# Learning The Concept of Area and Perimeter by Exploring Their Relation 

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#### Abstract

Learning the concept of perimeter and area is not easy for students in grade 3 of primary school. A common mistake is that students think that if the area is the same, the perimeter also has to be the same. It is difficult for them to understand that for a given area, there are many possibilities of perimeter and vice versa. When student are not aware of this relation they might confuse about the concept in their continuation of learning process. This research was conducted to study if it would support students' understanding of the concept of perimeter and area if we let them explore the relation between perimeter and area in the very first phase of the learning process. Design research was chosen as the method to study this issue and the three basic principles in The Realistic Mathematics approach were applied in this study to support the learning process of perimeter and area. Real life context such as picture frames was choosen in developing a sequence of learning line to reach the learning goal of perimeter and area. The partipants of this research were students and mathematics teacher of grade 3 in one of the elementary school in Surabaya. Two classes were taken to involve in the first cycle and second cycle respectively. The teaching experiment shows that the class activities such as making photo frame, measuring photo paper with sticky paper and arranging shapes with wooden matches are activities which can be used to reveal the relation of perimeter and area. From those activities students build their own understanding that in fact area and perimeter are not in one to one correspondence, they found that for the given area they might find different perimeter or vice versa. They also found the reason why they multiply length and width to count the area of rectangular or square shape from sticky paper activity. Somehow some students were found still struggle with their understanding of area and perimeter. They often simply count the area and perimeter but when it comes into comparing the area or perimeter they still struggle to differentiate between area and perimeter.


Keywords: Perimeter, Area, Relation between perimeter and area, Understanding


#### Abstract

Abstrak Belajar konsep perimeter dan daerah tidak mudah bagi siswa di kelas 3 sekolah dasar. Sebuah kesalahan umum adalah bahwa siswa berpikir bahwa jika daerah tersebut adalah sama, perimeter juga harus sama. Sulit bagi mereka untuk memahami bahwa untuk daerah tertentu, ada banyak kemungkinan dari perimeter dan sebaliknya. Ketika siswa tidak menyadari hal ini hubungan mereka mungkin bingung tentang konsep dalam kelanjutan mereka proses pembelajaran. Penelitian ini bertujuan


untuk mengetahui apakah itu akan mendukung pemahaman siswa tentang konsep keliling dan luas jika kita membiarkan mereka mengeksplorasi hubungan antara perimeter dan daerah dalam tahap pertama dari proses pembelajaran.
Desain penelitian dipilih sebagai metode untuk mempelajari masalah ini dan tiga prinsip dasar dalam Pendekatan Matematika Realistik yang diterapkan dalam penelitian ini untuk mendukung proses pembelajaran dari perimeter dan daerah. Konteks kehidupan nyata seperti bingkai foto yang dipilih dalam mengembangkan urutan belajar line untuk mencapai tujuan belajar dari perimeter dan daerah. Para partipants penelitian ini adalah siswa dan guru matematika kelas 3 di salah satu sekolah dasar di Surabaya. Dua kelas diambil untuk terlibat dalam siklus pertama dan siklus kedua masing-masing.
Percobaan menunjukkan bahwa kegiatan mengajar kelas seperti membuat bingkai foto, berukuran kertas foto dengan kertas lengket dan mengatur bentuk pertandingan dengan kayu adalah kegiatan yang dapat digunakan untuk mengungkapkan hubungan perimeter dan daerah. Dari kegiatan tersebut siswa membangun pemahaman mereka sendiri yang notabene luas dan keliling tidak dalam satu sampai satu korespondensi, mereka menemukan bahwa untuk daerah diberikan, mereka mungkin menemukan perimeter yang berbeda atau sebaliknya. Mereka juga menemukan alasan mengapa mereka mengalikan panjang dan lebar untuk menghitung luas bentuk persegi panjang atau persegi dari aktivitas kertas lengket. Entah bagaimana beberapa siswa ditemukan masih berjuang dengan pemahaman mereka tentang luas dan keliling. Mereka sering hanya menghitung luas dan keliling tetapi ketika datang ke membandingkan daerah atau perimeter mereka masih berjuang untuk membedakan antara luas dan keliling.

Keywords: Perimeter, Area, Hubungan antara perimeter dan area, Pemahaman

Perimeter and area become one of the interesting topics to be discussed since those two are very relevant to the real world. The concept of perimeter and area is not an easy thing to learn. Romberg (1997) states that a common difficulty regarding perimeter and area is to understand that for a given area, many perimeters are possible, and vice versa. In addition, it is frequently found that pupils mix the concepts of area and perimeter. They often tend to think that two figures with the same area also have the same perimeter (TAL Team, 2004). From an observation in a primary school in Indonesia, it was found that some pupils had counted the perimeters to answer the questions about areas (Fauzan, 2002).
Pupils are familiar with the concept of length since they were in grade 2 of primary school. For pupils who have a good understanding of perimeter as a special
application of length that measures the distance around a figure, they will be accustomed to finding perimeters where the length of every part of a figure is given and they just had to add all the given numbers but for those who do not have an adequate understanding of perimeter will find it difficult to deduce the length of the side when it was not stated explicitly (Kai Kow, 2006). Therefore it is important to embed a good understanding of perimeter for students on their early years of introduction in perimeter.

Different from perimeter, in fact, area is a more difficult concept for the students on their first year of introduction to area. Perimeter is in line with the concept of length which is known as linier measurement but area is not about length, it is about the whole surface that covers a shape. Understanding the attribute of area involves giving a quantitative meaning to the amount of bounded two-dimensional surface (Cross et all, 2009). Understanding of area measurement involves learning and coordinating many ideas (Clement \& Stephan, 2004), such as transitivity, the relation between number and measurement, and unit iteration operating in area measurement.

## Research Methods

There are some elements will be discussed in this chapter regarding the research methods used in this research.

## Research Design

In designing the research methodology, the researcher use the three phases of conducting a design experiment from Gravemeijer \& Cobb (2006), namely preparation and design, teaching experiment, and Retrospective analysis.

## Data Collection

For collecting the data of the research experiment, researcher will use video recording, interview, students' work and also field note.

## Validity and Reliability

The validity and the reliability from the result of the research to strengthen the quality of the research are described as follows,

In order to validate the research internally then it was expected that during this research, the sufficient data to see how students reasoning about their understanding of the concept of area and perimeter will be gain. It will include video recording of the whole lesson series, students' works, record of the interview and also field note. Externally, the validation will be seen from the generalization of the research whether it will appropriate to implement research for the population where the samples were taken.
Reliability
Reliability in this research will be seen from three ways, namely Data Registration, Track ability, and Inter subjectivity

## Result of The Research

The research was conducted in $3^{\text {rd }}$ grade of elementary school in Surabaya. We have 2 cycles of HLT implementation. Two classes were taken to participate in different cycles respectively. The first cycle consists of 6 random students from grade 3C. Those 6 students are Andre, Dilla, Dio, Farel, Ryan, and Salma. Different from the first cycle where we only take 6 students, for the $2^{\text {nd }}$ cycles, we take the whole class of 3B. The class consists of 28 students, 10 girls and 18 boys. Since cycle two was the revision of cycle one then for the following we will focus on discussing the result of the research in cycle 2 .

During the teaching experiment, we thoroughly observed the learning process of students in learning perimeter and area and its relation. Somehow we also paid attention on teacher's role during the class experiment. Some finding related to the elements in perimeter and area and its relation need to be analyzed further and clarified so it can come into the generalization of the learning process of perimeter and area.

## Pre-test

Pre-test was conducted to know the students understanding towards the subject that will be given. From the result of pre-test we can see how far the knowledge of students about perimeter and area. It is very acceptable that students in grade 3 do not have the understanding about the term 'perimeter' and 'area' yet because indeed it is new for them. The result of this pre-test also help us to redesign our HLT so it will as
much as possible be in line with the needs to support the students in their learning process of understanding perimeter, area and its relation.

## Introducing the term perimeter

The idea of choosing the fencing frame activity as the first activity along the series was under the consideration that grade 3 students have learned the length measurement in grade 2. It was expected that by starting the learning series with something that already accustom for the student then it will give good stimulation to go further on the learning series. Fencing activity indeed helps students to come into the terms perimeter as the outer part of the shape and how to count it. We found that it was not an easy task for students in grade 3 to describe perimeter in the words. Somehow from the class activity and class discussion we can see that students are able to build their own understanding about perimeter. They can show how they compare the perimeters of the frames. They can show which part of the shape known as perimeter. We can conclude here that students in grade 3 do not have to be really able to describe the meaning of perimeter in certain words. Their understanding can be seen from how they work on counting and comparing perimeter and their reasoning.

## Counting perimeter with non-standard unit and introducing the term 'area'

In this activity we start seeing that perimeter was learned together with area. When the students arranged the ice cream sticks to form a certain shape then teacher directs them to be aware that there is a region within the shape that they've created. Somehow the sense of perimeter seems to be stronger than area in this activity. Students seem busy in arranging the sticks and come into how many sticks they've spent for each shape somehow they do not really pay attention on the region within the shape unless the teacher asked them about. Teacher's question related with counting perimeter become a good support for students understanding especially for them who still have less understanding of perimeter from the previous activity which using ropes as the context. To build students awareness on area then teacher also post question about the region within the shape. Indeed some students seem struggled in this activity because they pay more attention on working with sticks and put aside on
the counting perimeter and awareness on region within the shape. This students need to be supported so that they will have the same understanding as their other friends.

## Comparing the area

Area comparison of some shapes which can be compared directly seems to be good starting point. This type of comparison is the simplest one for students. Pupils have no problems in comparing these kinds of shapes only by seeing them or put them as stacks. The idea of area was in fact already embedded in students mind as well. They can compare the areas by saying one is bigger or smaller than the other even though they do not have the idea of counting the area yet. Stepping into indirect comparison where the students use sticky paper as measurement unit then students starts to learn how to count the area. They start to have the sense of area as the number of sticky papers they use to cover the whole surface of photo paper. Somehow when it comes into counting perimeter by using the sides of sticky paper as measurement unit then some confusion were there. For some students, the side of sticky paper used as measurement unit of perimeter is still quite abstract. First they use sticky papers to count the area but then they use its side to count the perimeter. These students still need further support on their understanding about perimeter and area.
Two plane figures with the same area can have different perimeter or vice versa
The use of wooden matches and grid paper was really good support for students learning after some confusion in the previous learning with sticky paper. First of all wooden matches are something separated from the grid paper. Students will notice it as 2 separate things. Grid paper to represent the area counting and wooden matches along the side of the grid paper to represent the perimeter counting.
By using wooden matches and grid paper teacher tries to build awareness that for the given area it might have different perimeters and vice versa. As the example mentioned above that for 10 wooden matches, for instance, students can create different shapes with different areas as well.

The wooden matches and grid paper are real and make sense for students, so they do not find it difficult to come into more formal level where they no longer use wooden matches but draw it on grid paper. Drawing the shape on a grid paper was quite similar with arranging wooden matches so students are still able to recognize this pattern of work.

## Form the shape with odd number of wooden matches

Exploring the ideas of arranging the shapes from wooden matches in fact can be very broadening. After working with the same number of matches or the same number of square grids but different number of matches then the teacher try to explore to the possibilities of forming rectangles or square from odd number of wooden matches. In this activity all students involve with their design respectively and indeed they have the idea that it was impossible to form the rectangles and square with odd number of matches in the contrary it have to be even number. Somehow the students did not come into the awareness yet that the opposite side has the same number of matches. They have those ideas after the teacher direct the students to see the sides of the shapes. To come into the idea of formal calculation also still a long run. It seems that students need more time to explore the idea of odd wooden matches.

## Exploring the perimeter and area of the shapes other than rectangle and square

The perimeter and area exploration of the shape other than rectangle and square was important for grade 3 students. In their daily life they do not just face rectangular shape only but also other shapes including irregular one. By giving a chance for the students to explore the shape other than rectangle and square than we help the student to build their awareness about the concept of perimeter and area for any shapes.
Indeed the student's understanding about the concept of perimeter and area might be more in rectangle and square form somehow we already gave a preliminary knowledge about different shapes that available and need to be consider by them as well.

Starting with parallelogram with ice cream sticks and grid paper seems to be good choices before come into irregular shape with grid paper. In fact ice cream sticks were less abstract compare to ropes to be form as the shapes. With the parallelogram and rhombus we can lead the students to the shape other than rectangle and squares but also still in the coverage of the relation between area and perimeter where in the same area it might be some possibilities of perimeter.
The use of ropes as the tools to form any irregular shapes seemed does not really help to come into the idea of the relation between perimeter and area. It happen because the ropes it selves were not quite handy tools to be formed as the shape. Students more focus in deciding what shape to be formed and how to form it rather than realize
that each of them use the same length of ropes to form different shapes and different area. Indeed teacher also bring the discussion about it to the class, somehow we figure out that the students still struggle with it.

## Post-test

The result of this post-test more or less can give description about students' progress from their initial knowledge. Statistically comparing the result of pre-test and post-test indeed we see some improvement for some students but some other also still struggle with the concept of perimeter and area.

For instance the different acceptance about the concept of area and perimeter, most of the student can count and compare the area of two different shapes; somehow many students still struggle to compare the perimeter of two different shapes. It might be happen because the idea of area in this learning series seems more real for students to be put in their logic. Even though perimeter seems to be a very simple measurement but when we put together with area, student might get it more abstract compare to area.

Even thought the main consideration in this paper is about the process of students learning but at least from this post-test we also can briefly conclude that indeed those representations of numbers (see appendix) can support the argumentation previously about the development of students' learning process.

## Discussion

The relation of perimeter and area become the means in this research to support students' understanding about the concept of perimeter and area. Based on the aim of this research, the learning series was arranged as such so that perimeter and area were studied side by side along the learning series. The key principles of RME serve as a basis for the whole learning series in this research. We will elaborate how that key principles underlay the activities in this research.

## Guided Reinvention or Mathematizing

The first key principle of RME is guided reinvention or mathematizing. The following are the elaboration of teacher's role in guided reinvention or mathematizing.

1. Providing students an opportunity to present their idea

During the teaching experiment we figure out that teacher posed the questions below in order to provoke students to present their idea.

- "What is perimeter according to you?"
- "What did you use to measure perimeter?"

2. Stimulating social Interaction

Teacher should be able to stimulate social interaction among students. Teacher can provoke this interaction through pair work, group work or class discussion. It was proven in the observation that those students who rare to share their ideas in the class discussion were able to share it in the pair or group work. Another strategy to stimulate social interaction was posing appropriate question, as one of the question posed by the teacher during the teaching experiment below,
"Kiki said we can use hand span to count the perimeter, how about the others?"
Along the lesson series teacher also arranged group activities where students work in their group to cooperate in solving the problem and sharing ideas. As the discussion in one of the group during the teaching experiment below when one student try to respond to their friends answer.

Teacher: (talking to the group) So how you count this perimeter?
Rizky : (tagging each side of sticky paper) one, two,...seventeen
Aang : (knowing that Rizky's tagging was not synchronic with his counting) you wrong!
Teacher: this is how you suppose to count (tagging each side of sticky paper) one, two, so now you try
Arya : (tagging each side of sticky paper) one, two,... (he did skip counting)
Aang : you wrong, you missed one line. (he showed to the group how to count the perimeter correctly)

## 3. Connecting activities

During the teaching experiment we figured out a discussion where teacher try to relate the previous activities with the next activities, as the conversation below.

Teacher : "what tools can you use to measure perimeter?"(students mention ruler, hand span, ropes, pencil case etc)
Teacher : (show a circular shape) "what tools can be used to measure this shape?"
Meizar : ribbon
Teacher : how about this (show rectangular shape)
Rizky : ruler
Teacher : (repeated again for circular shape) so can we use ruler to measure the perimeter of this shape?
Students : no..

From the conversation we can see that teacher try to relate the knowledge of students about length measuring tools with counting perimeter. Teacher tried to bring the students to their awareness that not every shape can be counted with the same tools. Teacher also wanted to make ropes as a tool that make a sense for students in the next activities that will be conducted.

## 4. Eliciting the mathematical concept

One of the examples of transforming a concrete experience into mathematical concept during the class experiment was when the teacher tried to bring the idea of region within the boundaries into measuring area.
" Do you think the region within the boundary has quantity?"
By posing a question about quantity, teacher try to direct the students to the idea that if the region has quantity then it can be measured, the next question for students will be "How can I measure the quantity of this region?" Other finding during the teaching experiment that shows how teacher tried to elicit the mathematical concept was when students use sticky paper to count the area of the photo paper below


Teacher : what is the area of the shape?
Aang : (counting the square one by one) twelve
Teacher : do you have any other way than count it one by one
Aang : this is 4 right (pointing to the first row), then this three (pointing to the last coloumn) so four $\times$ three equal to twelve.

## 5. Asking for clarification

During the observation of teaching experiment we found several discussions when teacher tried to clarify students' answer.

Teacher : which one has the cheapest price s? (show 3 different rectangular shapes and stated that the smallest its area the cheapest its price will be )
Students : B (the biggest one)
Teacher : how can you decide B?
Students : It's the smallest
Teacher : How can you know B is the smallest? Aang, come here, show how you get $B$.

In the discussion above teacher try to get the clarification from students about their answer. This becomes one of the ways for the teacher to see students' ability in expressing their idea as the benchmark of their understanding.

## Didactical Phenomenology

In this research we helped the teacher to design the series of activities that reveal a contextual problem in which students might come with a wide variety of solutions.

A situation that is experientially real for students is used as the base for mathematical activity along the learning series. Some activities were chosen to fit the concepts of perimeter and area and are also experientially real for students; for instance, arranging shapes with wooden matches. From 10 wooden matches students can come with different shapes but often with rectangular and square form. For rectangular and square forms, to count the perimeter of the shapes, students might also come with different strategies. From the observation during the teaching experiment some students choose to count the number of wooden matches one by one, others choose to see only two sides, they add it up then multiply it by two since they know that the opposite sides has the same number of wooden matches.

## Self-developed model

The whole series of activities were intended to be able to support students in making their self-developed model to understand the concept of area and perimeter. Creating picture frame from ice cream sticks, in fact, help students make a very simple preliminary description about perimeter and area. The inner part is area and the outer part (i.e ice cream sticks) is the perimeter. With their own simple description they use to memorize in their mind what area and perimeter are.
Starting to count the number of square grids to count the area and also arranging wooden matches in square grid paper then students accustom with the square grid unit in counting area. As they have to count the area of shape made from ropes, the first idea they have is to put it on the grid paper so that they can count its area. Students are also familiar with counting perimeter with wooden sticks unit or use one side of the square grid. They accustom with certain model that they use in counting perimeter and area and use it in solving problem.

Learning perimeter and area side by side along the lesson series makes them accustomed to see both aspects of perimeter and area in any shapes. When they were asked about the different between two shapes, their answers mostly go into the similarities or the difference of its perimeter and area even though some of them also pay attention to the different form of the shapes. The fact that two shapes with the same area might have different perimeters and vice versa seems help the students to be aware that area is not depending on its perimeter and vice versa.

## Conclusion

The teaching experiment shows that the class activities such as making photo frame, measuring photo paper with sticky paper and arranging shapes with wooden matches are activities which can be used to reveal the relation of perimeter and area. From those activities students build their own understanding that in fact area and perimeter are not in one to one correspondence, they found that for the given area they might find different perimeter or vice versa. They also found the reason why they multiply length and width to count the area of rectangular or square shape from sticky paper activity. Somehow some students were found still struggle with their understanding of area and perimeter. They often simply count the area and perimeter but when it comes into comparing the area or perimeter they still struggle to differentiate between area and perimeter.

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