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EXPLORING MATHEMATICAL REPRESENTATIONS IN SOLVING ILL-STRUCTURED PROBLEMS: THE CASE OF QUADRATIC FUNCTION

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Abstract

Mathematical representation has an essential role in solving mathematical problems. However, there are still many mathematics education students who have difficulty in representing ill-structured problems. Even though the ill-structured-problem-solving tasks designed to help mathematics education students understand the relevance and meaningfulness of what they learn, they also are connected with their prior knowledge. The focus of this research is exploring the used of mathematical representations in solving ill-structured problems involving quadratic functions. The topic of quadratic functions is considered necessary in mathematics teaching and learning in higher education. It's because many mathematics education students have difficulty in understanding these matters, and they also didn't appreciate their advantage and application in daily life. The researchers' explored mathematical representation as used by two subjects from fifty-four mathematics education students at the University of Nusantara PGRI Kediri by using a qualitative approach. We were selected due to their completed all steps for solving the ill-structured problem, and there have different ways of solving these problems. Mathematical representation explored through an analytical framework of solving ill-structured issues such as representing problems, developing alternative solutions, creating solution justifications, monitoring, and evaluating. The data analysis used technique triangulation. The results show that verbal and symbolic representations used both subjects to calculate, detect, correct errors, and justify their answers. However, the visual representation used only by the first subject to detect and correct errors.

Keywords: mathematical representation, problem-solving, ill-structured problem, quadratic function

Abstrak

Representasi matematika memiliki peran penting dalam memecahkan masalah matematika. Namun, masih banyak siswa pendidikan matematika yang mengalami kesulitan dalam merepresentasikan masalah yang tidak terstruktur. Meskipun tugas penyelesaian masalah yang tidak terstruktur dirancang untuk membantu siswa pendidikan matematika memahami relevansi dan kebermaknaan dari apa yang mereka pelajari, mereka juga terhubung dengan pengetahuan mereka sebelumnya. Fokus dari penelitian ini adalah mengeksplorasi penggunaan representasi matematis dalam memecahkan masalah yang tidak terstruktur yang melibatkan fungsi kuadratik. Topik fungsi kuadrat dianggap perlu dalam pengajaran dan pembelajaran matematika di pendidikan tinggi. Itu karena banyak siswa pendidikan matematika mengalami kesulitan dalam memahami hal-hal ini, dan mereka juga tidak menghargai keunggulan dan aplikasi mereka dalam kehidupan sehari-hari. Para peneliti mengeksplorasi representasi matematika yang digunakan oleh dua subjek dari lima puluh empat mahasiswa pendidikan matematika di Universitas PGRI Kediri dengan menggunakan pendekatan kualitatif. Kami dipilih karena telah menyelesaikan semua langkah untuk menyelesaikan masalah yang tidak terstruktur, dan ada berbagai cara untuk menyelesaikan masalah ini. Representasi matematis dieksplorasi melalui kerangka kerja analitis untuk memecahkan masalah yang tidak terstruktur seperti merepresentasikan masalah, mengembangkan solusi alternatif, membuat justifikasi solusi, memantau, dan mengevaluasi. Analisis data menggunakan teknik triangulasi. Hasilnya menunjukkan bahwa representasi verbal dan simbolik menggunakan kedua subjek untuk menghitung, mendeteksi, memperbaiki kesalahan, dan membenarkan jawaban mereka. Di sisi lain, representasi visual hanya digunakan oleh subjek pertama untuk mendeteksi dan memperbaiki kesalahan.

Kata kunci: Representasi matematis, pemecahan masalah, masalah tidak terstruktur, fungsi kuadrat

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Mathematical representation means expressing a mathematical concept in various ways, such as languages, symbols, pictures, diagrams, models, graphs, or physical objects (Caglayan & Olive, 2010; Goldin & Beilock, 2010; Villegas, Castro, & Gutiérrez, 2009; Putri & Zulkardi, 2018; Saleh, et al. 2018). National Council of Teachers of Mathematics explains that the use of mathematical representation forms, such as charts, graphs, tables, and symbols, as well as the transition between the fore -mentioned forms, is an important aspect in mathematics learning (NCTM, 2000). Meanwhile, Nizarudin (2014) stated that mathematical representation is an internal generalization of mathematical ideas that are constructed as internal mental networks. Based on all the definitions mentioned, mathematical representations can be seen as external products and internal processes.

Mathematical representation as an external product means the expression of mathematical ideas; as an internal process, it is a technique of expressing mathematical ideas (Minarni, et al. 2016; Nizar, et al. 2018). Mathematical representations can be observed as they occur in the mind when performing mathematical activities. Mathematical representation must be emphasized in the mathematics learning process because students' ability to represent problems can help them solve mathematics problems.

Studies about the importance of mathematical representation in problem-solving were carried out by Bal (2014), Caglayan & Olive (2010), and Villegas, Castro, & Gutiérrez (2009). These studies found that students' representation ability is the main aspect for success in solving mathematical problems and understanding mathematical concepts. Therefore, one of the problems that support multiple representations is ill-structured problems.

Jonassen (2011) defined ill-structured problems like types of problems with conflicting goals, many methods of solutions, unexpected problems, scattered knowledge, and various representations. Furthermore, solving ill-structured problems will support students' mathematical representation ability (Byun, Kwon, & Lee, 2014; Ge & Land, 2003).

Several studies on ill-structured problems state that authenticity, complexity, and openness are properties of ill-structured problems (Hong & Kim, 2011; Jonassen 1997; Palm, 2008; Shin, Jonassen, & McGee, 2003). Authenticity means that the problem is by following everyday life, with problems that describe real-life outside of school (Palm, 2008; Muhtadi, et al. 2017; Saleh, et al. 2018). A problem can be said to be authentic if it covers the context of everyday life and is relevant enough to deduce an integral part of the actual situation. In terms of complexity, Jonassen (1997) considers it to mean the uncertainty of concepts, rules, and principles needed to solve problems, or how problems are managed, and also that the relationship between concepts and rules and principles is not established. In terms of openness, Jonassen (1997) states that first, several evaluation criteria must be in place to solve the problem; second, the clarity of the problem is not presented; third, students must express their opinions and personal beliefs about their interpretations of the problem.

In this study, the researchers focused on ill-structured problems involving quadratic functions because these are important components of mathematics education and many students find it difficult to solve because of the lack of accurate representation used to solve ill-structured problems. They didn’t successes when to build problem representations in problem-solving situations.

Quadratic functions area subject that students have difficulty in understanding (Akgün, 2011; López, Robles, & Martínez, 2016; Samo, 2009), and they also do not understand their advantage and application in daily life (Akgün, 2011). Quadratic-function problems have been used in many studies on representation and problem-solving (Bannister, 2014; Samo, 2009; Schoenfeld, 1992). Likewise, they are used in secondary education and early years of university in the fields of science and mathematics education. Therefore this material needs to be discussed further related to the reasons why many students fail to solve ill-structured problems with this topic. Related to this topic, Bosse, et al. (2014), in their study found that there are four activities done by the students in doing translation from the function graph to the symbolic. The activities are unpacking the source, preliminary coordination, constructing the targets, and determining equivalence. Bosse, et al. (2014) suggests that further studies, about mathematical representation translation besides the graph to the symbolic, are also needed in completing the research to examine in more detail in the translation process.

Therefore, the purpose of this study is to explore the types of representations and translation processes between representations, which are used by mathematics education students in solving ill-structured problems involving quadratic functions. The specific objectives in this study are to describe the types of representations used by mathematics education students in solving ill-structured quadratic-function problems and to explain mathematics education students’ translation between representations when solving ill-structured quadratic-function problems.

A lot of research has been done regarding students’ use of mathematical representations and their usefulness as a tool in problem-solving (Goldin, 1993; Moreno, Hegedus, & Kaput, 2008; Villegas, et al. 2009). Also besides, some mathematical representations can be used to develop an understanding of the processing of mathematical concepts at a deeper level (Anwar, Yuwono, Asari, Sisworo, & Dwi, 2016; Hiebert & Carpenter, 2000; Pape & Tchoshanov, 2001). While it is important for students to have various representations of mathematical concepts and to allow for flexible use of mathematical concepts in problem-solving, to successfully manage the information used in problem-solving, mathematical representations must be correct and related to each other (Villegas, et al. 2009).

Given the importance of problem-solving in mathematics education (NCTM, 2000), the study of mathematical representations created by students when they solve problems is an interesting topic in both education and research in the field of mathematics. NCTM (2000) recognizes that a suitable use of multiple representations contributes to building a set of flexible tools for problem-solving. One of the goals of learning in middle-level and high-level mathematics education is that students acquire a good understanding of concepts with different representations and that they use precise and accurate representations during problem-solving because this is important for successful problem-solving (Schloeglmann, 2004).

Zhang (1997) said that the success of problem-solvers is based on their ability to build problem representations in problem-solving situations. The process of selecting problem representations allows students to practice balancing the advantages and disadvantages of various forms of representations (Chapman, 2010), and use them as tools in problem-solving. Lesh, Post, & Behr (1987) describe the role of mathematical representation, and the translation between representations, in mathematics learning, especially in problem-solving. They use the term 'representation' in a limited sense, as an external expression of students' mathematical conceptualization. For this study, different forms of representation will be shown and the translation between representations and transformations will be explored in problem-solving performance.

Rahmawati, Purwanto, Subanji, Hidayanto, & Anwar (2017) state the importance of translating activities from one representation system to another, starting with problem representations. Therefore, researchers need to explore mathematical representations and translations between the mathematical representations in solving quadratic-function problems. Kieran (2004) considers the concept of quadratic function as a form of understanding algebra with multiple representations. Therefore, in this study, the researchers explore three types of representations. Firstly, verbal representations consist of representations expressed both in writing and verbally. Secondly, visual representations consist of images, diagrams, or graphs. Finally, symbolic representations consist of numbers, operations, relationship signs, and algebraic symbols.

Some experts explain ill-structured problems as having definitions, goals, and boundaries that are not clearly stated (Voss, 1988); having several solutions, solution paths, or no solutions at all (Kitchner, 1983); having a relationship between concepts, rules, and principles that is not appropriate (Jonassen, 1997). These problems contain unique human interpersonal activities. Ill-structured problems are a type of problem faced in everyday life, so they usually create a dilemma (Jonassen, 1997). Because an ill-structured problem is not limited by the content learned in class, the solution is unpredictable.

Several studies on structured problems define the characteristics of ill-structured problems such as authenticity, complexity, and openness (Byun, Kwon & Lee, 2014; Ge & Land, 2003; Hong & Kim, 2016; Jonassen 1997; Palm, 2009). A problem can be said to be authentic if it covers the context of everyday life and is quite relevant to deducing an integral part of the real-life situation. Jonassen (1997) said that complexity includes the uncertainty of concepts, rules, and principles needed to solve problems, or how the problem is regulated, and the fact that the relationship between concepts and rules or principles is not specified. In terms of openness, Jonassen (1997) states that: first, several evaluation criteria must be in place to solve the problem; second, the clarity of the purpose of the problem is not presented; and third, students must express their opinions and personal beliefs about the problem.

Several researchers propose an ill-structured-problem solving process that is different from structured-problem solving (Ge & Land, 2003; Jonassen, 1997; Sinnott, 1989). Sinnott (1989) proposes the following problem-solving model: the problem space design process, making choices and creating solutions, monitoring and storing, and non-cognitive factors. Ge and Land (2003) outline the four steps of the solving process, namely: representation of problems, developing solutions, developing justification, and monitoring and evaluation. Jonassen (1997) proposes the same process: representation of problems, generating solutions, justification, and monitoring, and evaluation. In several studies such as Palm (2008) also Ge and Land (2003), students were given a problem to solve. In doing so, they took into account the real-life situation when developing practical solving strategies and organizing the information. Thus, their minds were led to a new understanding of the problem, by evaluating and examining various alternatives to find the most appropriate solution.

Therefore, in this study, ill-structured problems are problems that are complex and open and involve quadratic functions. The matters of quadratic functions application in solving mathematical and real-life problems are widely used in mathematics education. Visual representations of real-world problems with quadratic-function concepts provide more understanding of the application of mathematics in life (Janakievska, Stopanska, & Bogatinosca, 2012).

METHOD

This exploratory study with a qualitative approach because an analysis based on the data obtained then developed a certain relationship pattern or become a hypothesis then the researcher does triangulation for analyzed data. Data was undertaken by fifty-four mathematics education students at the University of Nusantara PGRI Kediri attending the algebra course. Data was obtained through the completion of an ill-structured problem task about the quadratic function that it's graph through one point by thinking aloud method. Some notes were taken by researchers during the task-solving session. The researchers reviewed the participants' answer sheets and field notes to assess the mathematical representations obtained. Based on the results of the worksheet analysis, two subjects were selected due to their completed all steps for solving the ill-structured problem, there have different ways of solving these problems, and their good response when interviewed.

The ill-structured problem in this study is called Ill-Structured Problem Mathematics Test (TPMI) and it was given to the students as follows: "*Determine the quadratic function that it's graph through point K (2, 3)*." These problems include ill-structured problems because it had complexity from the limited information and have an openness method also solution. These problems can also explore multiple representations because the topic of function allows being presented in various forms of representations (López, Robles, & Martínez, 2016).

The research procedure conducted to obtain the data of mathematical representations used in solving an ill-structured problem. First, the participants were asked to solve the problem as outlined on the worksheet by different ways of solving the problem. At this stage, only two participants have different ways of solving the problem. Second, the researchers interviewed the subjects to verify that the test data is written based on the analysis framework, which is shown in Table 1.

Table 1. Framework for Mathematical Representation Analysis

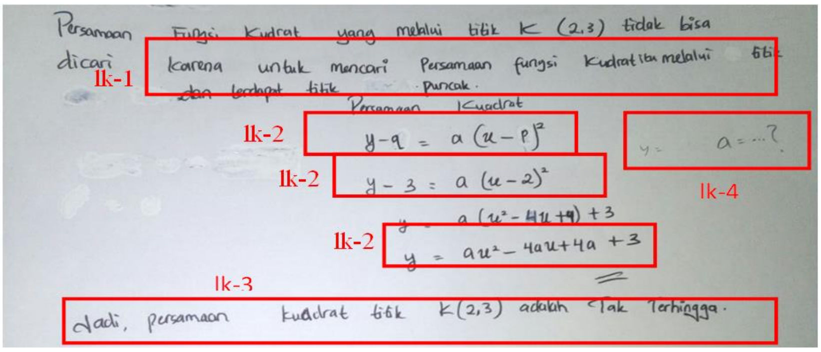
Type of Mathematical Representation	Description	Indicators for Mathematical Representation Analysis	Codes
Type 1: Verbal representation in solving ill-structured problems	Stating problems, problem-solving strategies, and monitoring results and discussions verbally.	Expressed problems similar to the text problem.	Vb11
		Expressed problems, change a few words with their own style of speech.	Vb12
		State the problem by writing the same text as the problem.	Vb13
		State the problem by writing a paraphrase text from the question.	Vb14
		Describe the strategies developed to get solutions.	Vb21
		Describe the strategies developed to get solutions and there are no solutions to paper problems	Vb22
		Describe the arguments and facts that support the solution chosen orally.	Vb31
		Describe the arguments and facts that support the chosen solution by writing text.	Vb32
		Describe the results of evaluation and effectiveness of solutions that are appropriate orally.	Vb41
Type 2: Visual representation in solving ill-structured problems	Stating problems, problem solving strategies, and monitoring results and discussion using embodiments of images, graphs, diagrams, number lines, and other mathematical drawings.	State the problem by making pictures, graphics, diagrams, number lines, and other mathematical drawings on the worksheet	Vs11
		Describe the strategies developed to get solutions and alternative solution to the problem by pictures, graphics, diagrams, number lines and other mathematical drawings on the worksheet.	Vs21
		Describe the arguments and facts that support the chosen solution by pictures, graphs, diagrams, number lines, and other mathematical drawings on the worksheet.	Vb31
		Describe the results of the evaluation and effectiveness of solution expressed by pictures, graphs, diagrams, number lines, and other mathematical drawings on the worksheet.	Vb41
Type 3: Symbolic representation in solving ill-structured problems	Stating problems, problem solving strategies, and monitoring results and discussion using numbers, variables and other mathematical expressions.	Express problems by using numbers, variables and other mathematical expressions on the worksheet	Sb11
		Describe the strategies developed to get solutions and alternative solutions to problems by using numbers, variables and other mathematical expressions on the worksheet.	Sb21
		Describe the arguments and facts that support the chosen solution using numbers, variables and other mathematical expressions on the worksheet.	Sb31
		Describe the results of the evaluation and the	Sb41

Type of Mathematical Representation	Description	Indicators for Mathematical Representation Analysis	Codes
		effectiveness of solution expressed by using numbers, variables and other mathematical expressions on the worksheet.	

In this stage, interviews were conducted to match the data with the written data from the students' work. Data validity checks were carried out using the triangulation technique, which is checking the consistency between the written result and the interview result. Data were considered valid if it showed consistency between the written and the interview data. If the data was invalid, then data collection was repeated until the data was valid.

RESULT AND DISCUSSION

Each subject's paper results and their transcript of think aloud were analyzed, described based on the framework in Table 1. The first subject, CML, used a more procedural method in solving the problem. CML realized that there was a lack of information in the problem, so she assumed that there were many correct answers. CML's reasoning showed in Figure 1.

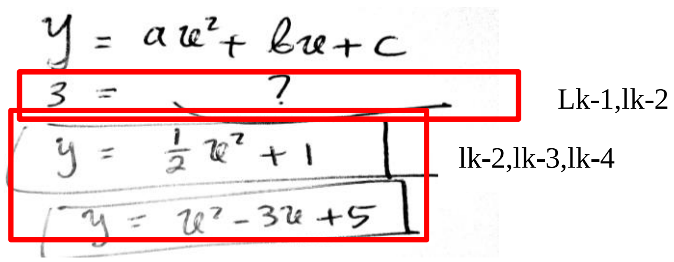


Time	Description of think aloud	Stages
00:01	Silent and read the problem.	lk-1
00:05	*"These question only have one point known, namely point K(2,3)."*	lk-1
00:11	*"Emm..it means a lot of solutions to this problem because the equation must go through a minimum of two points."*	lk-2
00:23	*"My first step is to write a general form of quadratic equations* $y - q = a(x-p)^2$*."*	lk-2, lk-3
00:32	*"Earlier q was known to be 3 and p was 2, and a is unknown, so I am writting* $y - 3 = a(x-2)^2$ *"*	lk-3
01:05	*"...then everything changed and moved to one segment, which resulted in* $y = ax^2 - 4ax + 4a + 3$*"*	lk-3
01:11	*"Hemm....but a=...He... I'm confused"*	lk-4

Figure 1. The Work Result of Subject-1 (CML)

Figure 1 stated that at the problem-representation stage (lk-1), CML's verbal representation expressed that the problem known was point K (2,3), and the quadratic function found passed through that point K. CML stated that the equation of the quadratic function that passes through point K cannot be found because the quadratic equation must go through one point and is known to be the peak point. According to CML, there must be a minimum of two known points to form a quadratic function. At the developing-solution stage (lk-2), CML used an algebraic equation $y - q = a(x-p)^2$ to represent symbolically the quadratic function in the problem. Then she reverted to verbal representation by stating that, "Earlier q was known to be 3 and p was 2, and a was unknown, then everything changed and moved to one segment, which resulted in $y = ax^2 - 4ax + 4a + 3$." At the argumentation stage (lk-3), CML chose a correct solution by representing symbolically and verbally that the number of quadratic function at point K (2,3) is infinite. However, CML could not provide an argument for her decisions, the monitoring and evaluation stage (lk-4), she reflected and used a symbolic representation by writing $a=?$

Similar to CML, the second subject, AZZ, also realized that there was a lack of information in the problem, thus supposing that there are many correct answers. AZZ chose a trial and error method by substituting points into predetermined equations. AZZ's work showed in Figure 2.



Time	Description of think aloud	Stages
00:01	Silent and read the problem.	lk-1
00:15	*"These question known, hemm...y* $= ax^2 + bx + c$ *so 3 = ..."*	lk-1, lk-2
00:21	*"The first alternative solution is* $y = \frac{1}{2}x^2, x$ *substituted with 2, then the result is 4; 4 divided by 2 is 2; because these result must be 3, 1 must be added, so this means* $y = \frac{1}{2}x^2 + 1$.	lk-2, lk-3, lk-4
00:23	*"...and then the second alternative solution is* $y = x^2 - 3x$*. It means -3 (2) +22 = -6 + 4, which is -2; so for the answer to be 3, this means 5 must be added* $y = x^2 - 3x + 5$*".*	lk-2, lk-3, lk-4

Figure 2. The Work Result of Subject-2 (AZZ)

Figure 2 shows that at the problem-representation stage (lk-1), AZZ represented symbolically the quadratic function in question as $y = ax^2 + bx + c$, and then used another symbolical representation by writing '3 = ?'. AZZ's verbal representation consisted of pointing the K point on the problem and declaring the y value to be 3 and x value to be 2. Next, at the developing-solution stage (lk-2), AZZ wrote $y = \frac{1}{2}x^2$ (symbolic representation) and then stated (verbal representation) that if x is substituted with 2, then the result is 4; 4 divided by 2 is 2; because these result must be 3, 1 must be added, so this means $y = \frac{1}{2}x^2 + 1$ (symbolic representation). AZZ carried out the justification (lk-3) and monitoring and evaluation (lk-4) stages simultaneously; for example, $y = x^2 - 3x$ means -3 (2) +22 = -6 + 4, which is -2; so for the answer to be 3, this means 5 must be added y $= x^2 - 3x + 5$.

In the results above, CML (first subject)'s stages to solve the ill-structured problem can be described as follows: 1) The subject used verbal representation to identify the problem; 2) the subject used translation from verbal representation to symbolic representation to describe the data; 3) the subject used translation between symbolic representation and verbal representation to compute the answer; and 4) the subject failed to uncover the necessary information about the scale to justify her answers.

AZZ (the second subject)'s solution stages to solve the ill-structured problem can be described as follows: 1) the subject used verbal representation to identify the problem, 2) the subject used translation from verbal representation to symbolic representation to describe the data, 3) the subject used symbolic representation to compute the answer, and 4) the subject used verbal representation to justify his answers. Both of subjects' translation-of-representation processes are shown in Figure 3.

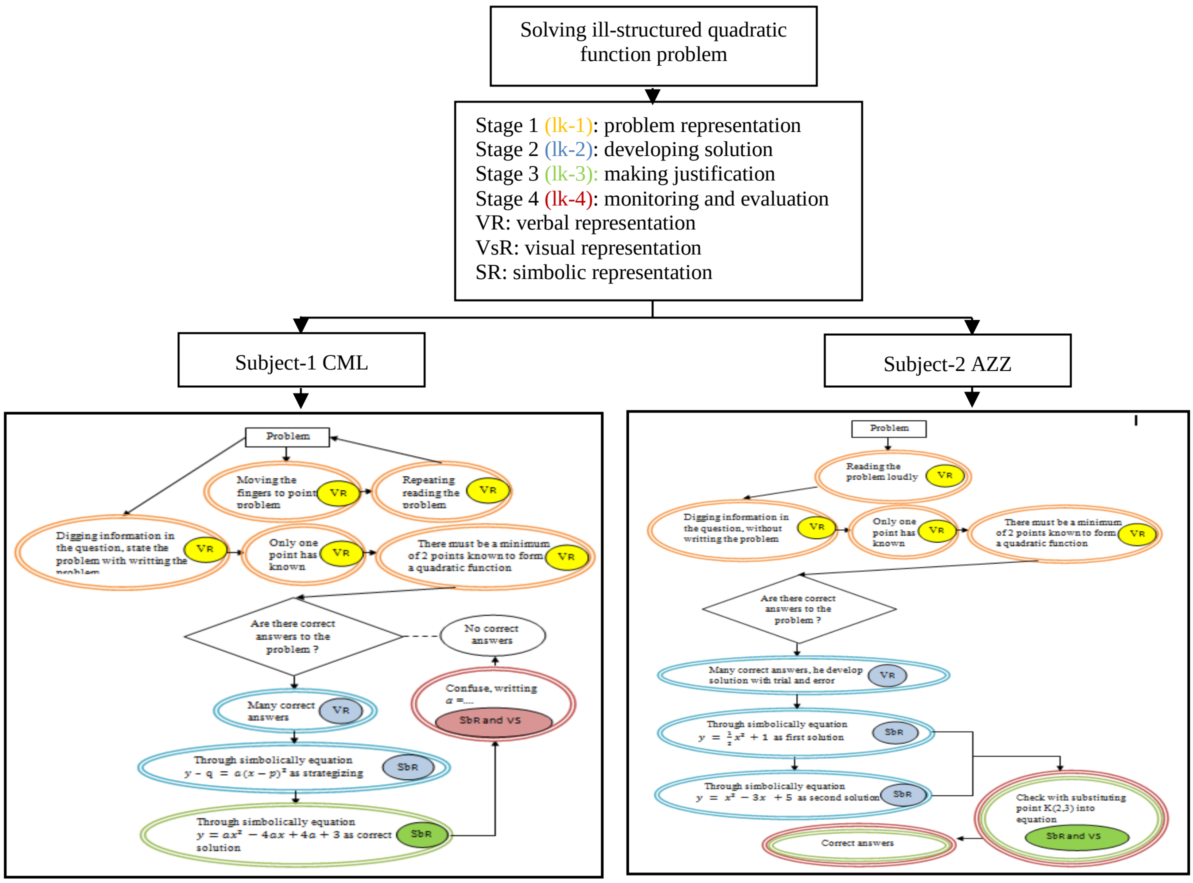


Figure 3. The Schema both of Subjects' Representation in Solving TPMI

The empirical data shows the mathematical representations created by the students when solving the ill-structured problem. They made their notations regarding concept understanding, but often had difficulty in transitioning their symbolic representations. There was a lack of understanding of symbol notations, which could be seen in CML's confusion when monitoring and evaluating to develop the final solution, and in the fact that AZZ rarely made verbal representations in writing but used the morally to describe the analysis and evaluation, he made in developing the final solution. Without using many types of representations and tending to use verbal representations, AZZ was able to give two solutions.

Based on the theoretical framework and results, we found that the two subjects had different methods of using mathematical representations to solve an ill-structured problem. It showed that the success of problem-solvers is based on their ability to build problem representations in a problem-solving situation (Zhang, 1997). The process of selecting problem representations causes differently characteristics when solving ill-structured problems, which are verbal-symbolic and symbolic-verbal problem solvers. These characteristics show that there is a strong relationship between success in solving ill-structured problems and translation of representation skills. The findings are by following Villegas et al. (2009), who states that there was a strong relationship between success in solving problems and skills in construction, use, and articulation of representation.

Another finding from this research is that in addition to the diversity of representations, the accuracy of the translation process between types of representations also determines the success of problem-solving. Pape & Tchoshanov (2001) said that translation representation reducing the level of abstraction of a representation. It is in line with Bosse et al., (2014), Rahmawati et al., (2017) and Swastika et al., (2018) said that the accuracy in determining the representation of sources and targets, prior calling of knowledge, and the mapping process between source representation and target representation are important aspects in the translation process.

CONCLUSION

This research analysed about the process of mathematical-representation translation and identified mathematical representations by applying ill-structured-problem solving activities to problem-solving learning approaches for mathematics education students. The finding of the present research that students' mathematical representation forms affect ill-structured-problem solving competencies. The schemas of mathematics education students' representations in solving ill-structured problems shows that they to tend to use verbal-symbolic and symbolic-verbal representations. Verbal and symbolic representations are used mathematics education students to calculate, detect, correct errors, and justify their answers, but visual representations are used mathematics education students to detect and correct errors. Furthermore, the studies will be carried out by increasing the number of subjects and extending the study period so that the pattern of translation of mathematical representation in solving ill-structured problems is obtained.

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