2010).

Three-Dimensional Geometric Thinking Test (3DGT)

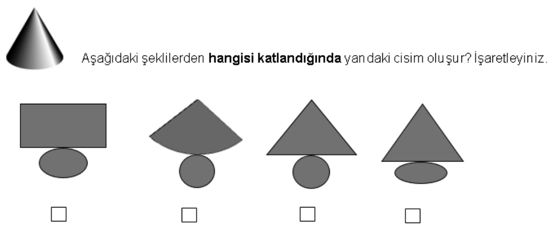
The 3DGT consists of 22 items. Together with the sub-items, the number of total items in the test is 45 as presented in Table 2. Of these items, 26 are open-ended and 19 are multiple-choice. In this regard, the 3DGT is a mixed test.

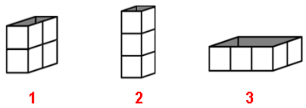
Table 2. Distribution of the 3DGT items across the three-dimensional geometric thinking components

Components of three-dimensional geometric thinking	Number of items	The total number of items together with the sub-items	The number of open-ended items
Recognition three-dimensional geometric objects	4	4	2
Recognizing the properties of three-dimensional objects and comparing objects according to their properties	7	18	14
Establishing the relationship of the two-dimensional and three-dimensional	4	4	1
Recognizing three-dimensional structures made up of identical objects	3	8	2
Calculating areas and volumes of three-dimensional objects.	4	11	7
Total	20	45	26

The reason for the existence of different numbers of items for each component in the test is that the number of the expected skills related to each component for the study group is different (NCTM, 2000). For example, a sub-skill of the component of establishing the relationship of the two-dimensional and three-dimensional, understanding of the two-dimensional plane form of a three-dimensional object requires not only knowing all the parts making up this object but also understanding the locations of these parts. Moreover, for the component of recognition of the properties of three-dimensional objects, besides knowing the numbers of corners, edges, faces, knowing the names of the faces are also an important sub-skill. Thus, the sub-skills captured by a component directly affect the number of the items related to this component. In Table 3, one sample question for each component is given.

Table 3. Sample items from the 3DGT and the related components

Component	Sample question for the component	English translation of the question
1. Recognition three-dimensional geometric objects	 	- *Which of the following resembles the object next? Please mark it.*
2. Recognizing the properties of three-dimensional objects and comparing objects according to their properties	 	- Corner - *One corner of the object next is shown in red.* - *How many corners do the flowing objects have? Please write under each object.*
3. Establishing the relationship of the two-dimensional and three-dimensional	 	- *When which of the following objects is folded, the object next is formed? Please mark it.*
4. Recognizing three-dimensional structures made up of identical objects	 	- *Look at this object.* - *Which of the following objects is the view of this object from a different perspective? Please mark it.*

Component	Sample question for the component	English translation of the question
5. Calculating areas and volumes of three-dimensional objects	**21.** Aşağıdaki **boş kapları** inceleyiniz.  Aşağıdaki soruları yanıtlayınız. Yanıtlarınızı **kutucukları işaretleyerek** belirtiniz. **1.** Yukarıdaki kaplar **tamamen** suyla doldurulursa **en fazla suyu** kaç numaralı kap alır? 1 2 3	*- Examine the following empty containers.* *- If the above boxes are completely filled with water, which box will have the maximum water?*

Application and Scoring of the 3DGT

Considering the age of the students, it was decided that it would be better for the researcher to give the instruction for the test and an instruction manual was prepared. The highest score to be taken from the test whose maximum completion time is 50 minutes and which is scored as true/false (1/0) is 45. In order to be able to get 1 point from the multiple-choice questions, it is necessary that the expected response option(s) be marked, and only this/these option(s) should be marked. In order to be able to get 1 point from the open-ended questions, the expected answer for the questions should be written/ drawn precisely.

RESULTS AND DISCUSSION

Results of the 3DGT Validity

During the test development process, the questions having a content validity higher than 0.54 were kept in the test (Lawshe, 1975). In this way, one question in the first draft test and two questions in the second draft test were discarded; yet, in the third draft test, all the questions were kept. The content validity of all the questions in the final test is higher than 0.54, which shows that the content validity of the 3DGT is statistically significant (Lawshe, 1975).

Results of the 3DGT Reliability

KR-20 reliability coefficient was found to be 0.88 in the first measurement, 0.88 in the second measurement and 0.87 in the third measurement. The reliability coefficient found in the last measurement shows that the reliability of the test is high (Büyüköztürk, 2007; Fraenkel & Wallen, 1993). Moreover, reliability coefficients were calculated for the components of three-dimensional thinking. These reliability coefficients were found to be 0.69, 0.76, 0.65, 0.63 and 0.70, respectively. In the evaluation of the reliability of measurement results, the standard error of the measurement was found to be $SH_{\text{Ö}} = 2.88$. This shows that the actual test score of a student getting 40 points from the test is, in fact, something between 37 and 43 by 68% likelihood; between 34 and 46 by 95% likelihood and between 31 and 49 by 99% likelihood (Büyüköztürk *et al.*, 2010).

Item Analyses of the First Application

Thirty- two questions in the first draft test whose KR-20 reliability coefficient was found to be 0.88 were found to be effective in discriminating the top and bottom groups ($r \geq 0.30$), 9 questions were found to be the questions that should be corrected ($0.19 < r < 0.30$) and 13 questions were found to be the questions that should be excluded from the test ($r \leq 0.19$). Seventeen questions in the test were found to be easy ($p > 0.60$), 17 were found to be difficult ($p < 0.40$) and 20 were found to be moderately difficult ($0.40 \leq p \leq 0.60$). The mean difficulty level of the questions in this test was calculated to be 0.52; that is, the test is moderately difficult.

Item Analyses of the Second Application

The results of the item analyses revealed that 12 items in the test cannot discriminate the students in the bottom group from the students in the top group ($r \leq 0.19$), 6 items should be corrected ($0.19 < r < 0.30$) and 35 items are effective in discriminating the bottom group students from the top group students ($r \geq 0.30$). As a result of these analyses, the items belonging to the component of determining the locations of three-dimensional objects relative to each other were excluded from the test as they were not found to be discriminating. When the difficulty indices were calculated, 29 items were found to be easy ($p > 0.60$), 14 items were found to be difficult ($p < 0.40$) and 11 items were found to be moderately difficult ($0.40 \leq p \leq 0.60$). The mean difficulty level of the test is 0.60, thus it can be said that the test is a moderately difficult test.

Item Analyses of the Third Application

As a result of the item analyses of the last draft test consisting of 45 items (See Table 4), it was found that 41 items in the test are effective in discriminating the bottom group students from the top group students ($r \geq 0.30$) as presented in Table 4. The discrimination indices of 4 items in the test were found to be in the range of $0.19 < r < 0.30$ (0.29, 0.27, 0.28 and 0.27). When the skills aimed to be evaluated with these items were taken into consideration, it was decided that keeping these items with discrimination indices close to 0.30 in the test would be more suitable by also seeking the opinions of experts. In the test, 23 items were found to be easy ($p > 0.60$), 12 items were found to be difficult ($p < 0.40$) and 10 items were found to be moderately difficult ($0.40 \leq p \leq 0.60$). The mean difficulty level of the items was found to be 0.58. Though the items in the test have different difficulty levels, the mean difficulty of the test was found to be around 0.50; which is good (Çepni *et al.*, 2008, Gronlund, 1977). Thus, it can be argued that the test whose mean difficulty index was found to be 0.58 is at the suitable difficulty level. As a result of these findings, no change was made in the third draft test and thus this test became the final form of the 3DGT.

Table 4. Item difficulty and discrimination indices of the test

Item	Difficulty index (p)	Discrimination index (r)
1	0.37	0.45
2	0.59	0.29
3	0.72	0.31
4	0.82	0.41
5	0.35	0.45
6	0.32	0.5
7a	0.82	0.45
7b	0.61	0.62
7c	0.65	0.63
8a	0.71	0.58
8b	0.72	0.59
8c	0.71	0.51
9a	0.42	0.44
9b	0.33	0.44
9c	0.65	0.34
10a	0.83	0.40
10b	0.57	0.31
10c	0.73	0.38
11a	0.54	0.47
11b	0.48	0.43
12	0.37	0.73
13a	0.61	0.62
13b	0.46	0.58
13c	0.28	0.56
13d	0.19	0.36
14a	0.42	0.59
14b	0.35	0.55
15a	0.69	0.51
15b	0.34	0.43
15c	0.21	0.38
16	0.52	0.48
17	0.35	0.66
18a	0.80	0.34
18b	0.79	0.48
18c	0.70	0.48
18d	0.69	0.44
18e	0.80	0.45
19	0.37	0.43
20a	0.64	0.40
20b	0.45	0.27
20c	0.72	0.38
21a	0.73	0.31
21b	0.65	0.56
22a	0.51	0.28
22b	0.76	0.27

CONCLUSION

In the current study, a valid, reliable and useful test (the three-dimensional geometric thinking test-3DGT) was developed to evaluate early graders' three-dimensional thinking. This test is believed to be used to elicit information about three-dimensional geometric thinking of these children and to compare this information depending on different variables. Moreover, this test would be useful to observe the development of the three-dimensional geometric thinking of students gaining new experiences with each year about three-dimensional geometric thinking.

The items related to the component of determining the locations of three-dimensional objects relative to each other were found to be very easy as a result of the item analyses thus not discriminating, they were discarded from the test after the second application. This component can be thoroughly developed in the medium childhood period and the exclusion of the questions related to this component should not mean that the students in this age group have completely mastered all the skills belonging to this component. Questions requiring more mental operations related to this component may yield different results.

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