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Early Fractions Learning of 3rd Grade Students in SD Laboratorium Unesa

Elisabet Ayunika Permata Sari, Dwi Juniati, Sitti Maesuri Patahudin

Abstract

Fractions varied meanings is one of the causes of difficulties in learning fractions. These students should be given greater opportunities to explore the meaning of fractions before they learn the relationship between fractions and operations on fractions. Although students can shading area represents a fraction, does not mean they really understand the meaning of fractions as a whole. With a realistic approach to mathematics, students are given the contextual issues of equitable distribution and measurements that involve fractions.

Keyword: fraction meaning, relation of fraction, *design research*, realistic mathematics education, equitable distribution, measurement

Abstrak

Makna pecahan yang bervariasi merupakan salah satu dari penyebab-penyebab kesulitan dalam pembelajaran pecahan. Siswa-siswa seharusnya diberi kesempatan seluas-luasnya untuk mengeksplorasi makna pecahan sebelum mereka mempelajari hubungan antar pecahan dan operasi pada pecahan. Walaupun siswa dapat mengarsir daerah yang merepresentasikan suatu pecahan, tidak berarti mereka benar-benar memahami makna pecahan secara menyeluruh. Dengan pendekatan matematika realistik, siswa diberi permasalahan kontekstual tentang pembagian adil dan pengukuran yang melibatkan pecahan.

Kata kunci: makna pecahan, hubungan antar pecahan, *design research*, pendidikan matematika realistik, pembagian adil, pengukuran

There have been a huge number of researches on fractions conducted for the reason of difficulties on learning and teaching fractions (Hasseman, 1981; Streefland, 1991). Children tended to be proficient in doing algorithm rather than in reasoning performance. As the consequence, students often do some mistakes that show their lack of understanding of meaning of fractions such as $\frac{1}{2}+\frac{2}{3}=\frac{3}{5}$. As experienced in teaching, developing the understanding of meaning of fractions is a complexity because fractions have many interpretations (Lamon, 2001 in Anderson & Wong, 2007) shown on the following table,

Table 1. Different Interpretations of Fractions ($\frac{3}{4}$)

Interpretations	Example
Part/whole	3 out of 4 equal parts of a whole or collections of objects
Measure	$\frac{3}{4}$ means a distance of 3 ($\frac{1}{4}$ units) from 0 on the number line
Operator	$\frac{3}{4}$ of something, stretching or shrinking
Quotient	3 divided by 4, $\frac{3}{4}$ is the amount each person receives
Ratio	3 parts cement to 4 parts sand

In grade 3, children start to learn the meaning of fractions and comparing fractions. The complexity of learning meaning of fractions seemed does not exist. Children could notate fractions represented in shaded area and also could shade area in geometrical shapes based on fractions given. Meaning of fractions as parts of a whole becomes the only focus of learning process. The problem is that such a meaning of fractions as parts of a whole is not enough to support children in solving problem of fractions that involved various kind representations.

In this article, the researcher wants to explicate children's development of multiple meanings or interpretations of fractions and kind of instructions that can support such developments. The instructions are the parts of teaching design experiment at grade 3 aimed to develop a local instructional theory on learning early fractions particularly about supporting children development in the meaning of fractions and the network of relationships around fractions.

Part-whole relationship is identified as a fundamental subconstruct of understanding of fractions and of all later interpretations of fractions (Kieren, 1988). Streefland (1991) employed contexts of fair sharing as the source of various kind of representations and interpretations of fractions. Dividing 3 pizzas among 4 children provoked children in Streefland's longitudinal study to elicit different representations of fractions ($\frac{3}{4}$, $\frac{1}{2}+\frac{1}{4}$, or $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$) as the starting point of further exploration for the relations around such different representations. Reys (2007) also proposed the similar problem about dividing 3 cookies among 5 people as a context for the quotient interpretation of fractions.

However, Streefland (1991, in Keijzer, 2003) also argued that fair sharing-regarding $\frac{3}{5}$ as three pizzas divided by five children-does not clearly present a fraction as one number or entity, but rather presents a fraction as (a ratio of) two numbers. There are some evidences that using a bar as a model and a number line as an abstraction of bar model can be profitably incorporated into a curriculum aimed at number sense (Keijzer, 1997 in Keijzer, 2003). Keijzer designed an experimental programme which measurements are used to encourage children in developing bar and number line model as the emergent model (Keijzer, 2003).

Measuring activity proposed by Keijzer was in line with the interpretation of fractions as a distance from 0 on number line. In his study, students measured all kind of objects in the classroom with a folded bar. A case study of an average-achieving student showed that this context evoked the student to use informal language of fractions based on the number of pieces of bar that was shortened to the formal notation.

Although there have been many findings on teaching and learning of fractions, more researches using context of Indonesian classroom are needed. Teacher centered learning is often found in Indonesian classroom. Students tend to be a passive learner instead of construct their own knowledge actively (Mujib, 2010). Actually since 2001, a movement to reform mathematics educations has occurred. Pendidikan Matematika Realistik Indonesia (PMRI) adapted from Realistic Mathematics Education in the Netherlands has been implemented in some primary schools in Indonesia.

Yet, the implementation of realistic mathematics as a movement to reform teaching and learning is quite complex. Even for the schools which have started to implement realistic mathematic, it is still in progress. According to Sembiring (2010), improving teacher competence in conducting mathematics teaching and learning based on realistic approach is one of challenges. Teachers still need a lot of supports such as model of teaching and learning using realistic approach.

Research Method

This article is a part of a thesis which the title is “Supporting Students’ Development of Early Fractions Learning”. In conducting the research, design research is chosen as the research method. There are three phases of conducting a design research (Gravemeijer & Cobb: 2006); preparing experiment, design experiment and retrospective analysis. Those phases were conducted in cyclical processes. The researcher applied two cycles which involved two groups of students in grade 3 SD

Laboratorium UNESA. For the teaching experiment, an instructional sequence based on realistic mathematics approach was designed and tested as a hypothetical learning trajectory.

In the first cycle, teaching experiment involved 6 students, namely group A which were chosen according to their achievement in the classroom. After had done the teaching experiment with those 6 students, the researcher did retrospective analysis and revised the instructional sequence to be implemented in the second cycle. All the third graders (28 students) from a classroom, namely group B, were involved in the second cycle. Besides the two (macro) cycles, daily micro cycles were presented in this research which involved new anticipatory thought experiment, the revision of instructional activities and modification of learning goals. During the first cycle, the instructional activities were revised in daily micro cycle based on students' pre-knowledge and ongoing learning process. As the data collection, the researcher did videotaping the learning process in the classroom or group of students, interviewing the students and the teachers, and collecting students' work.

The Results of The Study

Instead of giving the results of the study directly, the researcher will give an overview about the students' learning process corresponds to the instructional design in this research. The researcher will take some critical moments in which a new insight about students' thinking might be generated. The researcher will mix between the experiences in group A and B.

Before doing the teaching experiment in group A, the researcher gave a pre-test to the students. One of the questions is about partitioning object

Mother wants to cut a quarter of a brownies cake below.
Show a quarter part of the brownies cake!

All the students answered with shading area represented a quarter of cake as shown on the following figure

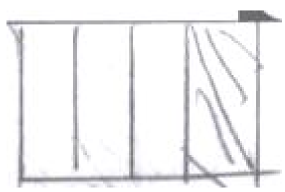

Figure 1. An Example of the Students Answer Pre-test Item

When the researcher posed a question *'Which part that you will give if I asked a quarter of the cake?'*, some students got confused in answering the question. One of the students answered that she would give the whole cake.

Considering such a fact, the activity of doing real partitioning by cutting then was developed to help the student in strengthening the understanding of part whole relationship. The context of fair sharing has provoked the students to have different interpretations of fractions. The absence of shaded parts gave the opportunity for different interpretations of fractions to emerge. Fraction $\frac{1}{4}$ as the result of dividing cake was interpreted as the parts of a whole cake and the division of one cake into 4 equal parts.

Nando	*: There are 4 parts. It means that* $\frac{1}{4}$ *is one part.*
The researcher	*: Where is 1 in the fruit cake?*
Nando	*: 1 means the whole cake*
The researcher	*: How about 4?*
Sasa	*: Here it is. If we cut it, then there will become 4.*
The researcher	*: Is there other opinions?*
Sasa	*: If we have cut the cake, then 1 will become this (taking 1 piece of cake)*

The more challenging problem about dividing 3 brownies cakes among 4 people was given to the students. The students had to cut the model of cake. The learning goal is that the students could produce simple fractions as results of fair sharing. In fact, although the students could partition brownies cake properly, the difficulty was about representing the pieces that each person got using fractions notations. The cutting-resulted pieces made the students could not see the whole of cakes. When the pieces were rearranged into the original shape of cake, then it was easier for the students to notate fractions.

Ignoring cutting activity in the context of brownies cake was helpful for the students in group B to notate results of dividing (Figure 1a and 1b). Interpretation of fractions as parts of a whole was dominant on the students' explanation although non-unit fractions as iterative of unit fractions were potential to emerge (Figure 1b). The difference between considering one cake as the whole and three cakes as the whole became one of focuses of class discussion.

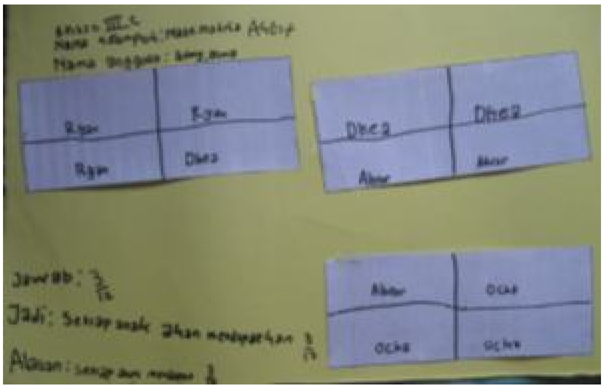

Figure 1a. Each Person Got $\frac{3}{12}$ Parts

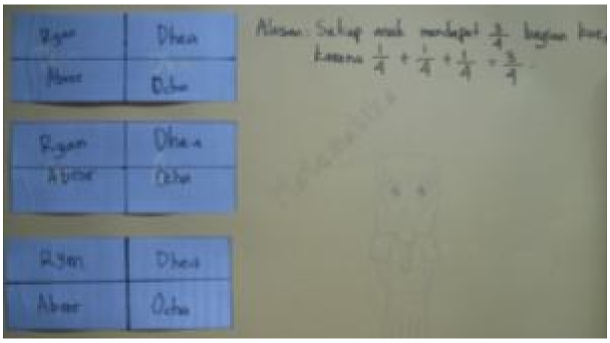

Figure 1b. Each Person Got $\frac{3}{4}$ Parts

Interpretation of fractions as results of division also could be facilitated by using discrete objects. Providing a stack of chocolate bar, the teacher could provoke the students to partition 12 chocolate bars in order to determine the number of chocolate bar that represented $\frac{1}{4}$ of 12 chocolate bar (Figure 2). The students then formalized the strategy by dividing 12 chocolate bars by 4. Interpretation of fractions as division became dominant in the knowledge construction.

Figure 2. The Student Partitioned a Stack of Chocolate Bars

Bridging the context of fair sharing and measuring activity which fractions transformed as a distance from 0, an activity of pouring a glass with $\frac{3}{4}$ full of tea was developed in group A. The researcher asked the students, *'If I want to pour this water into $\frac{3}{4}$ of this glass, how high is it?'*. There was only one student who was able to mark the glass with 4 strips then point the strip that showed $\frac{3}{4}$ (Figure 3). Another student even said that the first strip should be $\frac{2}{4}$ and $\frac{1}{4}$ should be below the first strip.

Figure 3. Students' Marked a Glass

Although there was a contribution from one student about the strategy of partitioning in measuring context, it did not appear in the problem of one glass full of tea that wanted to be distributed into two glasses equally (Figure 4) and in the problem of pouring two glasses full of tea into three empty glasses (Figure 5). The students did not apply partitioning in estimating the height of tea in the glasses.

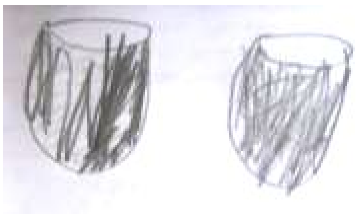

Figure 4. The Result of Distributing One Glass Full of Tea into One More Empty Glass

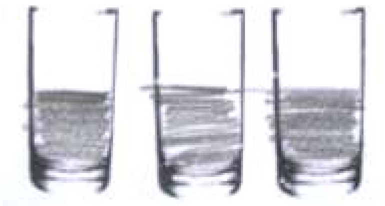

Figure 5. The Result of Dividing Two Glasses of Tea into Three Empty Glasses

As the students got difficult to find the position of certain fraction in the context of measuring, the students showed their lack of knowledge of partitioning to produce fractions in the context of measuring pencil. The learning goal of measuring pencil activity was that the students could produce fractions as results of measuring by iterating unit fractions. Folding paper into some equal parts was intended to represent unit fractions then would be used to measure the length of pencil. The fact was that the students had big difficulty in folding paper. The knowledge of students in partitioning model of cakes could not support the student to fold paper into equal parts. The answer of the students in post-test gave more evidence that the students could not grasp the interpretation of fractions in measuring activity (Figure 6).

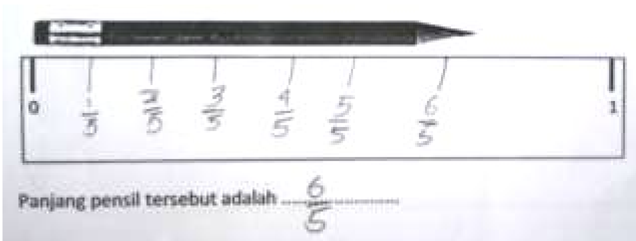

Figure 6. The Student's Answer in Post-test

The failure of measuring activity in supporting the students' understanding of interpretation of fractions particularly as a distance from 0 in number line might be caused by a gap between the context and the pre-knowledge of the students. The students' pre-knowledge of fractions was about representing fractions as parts of a whole using shaded parts in geometrical shapes. The refinement of measuring activity which then involved shading area was developed in the activity of the position of ants. The activity was delivered to the students in group B. The student had to mark the position of ants that stopped when walking to the pile of sugar. One of the questions was

Tom ant is walking to a pile of sugar. He has passed $\frac{1}{4}$ *path. Shade part of path that Tom has passed! Mark the position of Tom!*

Most of the students directly partitioned the path of ant into four to determine the position of ant. Two kinds of strategy applied in this problem were using estimation and standard units of measurement (Figure 7 and 8).

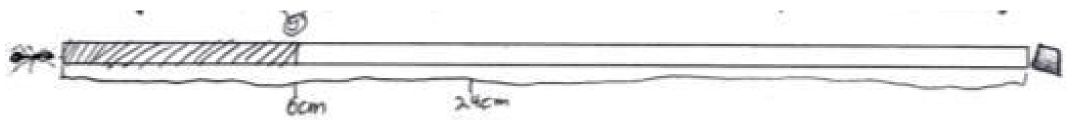

Figure 7. The Student' Answer using Standard Units of Measurement

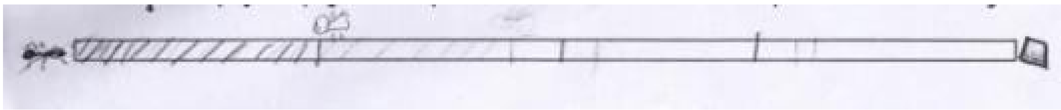

Figure 8. The Student' Answer using Estimation

Unit fractions resulted from this activity then were used to determine non-unit fractions as the position of ant. Those unit fractions were represented with the pieces of ribbon. There were pieces of $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ of the path of ant. Through using the pieces of ribbon, the students figured out that a certain times unit fractions built a non-unit fraction (Figure 9).

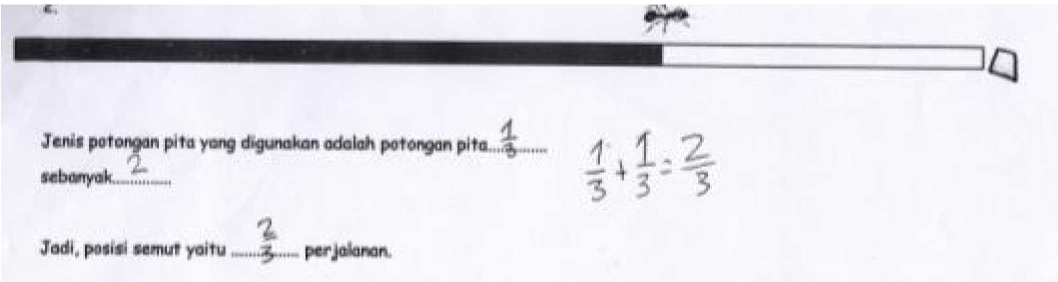

Figure 9. The Student's Answer on Determining the Position of Ant.

Discussion

Although the students could shade the parts properly, it did not guarantee that they understood the basic meaning of a fraction; parts of a whole relationship. Different contexts given to the students also showed that it does not enough for students only learning interpretations fractions as parts of a whole using shaded area. Avoiding students only focus on counting the number of shaded parts out of all parts, partitioning in real action could be delivered to give meaning of fractions notation. Related to the results of partitioning in fair sharing context, the interpretation of fractions as results of division is potential to emerge as recognized in the activity of sharing a cake. Sharing more than one object even could provoke more critical issue about a whole to be discussed. During notating the results of sharing objects, non-unit fractions as a certain time of unit fractions start to be learned by the students in the frame of the size of continuous object. Determining the number of discrete objects gives more emphasis on interpreting fractions as results of division.

The big challenge is about developing contexts that can support students to make a transition from understanding fractions as parts of a whole and as division to the understanding of fractions as a distance from 0 in number line (informal). Partitioning that was no longer explicit in measuring activity makes the students fail to produce fractions. It might be caused by a gap between students' knowledge about part whole relationship and the interpretation of fractions as a distance in a number line. In order to bridge such a gap, the measuring activity should be still strong related to partitioning continuous object. The object could be transformed gradually into bars that lead to number line as the abstraction. In fact, the activity of determining the position of ants just becomes parts of initial support for the students' understanding about interpretation fractions as a distance from 0 in number line (informal). There is a need of more contexts to encourage students' thinking of fraction in the framework of number line.

Conclusion

This study has shown that early fractions learning was not merely about shading a number of parts out of total numbers of parts on any shape. Understanding of meaning of fractions also requires the ability of solving problems related to fractions. On the other hand, through solving fractions problems, students even construct the building of knowledge about fractions. In this study, the researcher tried to deliver fair sharing and measuring activities. By giving such contextual situations, students were given more space to develop their understanding based on their previous knowledge. Students might not always succeed to achieve the learning goals, but the students' struggle could give us, as the teachers, an insight about how students might think. Such an insight was expected to give more information for improving teaching and learning process in the classroom.

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Elisabet Ayunika Permata Sari
State University of Surabaya, Surabaya, Indonesia
Email: ayoe_qthink@yahoo.co.id

Dwi Juniati
State University of Surabaya, Surabaya, Indonesia
Email: dwi_juniati@yahoo.com

Sitti Maesuri Patahudin
State University of Surabaya, Surabaya, Indonesia
Email: s_maesuri@yahoo.com.au