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A COMPARATIVE STUDY OF QUADRILATERALS TOPIC CONTENT IN MATHEMATICS TEXTBOOKS BETWEEN MALAYSIA AND SOUTH KOREA

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Abstract

This study compares Malaysian and Korean geometry content in mathematics textbooks to help explain the differences that have been found consistently between the achievement levels of Malaysian and South Korean students in the Trends in International Mathematics and Science Study (TIMSS). Studies have shown that the use of textbooks can affect students' mathematics achievements, especially in the field of geometry. Furthermore, to date, there has been no comparison of geometry content in Malaysian and Korean textbooks. Two textbooks used in the lower secondary education system in Malaysia and South Korea were referred. The topic of quadrilaterals was chosen for comparison, and the topic's chapter in the South Korean textbook has been translated into English. The findings show four main aspects that distinguish how quadrilaterals are taught between the two countries. These aspects include the composition of quadrilaterals topics, the depth of concept exploration activities, the integration of deductive reasoning in the learning content and the difficulty level of mathematics problems given at the end of the chapter. In this regard, we recommend the Division of Curriculum Development of the Malaysian Ministry of Education reviews the geometry content of mathematics textbook used today to suit the curriculum proven to produce students who excel in international assessments.

Keywords: Geometry, Quadrilaterals, Textbook, Malaysia, South Korea

Abstrak

Penelitian ini membandingkan konten geometri pada buku matematika di Malaysia dan Korea Selatan, untuk membantu menjelaskan perbedaan yang telah ditemukan secara terus menerus antara tingkat pencapaian siswa Malaysia dan Korea dalam hasil *Trends in International Mathematics and Science Study* (TIMSS). Sejumlah penelitian sebelumnya menunjukkan bahwa penggunaan buku teks dapat mempengaruhi prestasi matematika siswa, terutama pada bidang geometri. Selain itu, sampai sekarang, belum ada perbandingan konten geometris di buku teks Malaysia dan Korea Selatan. Dua buku teks yang digunakan dalam sistem pendidikan menengah bawah di Malaysia dan Korea Selatan dirujuk dan salah satu topik geometri yang diajarkan diidentifikasi. Materi Segiempat telah dipilih sebagai topik perbandingan dan bab topik dalam buku teks Korea Selatan telah diterjemahkan ke dalam bahasa Inggris. Hasil penelitian menunjukkan bahwa ada empat aspek utama yang membedakan bagaimana materi segiempat diajarkan pada kedua negara. Aspek-aspek ini termasuk komposisi topik Segiempat, kedalaman kegiatan eksplorasi konsep, integrasi penalaran deduktif dalam konten pembelajaran dan tingkat kesulitan masalah matematika yang diberikan pada akhir bab ini. Dalam hal ini, direkomendasikan kepada Divisi Pengembangan Kurikulum dari Kementerian Pendidikan Malaysia untuk merevisi isi geometri dari buku teks matematika yang digunakan hari ini untuk menyesuaikan dengan kurikulum yang telah terbukti menghasilkan siswa yang unggul dalam penilaian internasional.

Kata kunci: Geometri, Segi Empat, Buku Teks, Malaysia, Korea Selatan

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Classrooms use multiple educational resources. Perhaps the primary source used most often as reference material is textbooks. Textbooks refer to books designed and developed to translate the desired curriculum objectives. They are an important component of the education system and curriculum such that learning at school is synonymous with textbooks. However, according to Koedel, Li, Polikoff, Hardaway, and Wrabel (2017), though textbooks are the most widely used sources of education, studies on their impact on student learning are very limited. Therefore, Fan, Mailizar, Alafaleq, and Wang (2018) state that school textbooks have become subjects of international research. Rezat (2009) found that students use mathematics textbooks not only when they are told by the teacher, but also for self-learning. Also, students make their mathematics textbooks a medium for solving problems, consolidating, acquiring mathematical knowledge, and activities related to the interest in mathematics.

While according to Ceretkova, Sedivy, Molnar, and Petr (2008), textbooks also have several functions: (1) motivational function, i.e. well-written textbooks can stimulate the interest of students reading them, (2) communication functions, i.e. textbooks that can expand their vocabulary including technical terms, (3) regulatory functions, i.e. the curriculum divided into parts that can elaborate sequence logically, (4) application function, i.e. it consists of ideas using things that practice and express examples of real life, (5) integrated function, i.e. textbooks that are not tied to their subjects but contain links to other disciplines that lead to more complex cognitive processes, (6) innovative functions that introduce new knowledge, and (7) control and corrective functions which students use text, exercises and problems to test themselves, students discover what they understand or do not understand, and they are reviewing the matter. According to Lepik, Grevholm, and Viholainen (2015), textbooks are equally important resources for both students and teachers. Students use them to learn mathematics and teachers use them for planning and teaching mathematics lessons. Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) stressed that the structure of mathematical textbooks might have an impact on teaching in the classroom. They state that the shape and structure of textbooks affect different pedagogical models in mathematical classes.

COMPARISON OF GEOMETRY CONTENT IN TEXTBOOKS

Many studies compared mathematics textbooks as a whole, with some studies focusing on geometry. Among the earliest was the study by Kim (1993) comparing the geometry content of South Korean and US textbooks. His study found that geometry content in American textbooks was spirally arranged as the same topic emerged and extended to many grade levels. On the contrary, in Korean textbooks, geometry content was structured so that a concept or skill dominated each grade level. In Kelley (2013), eight American textbooks were studied from the 1980s and 2000s with the aim of identifying the differences between the two textbook groups in terms of approaches to teaching proof and writing of geometric proofing. All exercises in each text were encoded using parameters such as proof, type of proof, and reasoning task. It was found that new textbooks incorporated conjecture-based learning for the theorem and paid more attention to the evidence in the context of geometric reasoning. Hsu and Ko (2014) compared the geometry content of teaching materials in the mathematics textbooks of Taiwan, Finland, and Singapore. Content analysis was used, and mathematics problems were analysed. Problems were classified based on their cognitive type, representational form, and context. The study showed that most problems were classified as 'procedure without connection' with only a few problems under the 'doing mathematics' category. Most of the contexts of problems and representations in the three countries are non-contextual and visual forms. The obvious differences between the three countries are the presentation of problem examples and the ratios between examples of concepts and mathematics problems. From the aspect of problem delivery, Taiwan and Singapore provide a more detailed and focused process to help students solve the problems, but brief explanations and demonstrations are found in textbooks. The ratio between examples and mathematics problems is around 1:3 in Taiwan and Singapore and 1:25 in Finland.

Usiskin et al. (2008) conducted a micro curriculum analysis using a variety of textbooks in the United States on the concept of quadrilaterals. It discussed the issue of how a particular quadrilateral can be mathematically defined in the same way, and that definition can be inclusive or exclusive. Furthermore, the geometry thinking level in the van Hiele model has been used as the basis of analysis of primary and secondary school textbooks (grades K to 8) used in 42 States of the United States. Newton (2010) reported that learning objectives were in line with the general principles of the van Hiele theory, especially the principle that the level of geometry thinking is sequential. Fujita (2012) proposed an arrangement of plans to engage students with complex quadrilaterals definition hierarchical issues, which involve nurturing students' understanding of quadrilaterals concepts, encouraging them to seek inclusive relationships between definitions and properties, and critical and non-critical discussion based on what they already know.

Mironychev (2016) compared the sequence of topics in geometry courses in high school curricula in Texas, USA and Russia. Four textbooks used in Texas and Russia were selected for this comparison. The objectives of this research were to compare the sequence of topics in the course, determine how the sequence of topics corresponds to the objectives of the geometry topic, and understand why the course topics are structured in such ways. Mironychev (2016) used two approaches in developing geometry courses, namely (1) Topic approach - when parts of the book are arranged accordingly with object/ terms difficulties after consideration, and (2) Proof of evidence - when parts of the book were arranged according to theoretical evidence or form properties. His study found that in the Texas book (HISD), topics were arranged by object, without proof. In the books used in Moscow, the content was compiled in the order of proof. These books were used in geometry courses for different periods. For HISD, students learn this subject for only one year. Therefore, there were not many opportunities to explore the properties of geometry objects in sequence. The main focus was on applying the formula and the nature of the different calculation steps. In conclusion, the geometry textbook used in Texas is easy to use in short courses. They do not need in-depth analysis of geometry properties and are easily understood by students. Many calculating problems help to develop practical skills for applying the nature of learning in life. Russian geometry textbooks were more suited for advanced courses. They pay more attention to the subject's basics and logical relationships. They are suitable for high-level courses such as pre-university.

Wang and Yang (2016) compared the differences in the use of geometry in primary school textbooks between Finland, China, Singapore, Taiwan, and the United States. The results showed significant differences in representation, problem types, and question formats between mathematical textbooks of the five countries. In Singapore, mathematical textbooks focus primarily on visual forms combined with other forms of representation. There were significant differences between contextual and non-contextual geometry questions between the five mathematical textbooks. In particular, Chinese textbooks have the highest percentage of contextual problems. Mathematical textbooks from China and the US have more open-ended geometry questions.

The main objective of Silalahi and Chang (2017) was to identify geometry equations and differences by analysing the contents of junior high school mathematics textbooks (grades 7-9) in California, Singapore and Indonesia. They found that problem-solving questions were provided at the end of subtopics in geometry textbooks in California and Singapore but not in Indonesia. In contrast, the similarities from California, Singapore and Indonesia were that all three textbooks provided more non-application problems than applying questions. Yang, Tseng, and Wang (2017) analysed geometry problems in four series of high school mathematics textbooks from Taiwan, Singapore, Finland, and the United States. The analytical framework developed for the analysis of mathematics text problems has three dimensions: representational form, contextual feature, and type of feedback. The findings showed that Taiwan and Singapore textbooks contain more problems in combination, while Finnish and American textbooks contain more problems in both oral and visual forms. The problem distribution across various forms of representation is more balanced in Finnish and Singaporean textbooks than in Taiwanese and American textbooks. Most problems are non-application and close-ended compared to the application and open-ended problems. The Taiwanese textbook contains the lowest actual situation problems, rather than the American textbook that has the highest open-type problem. Wong (2017) discussed the opportunity for students to study the proving and reasoning of the geometry topics of the school's mathematical textbook in Hong Kong. The results showed that the Hong Kong Education Ministry took a traditional approach where the proof was taught mainly in geometry, and two-column proofing was emphasised. Overall, the results show that proofing plays a marginal role in mathematics schooling in Hong Kong.

Cao (2018) compared 3-D geometry content in American and Chinese textbooks. His study showed that the main topic of 3-D geometry in the US curriculum is the volume and surface area of a prism, pyramid, sphere, and real-world objects. The US curriculum emphasises connecting 3-D geometry to the real life of students through mathematical modelling. In China, the main topics required in the curriculum are abstract reasoning in spatial positional relationships, parallel relationships, perpendicular relationships and angles, and combining algebraic methods with spatial vectors. Volume and surfaces of three types of polyhedrons (prisms, pyramids, and pyramid frustums), and four types of solid revolutionary (cylinders, cones, circular frustums, and spheres) are required, but few are found in the Chinese curriculum as opposed to abstract reasoning. Both countries have very different topics in 3-D geometry texts. In the United States, the 3-D main geometry topics taught at school are volume, surface area, and categorisation of objects like a prism and real-world or composite solids. On the contrary, in China, volume and surface area are not the main focus. On the other hand, spatial position relationships, parallel relationships, perpendicular relationships and angles based on abstract graphs, as well as real-world or composite solids and prisms become the main 3-D geometry topic. The findings revealed that the topics found in Chinese texts are quite complex and have a broad spectrum. Also, the content load and cognitive demand are higher than the US text.

LACK OF COMPARATIVE MATHEMATICS EDUCATION STUDIES BETWEEN MALAYSIA AND SOUTH KOREA

Many countries have made South Korea the basis of comparison in mathematics education, particularly in the areas of curriculum, pedagogy and assessment due to the excellent performance of South Korean students in the subject. The many studies on mathematical curriculum include Kuang, Yao, Cai, and Song (2015) concerning the difficulty level of primary school mathematics textbooks in South Korea, and other countries such as France, Russia, Japan and China; Cao, Wu, and Dong's (2017) study on the difficulty level of mathematical textbooks in junior high schools in China, USA, South Korea, Singapore and Japan; Son and Senk's (2010) analysis of the development of the concept of multiplication and division of fraction in two curricula: Everyday Mathematics (EM) from the United States and Korean mathematics curriculum; and Kim's (2012) study of non-textual elements in South Korean and US mathematical textbooks using a conceptual framework that includes accuracy, connectivity, contextuality and conciseness; Shin and Lee's (2018) study on how mathematical textbooks in Korea and the United States helped in the development of student learning from the aspects of recursive partitioning, common partitioning, and distributive partitioning. Studies were also conducted for algebraic learning. Hwang (2004) concluded several elements are distinguishing the South Korean mathematics curriculum with that of the British. He identified that in South Korea, algebraic content is exposed only once to the students, while in England, the algebraic content tends to be repeated or evolving at every level. The algebraic curriculum in England emphasises approximation, mental calculations, and the use of calculators. Consequently, the English mathematics curriculum is less concerned with writing methods and introduces the written approach slightly later than the Korean curriculum. The English curriculum uses a more flexible approach through rounding, mental methods, calculator usage, ratio, and proportion, while the South Korean curriculum emphasises formal and abstract mathematical knowledge and the understanding of certain mathematical concepts. All mathematical content implemented in England and South Korea is provided in the national mathematics curriculum.

Choi and Park (2013) compared the curriculum standards, textbook structure, and textbook items for geometrics topics between the U.S. and Korea. The study found that the Korean curriculum standards do not focus on real-life situations and the textbooks used in the study only included a few real-life application problems. The study also found that the American CMP textbooks begin each section with real-life examples and activities that can familiarise students with abstract ideas, while Korean textbooks introduce real-world situations related to the lessons without any activity or examples that promote student engagement in actual real-world problem-solving situations. Only a small number of life-related problems are found at the end of each part of the textbook. On the topic of probability, Han, Rosli, Capraro, and Capraro (2011) found that Malaysian, South Korean and American textbooks are routine, open-ended, and non-contextual.

From the pedagogical perspective, several comparative studies compared pedagogical practices and implementation in South Korean mathematics classes despite the study of Mustafa, Evrim, and Serkan (2016) indicating that pedagogical practices are not related to South Korean students' achievement in mathematics. In Siraj-Blatchford and Nah (2013), the pedagogical practices in mathematics classes in England and South Korea were compared in the areas of cultures, classroom activity observations and document analysis. Teachers in both countries use integrated activities to teach mathematics. In England, mathematics classes are more structured, more dominated by teachers and less holistic, while the classes are more structured and didactically independent in South Korea. Mathematics education in the UK is more systematic, using more individualised approaches and incorporates a wide range of hands-on activities and comprehensive outdoor activities, while in Korea, mathematical activities are more group-oriented, use limited material and less outdoor activities. Leung and Hew (2013) examined the use of counterexamples, which play a role in encouraging deductive reasoning skills in mathematics learning process among South Korean and Hong Kong students. O'Dwyer, Wang, and Shields (2015) examined eighth-grade teaching practices in the United States, South Korea, Japan and Singapore that support students' conceptual understanding as well as studied the relationship between practice and mathematical tests.

However, in the context of mathematics education, not many comparative studies exist on the differences between Malaysian and South Korean in the perspective of curriculum, pedagogy and assessment. Studies have shown that for mathematics education, most comparative studies were carried out between Malaysia and Singapore. Ibrahim and Othman (2010) and Ahmad (2016) were among the studies which compared the Malaysian curricula with its Singaporean counterpart. Ibrahim and Othman (2010) concluded that there was a need for the Malaysian mathematics curriculum to be revised to enable mathematical literacy among students and for them to be able to apply mathematics into other disciplines at higher educational levels. Han, Rosli, Capraro, and Capraro (2011) examined the analysis of Malaysian, South Korean and US textbooks on the topic of probability. Ismail and Awang (2008) and Thien and Ong (2015), on the other hand, studied the factors that contribute to the success of Singaporean students in the field of mathematics.

Based on the literature review, most comparative studies on components of mathematics curriculum as well as pedagogy and assessment in mathematical classes were conducted on South Korea and other countries but not Malaysia. The only comparative study in the curriculum perspective in which Malaysia was compared to South Korea was Han, Rosli, Capraro, and Capraro (2011). Most comparative studies in mathematics curriculum components were carried out between Malaysia and Singapore.

A COMPARISON OF GEOMETRY CONTENT DOMAIN ACHIEVEMENTS IN TIMSS BETWEEN MALAYSIA AND KOREA

Malaysia joined the Trends in International Mathematics and Science Study (TIMSS) assessment for the first time during the second cycle in TIMSS 1999. To date, Malaysia has participated in six TIMSS cycles in TIMSS 1999, 2003, 2007, 2011 and 2015 and 2019. Malaysia's participation is to foster effective science and mathematics learning among students compared to their peers in other countries. Since Malaysian participation in TIMSS, the best achievement of Malaysia was during the first participation in TIMSS 1999 at 519 points and above the TIMSS average score of 500 points. However, there were declines in performance after TIMSS 1999 in the next three cycles, whereby Malaysian students scored 508 points in TIMSS 2003 which is still above the TIMSS average score. In TIMSS 2007 and 2011, the score of the mathematics achievement was 474 and 440 respectively. However, an increase of 25 points to reach 465 points in TIMSS 2015 renders the fifth round of the assessment the fourth highest ever since TIMSS 1999. Even though there was an increase in points, Malaysia's performance is still on the Low-Level Benchmarking and is below the TIMSS average score.

South Korea has been involved in the TIMSS since its establishment. South Korea has achieved remarkable achievements throughout the involvement of TIMSS. Throughout the participation in TIMSS, South Korea is one of the top three countries with an average score of achievement for grade 4 and grade 8 students in mathematics subjects compared to other countries. In TIMSS 1999, assessment is only done for grade 8. Based on the findings, if we compare with the minimum score set by TIMSS of 500, the average score of South Korean Mathematics is at a very satisfactory level. When referring to the measurement level in TIMSS, the average score of South Korean Mathematics is in the category of high international benchmarking. Thus, we can conclude that South Korean students can apply basic mathematics knowledge in difficult questions and non-routine problems. TIMSS has organised into two domains namely, (1) content domain which refers to the subjects to be evaluated in mathematics, and (2) cognitive domain which focuses on the thinking processes expected from students as they engage in mathematical content. Figure 1 shows that from TIMSS 1999 until TIMSS 2015, 8^{th}-grade South Korean students outperformed Malaysian 8^{th}-grade students in geometry domain.

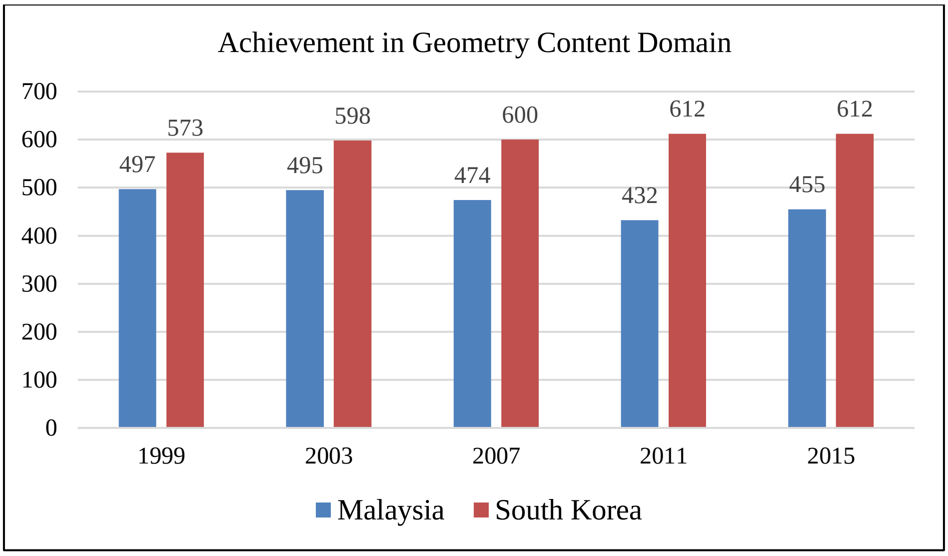


Figure 1. Achievement in Geometry Content Domain Between Malaysia and South Korea

Figure 2 shows a question in geometry domain in which 87% of South Korean students answered the question successfully as opposed to only 32% of Malaysian students. The percentage of Malaysians is not just below the international average, but also among the lowest countries. In this regard, the study compares one of the geometry topics in a textbook at the lower secondary level in Malaysia and South Korea. In the TIMSS 2011 report (Mullis et al., 2012), mathematics teachers of both countries reported that they use mathematical textbooks as a major source in mathematical classes.

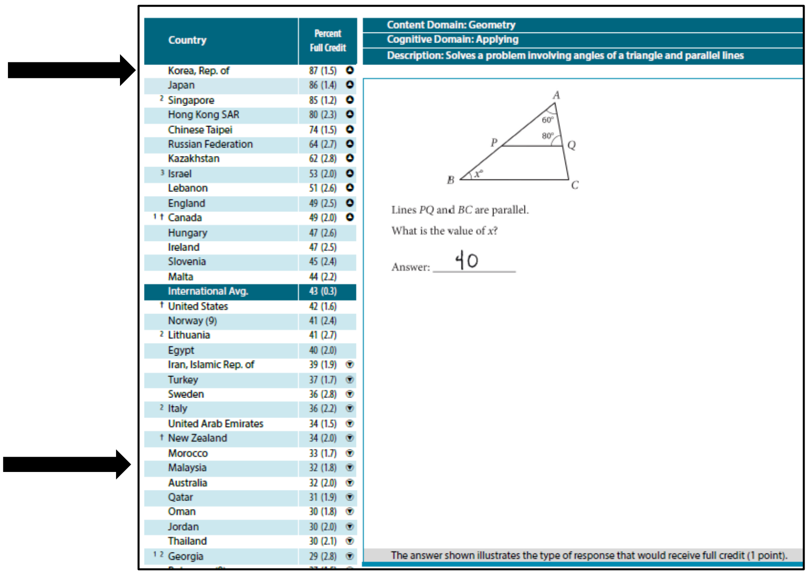


Figure 2. A Sample of Domain Geometry Question in TIMSS 2015

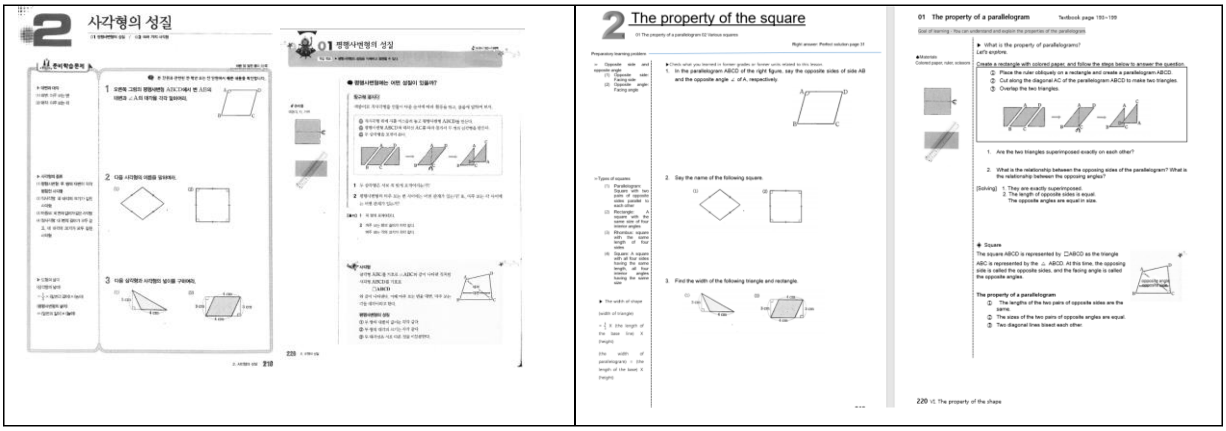
Source: Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016)

Previous studies have found a significant relationship between the textbook used and students' achievement in mathematics (Tornroos, 2005). The Hadar Study (2017) discussed the correlation between learning opportunities provided in mathematical textbooks and student achievement in national examinations. The findings show that students who use different textbooks have different scores in the national examination. If a textbook gives the students the opportunity to engage in a task that requires a higher level of understanding, students who use this book will obtain a higher score. Xin (2007) examined the potential impact of learning opportunities provided in a USA mathematical textbook and a Chinese textbook on the achievement of student problem-solving. Additionally, Xin studied the learning opportunities provided in textbooks by analysing the problematic distribution of problems across a wide range of problems, as well as the potential impact of learning opportunities on students' ability to solve arithmetical problems. The research shows a positive correlation between the presentation of problem task in textbook and students' mathematical problem-solving skills.

Furthermore, no previous study compared geometry content in mathematical textbooks between Malaysia and South Korea. Choi and Park (2013) analysed the comparison of geometry education related to curriculum standards, textbook structure and items in textbooks between the United States and Korea. While Hong and Choi (2018) analysed and compared the opportunities of reasoning and proofing activities in geometry lessons from American and Korean textbooks to understand how the textbook provides students with the opportunity to engage in reasoning and proving activities. Therefore, this study compares Malaysian and Korean geometry specifically for quadrilaterals topic content in mathematics textbooks to help explain the differences that have been found consistently between the achievement levels of Malaysia and Korean students in the TIMSS especially in geometry content domain.

METHOD

The textbook used as a comparison is the main mathematics textbook used in the education system in both countries. Quadrilaterals topic was selected because education systems in both countries teach the same topic, as well as comparable content. As shown in Figure 3, for South Korea, the content of the selected topic was then translated into English. For Malaysia, the English version of the mathematics textbook was used in this study. This study adapted the framework by Morgan (2004), which looks at content and structure, while also referring to Gracin (2018) who looked at content, cognitive demands, question types and mathematical activities. In this regard, this study examines the composition of quadrilaterals topics, the depth of concept exploration activities, the integration of deductive reasoning in the learning content and the difficulty level of mathematical problems given at the end of the chapter. The van Hiele Model and the Revised Bloom Taxonomy are the basis of comparison in this study. The van Hiele model has been a subject of continuous academic research in geometry and has been applied in various geometry studies (Battista, 2002; Bruni & Seidenstein, 1990; Clement & Battista, 1992; Halat, 2008; Noraini, 2005). Many researchers have recognised the geometry model of van Hiele (Fuys & Liebov, 1997; Usiskin, 1982). Battista (2002) also noted that students' thinking patterns on two-dimensional geometry is clear and best described by van Hiele's geometry thinking model. Researchers argued that lower secondary students are usually able to achieve up to three levels of van Hiele's geometry thinking of informal deduction (Husnaeni, 2006; Saifulnizan, 2007; Usiskin, 1982). NCTM (1989, 2000) emphasised that the van Hiele model can be applied in to effectively teach geometry. NCTM also emphasised the importance of structured learning as proposed in the van Hiele model. The revised Bloom Taxonomy (Anderson & Krathwohl, 2001) is often used as a framework in differentiating the difficulty of questions, especially in mathematics subject. There are six levels in the taxonomy which are knowing, understanding, applying, analysing, evaluating and creating.



Korean language English language

Figure 3. Translation of Quadrilaterals Content Topic in South Korean Textbook

RESULT AND DISCUSSION

The Content Arrangement of Quadrilateral Topics

The content of quadrilaterals in the South Korean mathematics curriculum is based on the van Hiele model. Based on Figure 4, the content of this topic is collected by asking students to recognise the names of the quadrilaterals in South Korean textbook. This is in line with the first level in the van Hiele model which is known as visualisation. At this point, students recognise geometrical shapes. For example, students can identify the names of the quadrilateral group such as rectangle, square, parallelogram, trapezium and so on. However, no such activity is found in Malaysian mathematics textbooks.

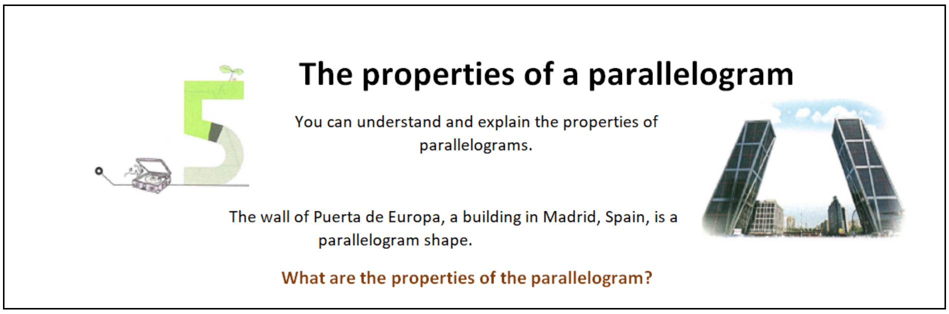
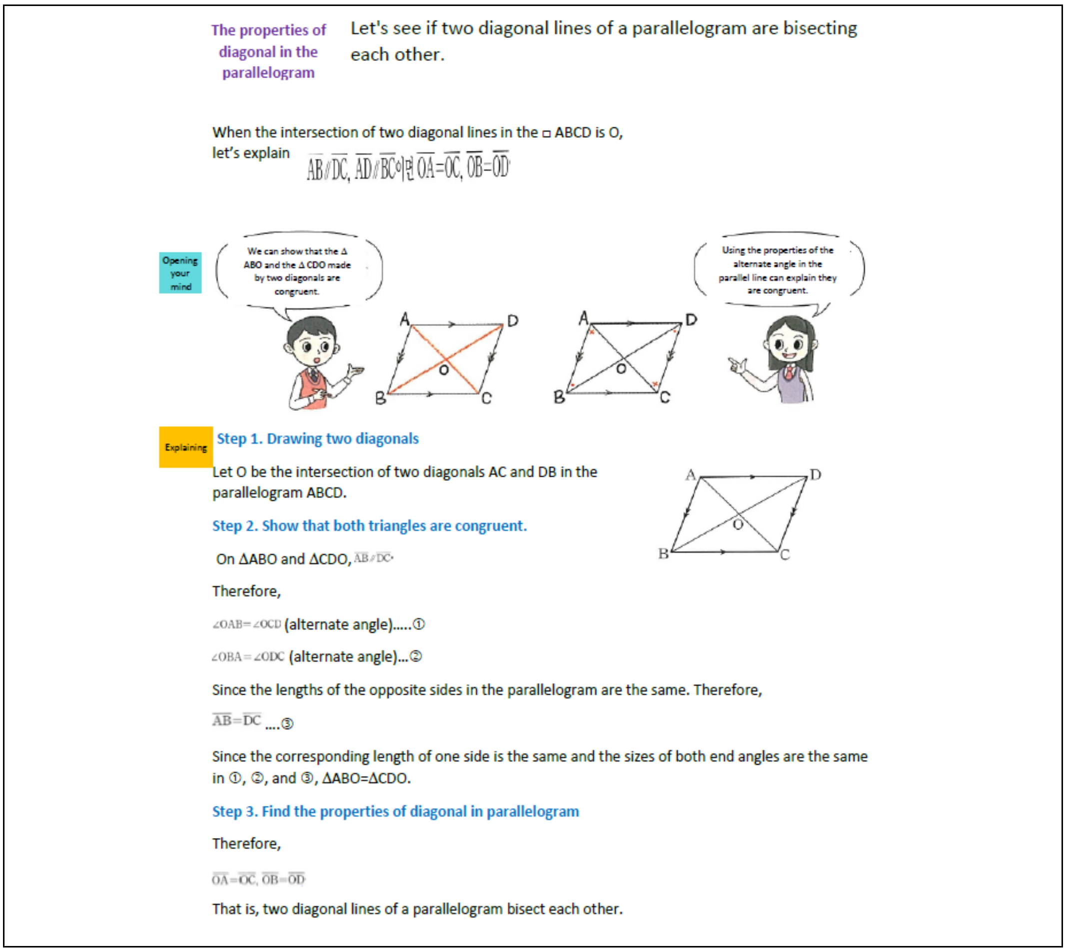


Figure 4. Students are introduced with the shape (Ministry of Education Korea, 2018, p. 165)

The subsequent activities provide students with the opportunity to explore a variety of quadrilaterals. Many of the activities provided in South Korean textbooks are hands-on by using manipulative materials. This is in line with the second level of the van Hiele model which is analysis. At this level, students will be able to recognise the characteristics of shape through observational and experimental activities. Figure 5, for example, shows the activity in which students are to look for properties of parallelogram. According to the National Academies of Sciences, Engineering, and Medicine (2015), the current curriculum focuses on manipulative activities in which students can achieve intuitive ideas about the topic they are currently learning as well as enhance their creativity. Such activities provide more time for creativity and foster a positive attitude towards mathematics. This is important as the TIMSS 2015 results have shown that even though South Korean students displayed encouraging results in mathematics, they scored among the lowest attitudes towards the subject.


The properties of diagonal in the parallelogram

Let's see if two diagonal lines of a parallelogram are bisecting each other.

When the intersection of two diagonal lines in the ▱ ABCD is O,
let's explain $\overline{AB} /\!/ \overline{DC}$, $\overline{AD} /\!/ \overline{BC}$이면 $\overline{OA}=\overline{OC}$, $\overline{OB}=\overline{OD}$

Opening your mind

We can show that the Δ ABO and the Δ CDO made by two diagonals are congruent.

Using the properties of the alternate angle in the parallel line can explain they are congruent.

Explaining

Step 1. Drawing two diagonals

Let O be the intersection of two diagonals AC and DB in the parallelogram ABCD.

Step 2. Show that both triangles are congruent.

On ΔABO and ΔCDO, $\overline{AB} /\!/ \overline{DC}$

Therefore,

$\angle OAB = \angle OCD$ (alternate angle).....①

$\angle OBA = \angle ODC$ (alternate angle)...②

Since the lengths of the opposite sides in the parallelogram are the same. Therefore,

$\overline{AB}=\overline{DC}$ ....③

Since the corresponding length of one side is the same and the sizes of both end angles are the same in ①, ②, and ③, ΔABO=ΔCDO.

Step 3. Find the properties of diagonal in parallelogram

Therefore,

$\overline{OA}=\overline{OC}$, $\overline{OB}=\overline{OD}$

That is, two diagonal lines of a parallelogram bisect each other.

Figure 5. Exploring the characteristics of a quadrilateral (Ministry of Education Korea, 2018, p. 168)

Once students have understood the characteristics of quadrilaterals, they would be able to make connections between them. Figure 6 illustrates the relationship between quadrilaterals. According to the van Hiele model, the third level is an informal deduction. In this level, students can see or prove the relationship between shapes and create a relationship.

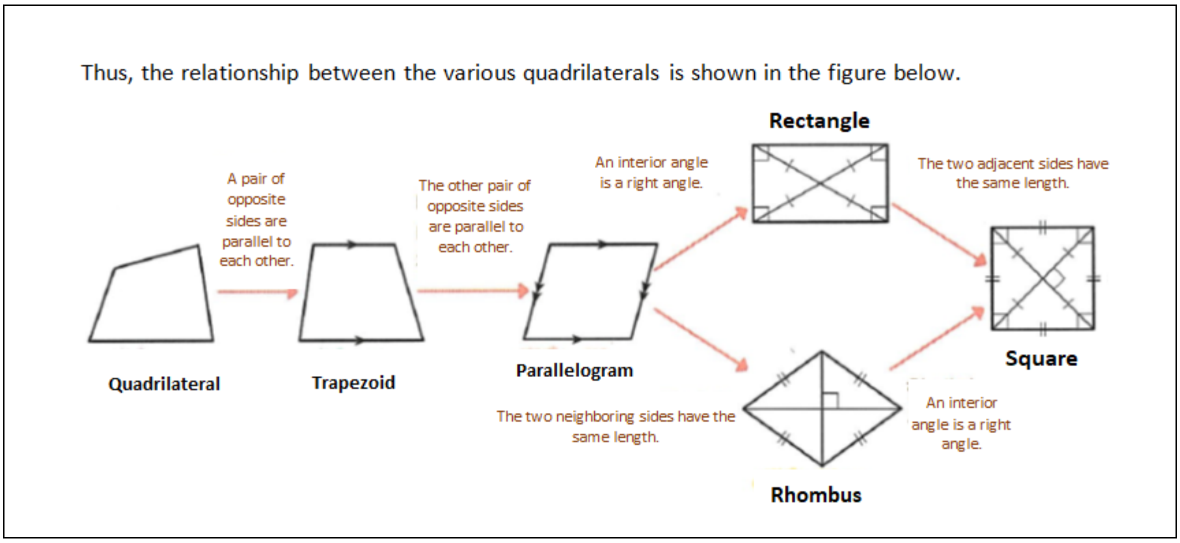


Figure 6. Relationship between the quadrilaterals (Ministry of Education Korea, 2018, p. 182)

Judging from the Malaysian textbooks, quadrilaterals are not compiled based on the van Hiele model. No quadrilateral identification activities were noted in the beginning of the chapter. Students are exposed to the activities of finding quadrilaterals' properties through dynamic geometry software as shown in Figure 7.

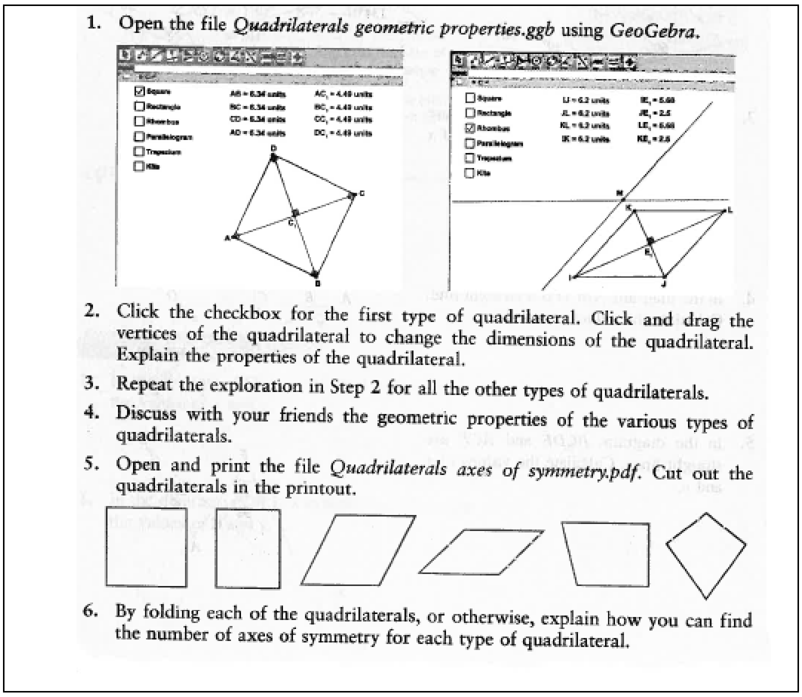
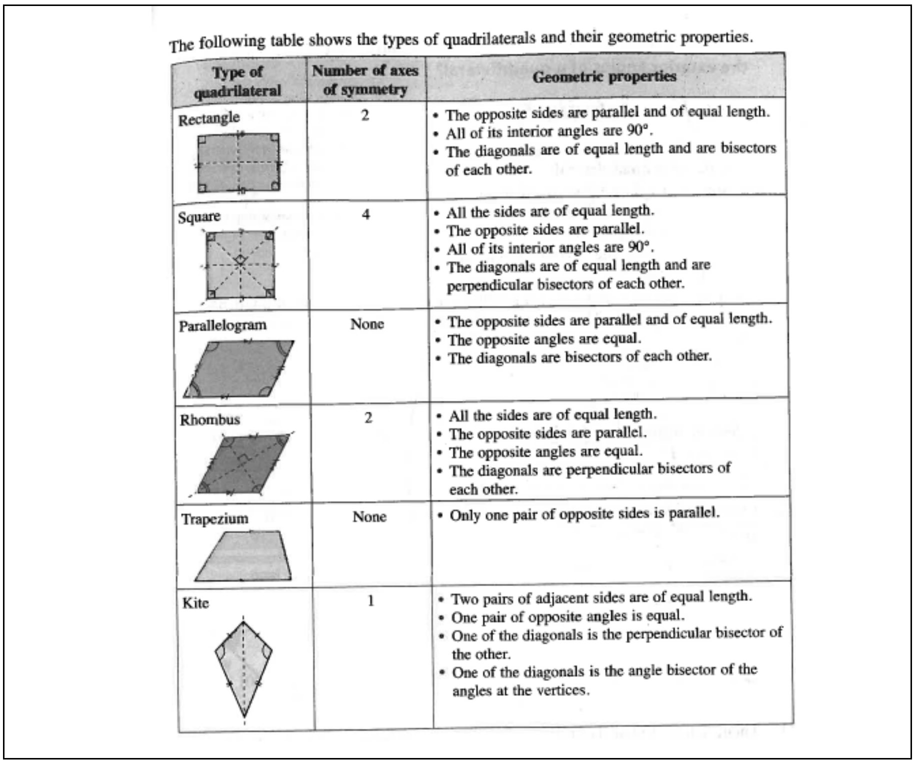

1. Open the file *Quadrilaterals geometric properties.ggb* using *GeoGebra*.
2. Click the checkbox for the first type of quadrilateral. Click and drag the vertices of the quadrilateral to change the dimensions of the quadrilateral. Explain the properties of the quadrilateral.
3. Repeat the exploration in Step 2 for all the other types of quadrilaterals.
4. Discuss with your friends the geometric properties of the various types of quadrilaterals.
5. Open and print the file *Quadrilaterals axes of symmetry.pdf*. Cut out the quadrilaterals in the printout.
6. By folding each of the quadrilaterals, or otherwise, explain how you can find the number of axes of symmetry for each type of quadrilateral.

Figure 7. Activity of identifying characteristics of quadrilaterals using the GeoGebra application (Ministry of Education Malaysia, 2016, p.212)

Figure 8 describes the properties of quadrilaterals. Information presented in such a way could encourage facts memorisation among students. According to Boyraz (2008), Brahier (2005), and Faucett (2007), for geometry content, most textbooks in encouraging memorisation and discouraging effective learning. Active activities involving students are limited (Nik Azis, 1992). No activity in the form of a conjecture investigation is included in textbooks and theorems are only delivered by the teacher through textbooks (Gillis, 2005).



The following table shows the types of quadrilaterals and their geometric properties.

Type of quadrilateral	Number of axes of symmetry	Geometric properties
Rectangle	2	• The opposite sides are parallel and of equal length. • All of its interior angles are 90°. • The diagonals are of equal length and are bisectors of each other.
Square	4	• All the sides are of equal length. • The opposite sides are parallel. • All of its interior angles are 90°. • The diagonals are of equal length and are perpendicular bisectors of each other.
Parallelogram	None	• The opposite sides are parallel and of equal length. • The opposite angles are equal. • The diagonals are bisectors of each other.
Rhombus	2	• All the sides are of equal length. • The opposite sides are parallel. • The opposite angles are equal. • The diagonals are perpendicular bisectors of each other.
Trapezium	None	• Only one pair of opposite sides is parallel.
Kite	1	• Two pairs of adjacent sides are of equal length. • One pair of opposite angles is equal. • One of the diagonals is the perpendicular bisector of the other. • One of the diagonals is the angle bisector of the angles at the vertices.

Figure 8. Various characteristics of quadrilaterals presented in a table (Ministry of Education Malaysia, 2016, p.213)

The content of this topic is then formulated in the form of classification as shown in Figure 9 without specifying the relationship between one quadrilateral and another.

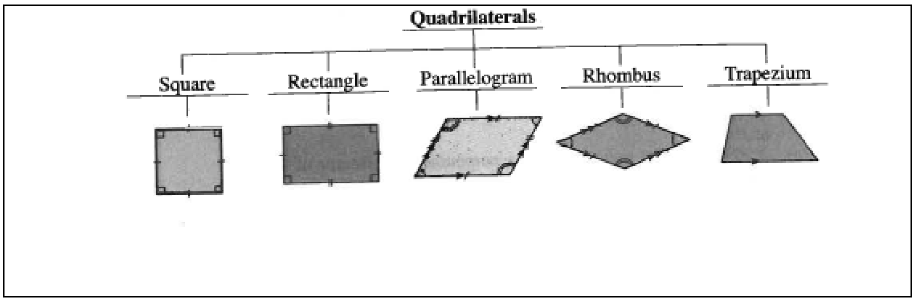
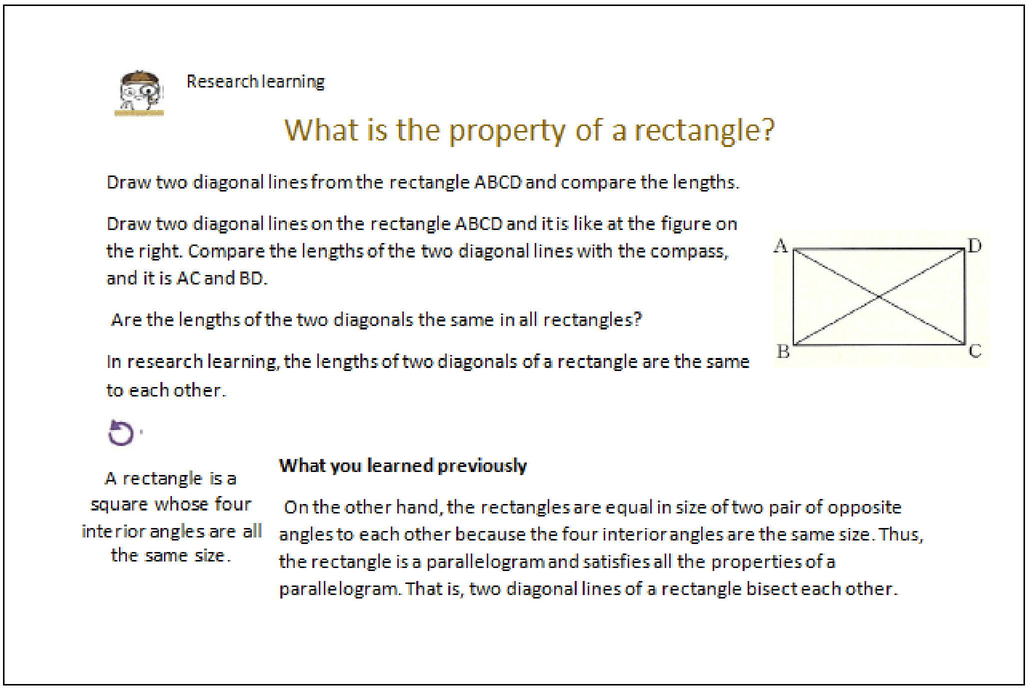


Figure 9. Classification of Quadrilaterals (Ministry of Education Malaysia, 2016, p.219)

Depth Concept Exploration Activities

The second aspect that distinguishes the content of this topic is the depth of the activities of a given concept. Compared to Malaysia which only provides the look of quadrilaterals properties using dynamic geometry software, as shown in Figure 10, the curriculum in South Korea provides immersive exploration activities with questions that test their thinking skills in each quadrilaterals concept. Ozlem and Jale (2011) showed that the learning process enriched with hands-on activities could improve students' achievement compared to traditional methods. Many studies show that hands-on learning, if often integrated into the learning process, can enhance students' cognitive achievements (Scharfenberg & Bogner, 2010; Thompson, & Soyibo, 2002; Revina, et al. 2011). In the South Korean example, hands-on activities are provided for each type of quadrilaterals. The learning method of discovery is a learning practice that involves students actively, is process-oriented and more focused on self-learning (Agus, Dian, & Ajat, 2017). Based on the results of the study by Sinambela, Napitupulu, Mulyono, and Sinambela (2018), there is a positive impact on learning methods through the discovery of the students' understanding of the mathematical concept, while Yunita, Wahyudin & Sispiyati (2017) concluded that the discovery method would enhance mathematical problem-solving skills. The findings of Balim (2009) study using the findings of learning discoveries showed that there is a significant difference in academic achievement among the students in the experimental group compared to the students in the control group concerning academic achievement, learning retention score, and the perception score on the study skills either at the cognitive or effective levels.


Research learning

What is the property of a rectangle?

Draw two diagonal lines from the rectangle ABCD and compare the lengths.

Draw two diagonal lines on the rectangle ABCD and it is like at the figure on the right. Compare the lengths of the two diagonal lines with the compass, and it is AC and BD.

Are the lengths of the two diagonals the same in all rectangles?

In research learning, the lengths of two diagonals of a rectangle are the same to each other.

A rectangle is a square whose four interior angles are all the same size.

What you learned previously

On the other hand, the rectangles are equal in size of two pair of opposite angles to each other because the four interior angles are the same size. Thus, the rectangle is a parallelogram and satisfies all the properties of a parallelogram. That is, two diagonal lines of a rectangle bisect each other.

Figure 10. Discovery activities in South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 177)

When students discovered and understood the properties of the rectangle, they will then be given low-level questions and questions that can improve their high-level thinking skills. Students are given a simple question (see Figure 11) followed by difficult questions (see Figure 12).

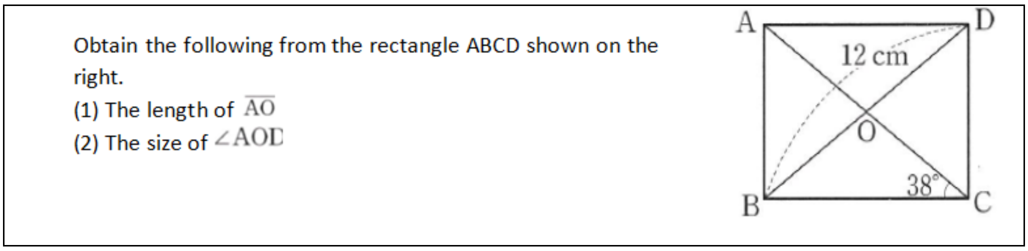


Figure 11. Low-level question for rectangle concept (Ministry of Education Korea, 2018, p. 178)

However, no such activity is found for this topic in Malaysian textbooks. After the introduction of the quadrilaterals concept, Malaysian textbooks introduced interior and exterior angles of the quadrilaterals while in the South Korean context, the concept is introduced together with each type of quadrilateral.

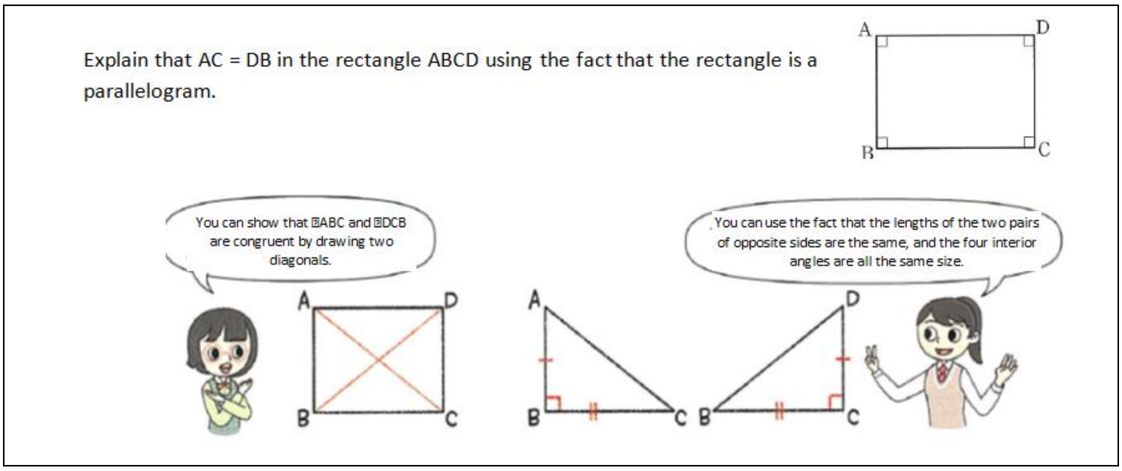
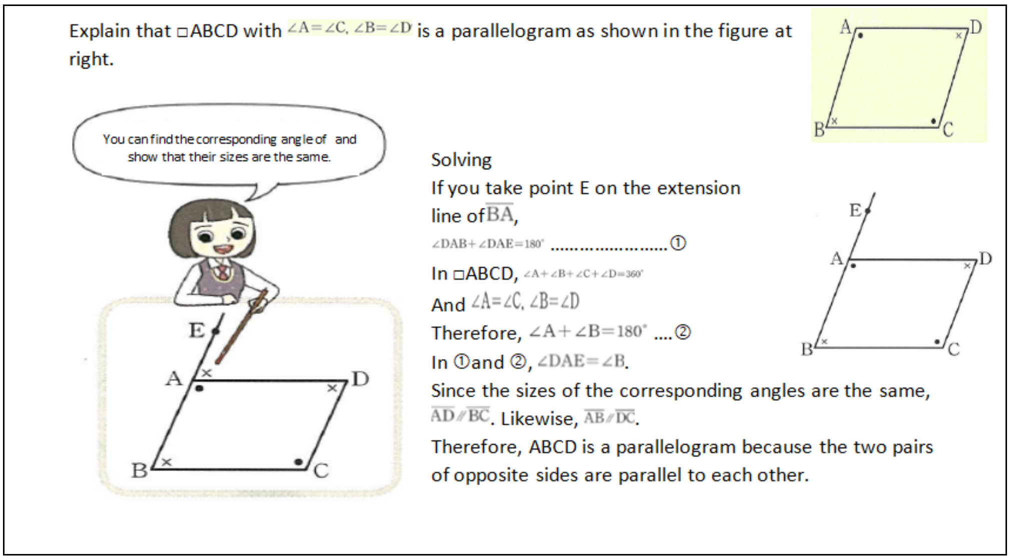


Figure 12. High-level question for rectangle concept (Ministry of Education Korea, 2018, p.177)

Integration of Deductive Reasoning in Learning Content

Although according to Husnaeni (2006), Saifulnizan (2007) and Usiskin (1982), lower secondary students are usually able to reach up to three levels of geometry thinking based on van Hiele's model of informal deduction, the South Korean textbooks extends to the fourth level of formal deductions. The fourth level of the van Hiele model is the deduction level. Students at this level understand the meaning and importance of deduction and the role of postulate, theorem and evidence. They are able to prove themselves of their understanding. They also understand that the proving process can be done in more than one way and the proof is not obtained by memorisation (Crowley, 1987). The most fundamental reasoning is logical reasoning that consists of inductive reasoning and proven reasoning. Inductive reasoning is one of the reasoning processes which requires students to engage in gathering, interpreting and generalising information. Whereas for deductive reasoning, students can analyse, describe the relationship between forms and prove the theorem deductively. Students also understand that the process of verification can be done in more than one way and the proof is not obtained by memorisation (Crowley, 1987; Prahmana & Suwasti, 2014). Figure 13 shows the reasoning method provided to prove that the given shape is a parallelogram.



Explain that □ABCD with $\angle A=\angle C$, $\angle B=\angle D$ is a parallelogram as shown in the figure at right.

You can find the corresponding angle of and show that their sizes are the same.

Solving

If you take point E on the extension line of $\overline{BA}$,

$\angle DAB+\angle DAE=180^\circ$ ....①

In □ABCD, $\angle A+\angle B+\angle C+\angle D=360^\circ$

And $\angle A=\angle C$, $\angle B=\angle D$

Therefore, $\angle A+\angle B=180^\circ$ ....②

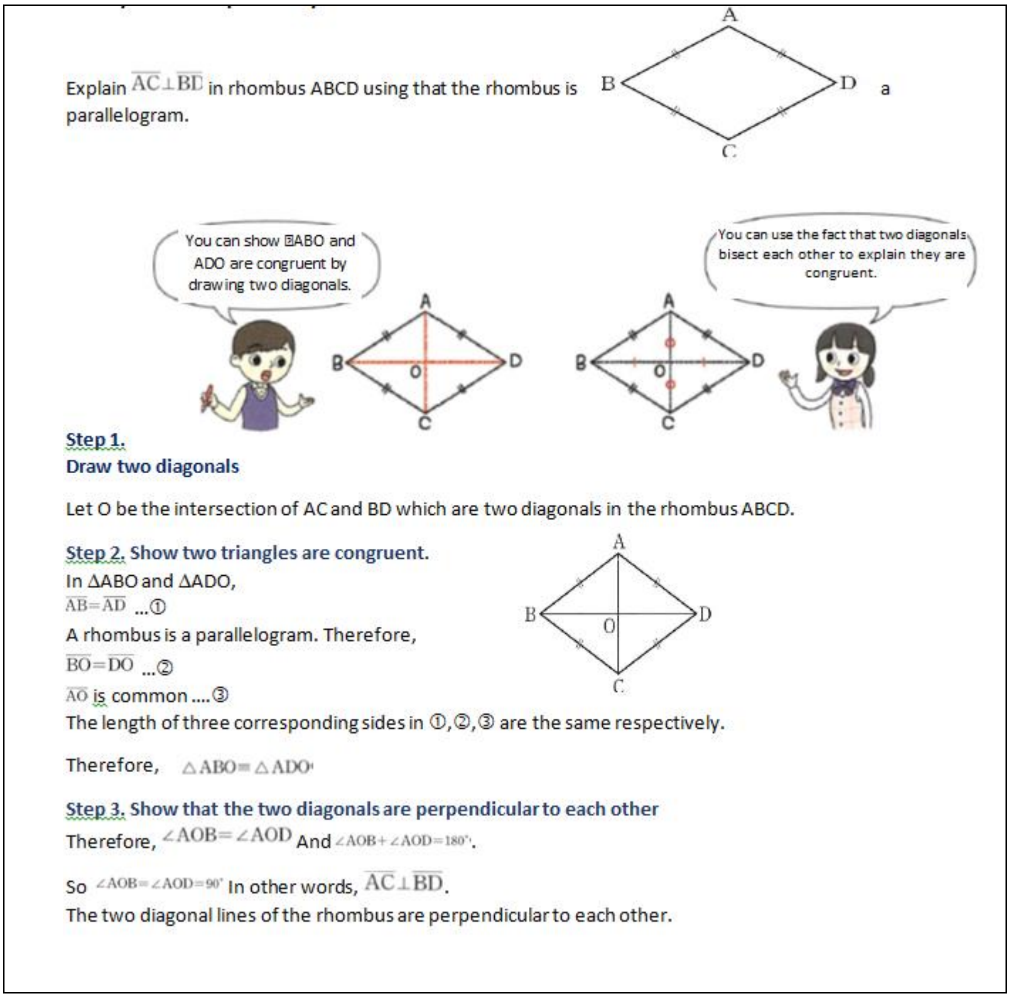
In ①and ②, $\angle DAE=\angle B$.

Since the sizes of the corresponding angles are the same, $\overline{AD}/\!/\overline{BC}$. Likewise, $\overline{AB}/\!/\overline{DC}$.

Therefore, ABCD is a parallelogram because the two pairs of opposite sides are parallel to each other.

Figure 13. Proof of parallelogram (Ministry of Education Korea, 2018, p. 173)

Figure 14 helps the students perform deductive reasoning to prove that the diagonals are perpendicular in a rhombus.



Explain $\overline{AC}\perp\overline{BD}$ in rhombus ABCD using that the rhombus is a parallelogram.

You can show △ABO and ADO are congruent by drawing two diagonals.

You can use the fact that two diagonals bisect each other to explain they are congruent.

Step 1.
Draw two diagonals

Let O be the intersection of AC and BD which are two diagonals in the rhombus ABCD.

Step 2. Show two triangles are congruent.

In ΔABO and ΔADO,

$\overline{AB}=\overline{AD}$ ...①

A rhombus is a parallelogram. Therefore,

$\overline{BO}=\overline{DO}$ ...②

$\overline{AO}$ is common ....③

The length of three corresponding sides in ①,②,③ are the same respectively.

Therefore, $\triangle ABO\equiv\triangle ADO$

Step 3. Show that the two diagonals are perpendicular to each other

Therefore, $\angle AOB=\angle AOD$ And $\angle AOB+\angle AOD=180^\circ$.

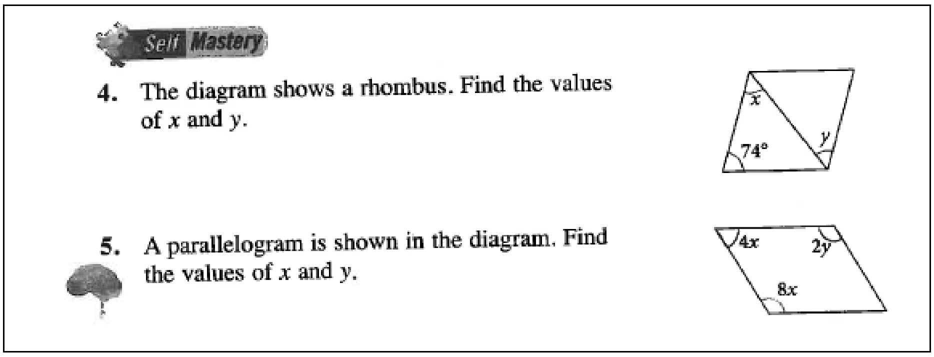
So $\angle AOB=\angle AOD=90^\circ$ In other words, $\overline{AC}\perp\overline{BD}$.

The two diagonal lines of the rhombus are perpendicular to each other.

Figure 14. Proof of rhombus (Ministry of Education Korea, 2018, p. 179)

The Difficulty Level of Mathematics Problems Provided at Chapter End

For this, both Malaysian and South Korean curriculum end the learning with mathematical problems related to the topic. Based on the analysis, the problems provided in the Malaysian textbook for quadrilaterals are directed at asking students to look for values of angles in a particular form. For example, as shown in Figure 15, students are asked to find the internal angle of the rhombus and the parallelogram.


Self Mastery

4. The diagram shows a rhombus. Find the values of x and y.

5. A parallelogram is shown in the diagram. Find the values of x and y.

Figure 15. Problems at “applying” level in the Malaysian mathematics textbook (Ministry of Education Malaysia, 2016, p.221)

The six levels in the Revised Bloom Taxonomy introduced by Anderson and Krathwohl (2001) are remembering, understanding, applying, analysing, evaluating and creating. If the student runs or uses a specific procedure to get an answer, then the problems are only at the level of application. There are also questions of understanding level provided in Malaysia textbook as in Figure 16. According to Anderson and Krathwohl (2001), constructing the meaning of various types of functions in writing, graphics or activity such as interpreting, proving, classifying, formulating, concluding, comparing, or explaining is problems at the understanding level.

2. Determine the types of
   (a) quadrilaterals which have two axes of symmetry.
   (b) triangles which do not have an axis of symmetry.
   (c) quadrilaterals with all the sides having the same length.
   (d) quadrilaterals with all the interior angles being 90°.

Figure 16. Problems at “understanding” level in the Malaysian Mathematics textbook (Ministry of Education Malaysia, 2016, p.221)

The problems presented for this topic in South Korean textbooks are more diverse and challenging. Undeniably, problems that require students to look for the angle, length or width of a shape which is the problem for application level as shown in Figure 17 were also present.

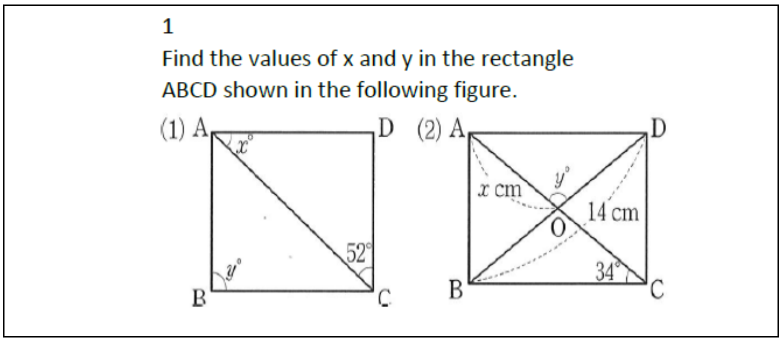


Figure 17. Problem at "applying" level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

Problems for analysing and evaluating levels are also provided in South Korean mathematics textbooks.

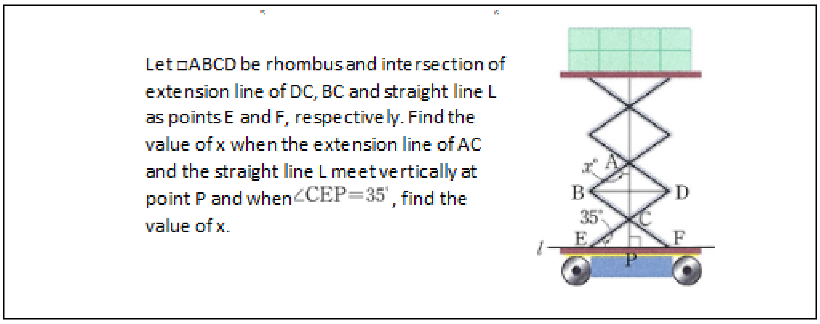
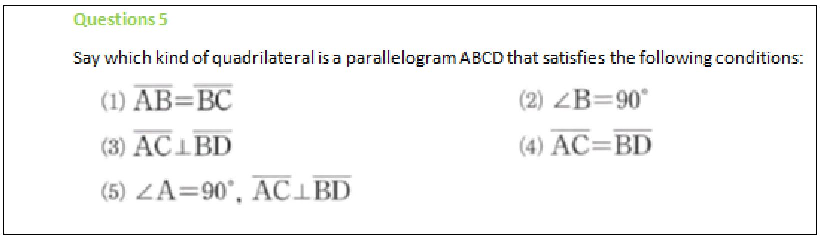


Figure 18. Problem at "analysing" level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

Figure 18 shows one of the samples of problem at "analysing" level. According to Anderson and Krathwohl (2001), mathematics problems at the analysing level involve convincing concepts to their sections, determining how they are related to each other or how they are interrelated and how they complement the overall concept. Thinking skills at this level include comparing and distinguishing between components or parts. The mathematics problem shown in Figure 19 is a problem for evaluating level which is the second highest level based on the Revised Bloom Taxonomy. According to Anderson and Krathwohl (2001), at the evaluating level, students make decisions based on criteria and standards through checks and criticisms. Criticisms, suggestions, and reports are some of the methods that can be done to indicate that evaluation process.


Questions 5

Say which kind of quadrilateral is a parallelogram ABCD that satisfies the following conditions:

(1) $\overline{AB}=\overline{BC}$ (2) $\angle B=90^\circ$

(3) $\overline{AC}\perp\overline{BD}$ (4) $\overline{AC}=\overline{BD}$

(5) $\angle A=90^\circ$, $\overline{AC}\perp\overline{BD}$

Figure 19. Problem at "evaluating" level in the South Korean mathematics textbooks (Ministry of Education Korea, 2018, p. 183)

CONCLUSION

There is a difference in the arrangement of the content for quadrilaterals between the Malaysian and South Korean curriculum. Geometric content, especially in the topic of Quadrilaterals in South Korean mathematics textbooks, is based on the van Hiele model which begins with visualisation, analysis and informal deductions. This means the content of Quadrilaterals in South Korean textbooks is based on the van Hiele model, which is the best model in the arrangement of geometric content. Students should first recognise shapes before they can analyse and find relationships between shapes. According to Noraini (2005), van Hiele states that the geometric thinking of secondary school students is below the expected levels of student thinking for this age cohort. Accordingly, the arrangement of geometric content should begin with visualisation. This model was also proposed by NCTM (1989, 2000) as the best model for effective geometry learning. NCTM also emphasised the importance of structured learning as proposed in the van Hiele model. Many studies show that the geometric content arrangement based on the van Hiele model supports increases in the level of geometric thinking of students. While many computer-assisted learning based on van Hiele models (Chew, 2009; Abdulah & Zakaria, 2013; Muhtadi, et al. 2018; Sukirwan, et al. 2018; Ahamad, et al. 2018) have a positive impact on students' geometry thinking, activities in South Korean textbooks emphasise hands-on manipulative-based activities. Whereas the composition of the content of the topics in the Malaysian curriculum is structured starting with technology-based activities to look for the characteristics of the quadrilaterals followed by the description of the properties in the table and then emphasises the concept of interior angles and the exterior angles of the quadrilaterals. Additionally, every quadrilateral concept in the South Korean curriculum is presented in the form of immersive and engaging activities. According to Nik Azis (2008), the ultimate goal of mathematical learning based on constructivist support based on pragmatism philosophy is the construction of mathematical strength by all students which involves some special abilities that each student needs to develop such as the ability to explore and reason, solve problems, relate ideas mathematics, communicating mathematics and developing self-beliefs about mathematics.

Maheshwari and Thomas (2017) showed that students had high motivation levels and achieved higher mean scores when they were taught using a constructivist teaching approach compared to non-constructivist teaching approaches. South Korean textbooks, especially in quadrilaterals, include deductive aspects of mathematical reasoning. According to Thompson, Senk & Johnson (2012), if the opportunity to address and prove that it is not available in textbooks, it is impossible for reasoning and proven activities to be implemented in mathematics classes. In this regard, secondary mathematics curriculum developers with the Malaysian Ministry of Education (MOE) should consider this in future curriculum reviews. In South Korea's curriculum, although many reasoning and proving activities can be found in the textbooks, according to Hong and Choi (2018), the reasoning and proving opportunities in South Korean textbooks are slightly different as they provide some problems to be proven reasonably while more statements are proven deductively. Many general statements are proven deductively to give students the opportunity to read and familiarise themselves with deductive prooving. From the aspect of the difficulty of mathematical problems provided especially for this topic, the problems presented in South Korean textbooks are seen as more diverse and challenging. Mathematical problems in Malaysian textbooks are still at the level of applying and below based on the Revised Bloom Taxonomy (Anderson & Krathwohl, 2001). Therefore, we recommend the Division of Curriculum Development of the Malaysian Ministry of Education reviews the geometry content of mathematical textbooks used today to suit the curriculum proven to produce students who excel in international assessments.

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