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Developing Student's Notion of Measurement Unit For Area

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Abstract

Many researchers found that students have difficulties in understanding area measurement. Students mostly focus on applying formula to find the area of certain shapes without knowing what the area is and why the formula works. It is important for the students to know what attribute being measured and to construct the unit for area measurement. Therefore, the aim of this research is to develop activities that support students to know the attribute of area and the notion of measurement unit in learning area measurement. For this purpose, design research is chosen for achieving the research goal. Realistic Mathematics Education (RME) underlies the design of context and activities. A teacher and students in grade 3 in elementary school (SDN 21) in Palembang Indonesia will be involved in this research.

Keywords: Area measurement, unit, design research, RME

Abstrak

Banyak peneliti menemukan bahwa siswa mengalami kesulitan memahami pengukuran luas. Siswa kebanyakan hanya fokus pada penerapan rumus untuk menentukan luas bangun datar tertentu tanpa mengetahui pengertian luas dan tidak mengerti mengapa rumus tersebut dapat digunakan. Sehingga dirasa sangat penting bagi siswa untuk mengetahui karakteristik luas dan mengkonstruksi satuan luas. Oleh karena itu, tujuan dari penelitian ini adalah untuk membangun kegiatan kelas yang mendukung siswa mengenal karakteristik luas dan mengagas satuan untuk mengukur luas. Design research dipilih sebagai metode penelitian guna mencapai tujuan penelitian. Rancangan konteks dan aktivitas dilandasi oleh Realistic Mathematics Education (RME) atau yang lebih dikenal dengan Pendidikan Matematika Realistik Indonesia (PMRI). Guru dan siswa kelas 3 SDN 21 Palembang Indonesia dilibatkan dalam penelitian ini.

Kata Kunci: Pengukuran Luas, Satuan, Design Research, RME

Introduction

People deals with measuring situations every days such as measure how much ingredient put when cooking, how far the school from the student's house or how large a garden. Interpreting the number is important to communicate the result of measuring to the other. It means that competence in measuring is needed to teach in primary school. According to Reys et al (2007) measurement should be included in

school mathematics because of its power to help students to see the usefulness of mathematics in everyday life. Measurement also can be used to help students learn other topics in mathematics.

This research focuses on supporting the third grade students in learning area measurement. Area measurement is based on partitioning a region into equally sized units which completely cover it without gaps or overlaps. It is often that teaching tends to focus on numerical results and ignored the idea of the unit. Many findings show that understanding area measurement is difficult. Research in the field of mathematical education often reveals poor understanding of the processes used for area measurement of plan figure (Zacharos, 2006).

For young children, measuring area of an object can be difficult if they do not understand the concept of area (Reys et al, 2007). Heuvel-Panhuizen (2005) also stated that area is a physical quantity that is harder to comprehend for children since area is discussed in hidden term. Battista (Keijzer, 2008) mentioned that there is no obvious instrument for measuring area. It means that it is important to construct the unit for area and the measurement procedures. According to Gravemeijer, et al (2007) students are expected to master an extensive system of units of area, but they appear to have serious difficulty with applying this knowledge. It is preferable to make students familiar with a number of units and applications that are relevant to daily life, and to place the emphasis on developing concept, rules and procedures.

Considering the difficulties of the students about area measurement, we develop classroom activities that support students to perceive measurable attribute and the notion of measurement unit in learning area measurement. So in this research a series of learning activities are designed to bring students in developing a unit as a means of measuring area. Realistic Mathematics Education (RME) underlies the design of the activities. The research question is *How can comparing and covering activities bring students to develop the notion of a measurement unit for area?*

Theoretical Framework

1. Area measurement

Area is an amount of two-dimensional surface that is contained within a boundary (Clements and Sarama, 2009). According to Simon and Blume (1994) in Zacharoz (2006), the study of area involves two steps: considering the area as a quantity and

evaluating that quantity. To measure the quantity of area a unit must be chosen so that the number of those units that is takes is the size of an object (Cross et al, 2009). Physical quantity can be seen while the experiences offer the students to compare area of objects (Heuvel-Panhuizen, 2005). In this way, intuitive awareness of measuring emerges in many students. Exact statements to quantify the area can be made by using a unit of measurement used. In here, the students start to develop the need of unit to measure when they have to evaluate the quantity.

To measure with understanding, children should know what attribute they are measuring. This has to come from the experiences of the attribute and it requires careful development of language. Three types of comparisons which can build understanding of attributes are perceptually, directly and indirectly through references. For area, it can be compared by sight (perceptually) if the differences are large enough and the shapes are similar enough. Direct comparison means that children compare two regions where one of the regions can fit within the other. They can cut out the region to easily compare without changing the area. According to Lehrer students have difficulties accepting that when they cut a given region and rearrange its parts to form another shape, the area remain the same (Clements, 2004). When children have some idea of conservation of area that a region can be cut and rearrange without changing the area, they can give many solutions to find the area. If the objects cannot be moved to place one on top of the other, children can trace the objects and use this representation to make an indirect comparison.

Moreover, finding the area of a region can be thought of as tilling a region with two-dimensional unit of measure (Clements and Stephan, 2004). Cavanagh (2007) stated that area measurement is based on partitioning a region into equally sized units which completely cover it without gaps or overlaps. However, students probably are not thinking about measuring as covering space (Clements and Stephan, 2004). Therefore, students should be involved to investigate covering regions with a unit of measure which completely covers it without gaps or overlaps and quantifying that covering. Furthermore, Zacharoz (2006) suggested that area be measured using two-dimensional units such as plane figures (e.g. squares and rectangles) and a square unit is usually selected for overlapping rectangles and other figures with right angles.

2. Realistic Mathematics Education

The designing of local instructional theory in this research is inspired by Realistic Mathematics Education. Realistic Mathematics Education has been developed in the Netherlands since 1970s (de Lange, 1996). This approach has been implemented in Indonesia since over the last 10 years that is called Pendidikan Matematika Realistik Indonesia (PMRI). Many schools have been involved in developing PMRI including the school that we will work with.

According to Freudenthal, mathematics must be connected to reality through problem situations. The term "*reality*" means that the problem situation must be experientially real for students. In realistic mathematics education (RME), students should be given opportunity to reinvent mathematics based on their own strategies.

The local instructional theory in this research in line with five tenets of realistic mathematics education that have been defined by Treffers (1987, in Bakker, 2004) as following:

a. *Phenomenological exploration or the use of contexts*

   In this study, students are involved to explore mathematics from a real situation that has meaning to develop basic concepts of mathematics. In learning area measurement, comparing the size of objects around can be the starting point in learning process.

b. *Using models and symbols for progressive mathematization*

   The second tenet of RME gives a bridge from a concrete level to more formal level. Models, symbols, schemas, and diagrams can support the development of students' thinking from concrete level to formal level. Using non standard unit measurement that is chosen by students can be a model to support the students' thinking.

c. *Using students' own constructions and productions*

   Students can use their own strategies to solve a problem that have meaning for themselves. Hence, using students' constructions and productions is promoted as an essential part of instruction. Students can use their own production when they choose their own unit in covering shapes.

d. *Interactivity*

   The learning process of the students is not only as an individual process but also as social process. In this research, we ask students to work with

small group so that they can share their ideas to their friends and can learn from each other in discussion. In class discussion, it could also encourage more interactions among every element in the class.

e. *Intertwinement*

It is important to consider an instructional sequence in its relation to other domains. When students learn about area measurement, it is also support other domain such as multiplication and geometry.

Methods

As the main goal of this research, we develop classroom activities that support students to identify the attribute of area and the notion of measurement unit in learning area. Design research is chosen for achieving the research goal and answering the research question. Three phases of design research are discussing in this research. According to Gravemeijer and Cob (2006), the three phases of conducting a design experiment, as following:

1. Preliminary design;
2. Teaching experiment;
3. Retrospective analysis.

In the preliminary phase, Hypothetical learning trajectory will be designed. It consists of teaching and learning activities, learning goals for students and conjectures of students' thinking process. The conjectures in HLT will be tested in teaching experiment in order to improve the design. Data gathered from teaching experiment will be analyzed in the retrospective analysis.

A teacher and students in grade 3 in SDN 21 Palembang in Palembang were involved in this research. This school has been involved in PMRI project since 2010. The students were about 8 or 9 years old and they have learnt about linear measurement and multiplication in grade 2. This research was carried out in two cycles namely pilot experiment and teaching experiment. In the pilot experiment the sequences of activities were tried out to small group that involved 9 students. The design of HLT was tried out to see how it works and to investigate the students' thinking about the problems so that we can improve the HLT. The improved of initial HLT then was tested to another class in the second cycle. This involved whole class that consisted of 36 students.

Video recording and students' written works will be used to investigate the learning process of the students. We also conduct interview with the students to get deeper information of their thinking process. Reliability of the data was gathered in qualitative reliability. The data gathered cross interpreted with colleagues and supervisors in order to reduce the subjectivity of the researcher's point of view.

Hypothetical Learning Trajectory

A design and research instrument that proved useful during all phases of design research is called hypothetical learning trajectory (HLT) (Barker, 2004). An HLT consists of the goal for students' learning, the mathematical tasks that will be used to promote students learning, and hypothesis about the process of students' learning (Simon and Tzur, 2004). As the main goal of this research we design series of activities to support students in developing measurement unit for area.

After tried out the activities in the pilot experiment, we modified some activities and some tasks included the mathematical goals for students to improve the HLT. The overview of hypothetical learning trajectory is on the following table:

Contents areas	**Goals**	**Activities**	**Conjectures**	**Concept**
Identifying the attributes	• Students are able to identify the attribute of area • Students are able to compare and order the area	Telling the size of cakes	Cutting and pasting	Conservation
	• Students are able to compare area by using same kind of unit	Choosing the chocolate	• Cutting and pasting • Counting with the same unit	Identical unit
Comparing Area	• Students are able to use non standard units to compare the area of shapes	Cookies in baking trays	Using small paper to measure	Partitioning, Unit iteration
Non-standard unit	• Students are able to explain there is inverse relationship between the number of units	Unit investigation	Arranging many kind of paper	Structuring array, unit iteration

and the size of the unit

Research Findings

The first activity concerns with students' awareness of physically quantity of area. The starting point was designed by a situation that involves the area in which students have to compare and order the size of cakes. This activity showed that some students have difficulty in describing why the object is larger than another. In building their vocabulary to describe the area and a sense of the attribute, first they were shown two cards that were different in size, one is big and another is small. Various reasons were told by students in explaining why they think the big one is big. Such as the side of the card is bigger than another, the angle is greater than another one and the big one is larger than another.

There also a student used ruler to compare. He measured the length and the width, and added them up to get his measurement. In this level, he added the length and the width to reveal the quantity of area. There are also students argued that the figures have different number of angle which influences the area of shapes. It seems the concept of angle is not fully understood by students.

Meanwhile some students were aware to the physical quantity of area. Students compared figures given by putting the one to the top of the other and then looking at which piece sticks out. Some of them also perceive the idea of conservation of area in which they cut and rearrange the shape to determine which one is bigger by looking at the out-sticking parts.

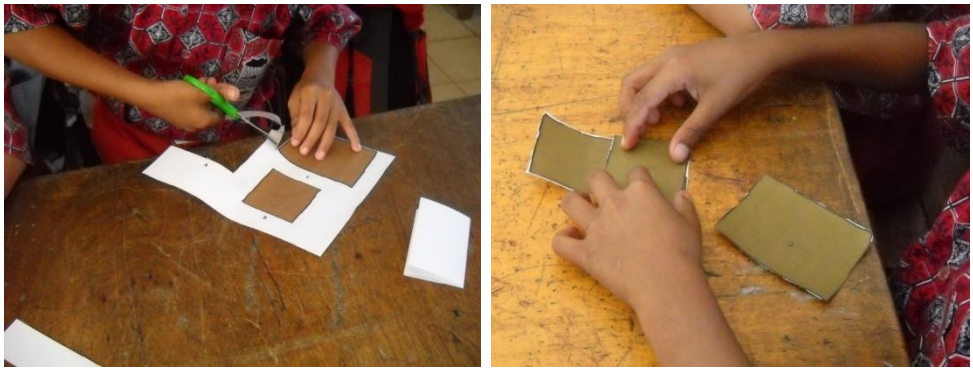

Figure 1. Students cut and paste the figures

The awareness of the attribute of area was made for the next lesson but they also worked with measurement units. It is expected that they can compare using identical units. In this lesson students had to choose which chocolate to buy by comparing their sizes. Students compared the chocolate by putting the one to the top of the other and also count the unit in each chocolate. It is expected that can compare their strategy to compare. Almost all students immediately guess which chocolate is bigger. When asked to prove their guess, most students tend to cut and paste the figure like what they did in the first activity. Only few students recognize the existence of the unit in the figure. The students compare the number of slabs in each chocolate and realize that slab is not identical and it is difficult to compare by using unidentical units. This is because the figure of the chocolate is too easy compared by sight so that students did not pay attention to the units in the chocolates.

In the third activity, the students had to compare baking trays that can be put more cookies. It is expected that they can use the unit to compare. In this lesson, students are able to use their own unit to cover the shape in comparing the area. They can partition the region and structure the units into arrays. However, some students choose unit that physically resemble with the region they were covering. They used different units for different baking tray. In this manner, they only focus on the process of repeatedly using a unit and it seems they did not use the unit to compare. It might be because the question is which baking tray that can be put more cookies. Therefore, they did not pay attention to the size of baking tray. The question should be which baking tray is bigger so that they can think how to compare by using same kind of units. Some students were not aware of gaps and overlap in covering. They ignore the leftover paper and count all units include the leftover parts as area. In this level, these students did not get what is the area since they only focus on counting the unit. Students' work is in the following figures.

Figure 2. Students cover the baking tray

Experience in covering with non standard units helps students to develop the concept of unit iteration and structuring arrays with row and column structuring. They become familiar with the process of measuring that would be used in the next activity.

In the fourth activity, students have to find the area of a baking tray with different kinds of unit by covering the baking tray. Afterwards, they have to compare the result. The students can use their experience in the previous activity to cover the shape with no gaps or overlap. This experience helps them in counting the units used because they know that the cutting parts originate from one unit. In this way they can accept the concept of conservation of area. This is obvious from the fact that the process of measuring improve in this activity. In the previous activity students ignore the half part by counting it as one. But in this activity they consider about the accuracy of counting. They also find that different unit will affect the result of measuring and they realize the inverse relationship of the unit, the larger the unit the fewer are required and vice versa.

Discussion and Conclusion

Based on the retrospective analysis, we have investigated the activities that lead the students to learn area measurement. The activities bring students to develop the notion of a measurement unit for area. Experiences with physical quantity of area spontaneously gained by comparing activities in which students can aware the attribute of area and develop the range of words that can be used to discuss it. Based on observation, comparing activities lead students to be acquainted with physical quantity of area. The words such as big, large, small when they explain figures indicate that they focus on the attribute of area. Some students showed their ability of the concept of conservation area by reshaping the figure to compare the area. The students who did not reach this level tend to compare by sight after they cut and paste the figure. This strategy indicates students still estimate to compare the area.

The experience with non standard unit is provided to the students so that they start to develop the need of unit to measure when they have to evaluate the quantity. We found that covering activity leads students to mentally partition a region into small units and allows students to focus on the process of repeatedly using non standard unit as a tool to measure. It also provokes the students to construct the idea of the inverse relationship between the size of unit and the number of unit used in covering the

region. The students perform their ability in structuring array of the unit while covering. Most students are able to arrange the unit given in such a way fit to other unit without gap or overlaps. However, some students find difficulties in dealing with overlapping space. These students only focus on counting the unit without pay attention to the region they want to cover. After experiencing gaps and overlaps in the iteration process, gradually students can perceive that entire region must be covered by the units and the area is the number of units within boundaries. Through covering activity, the students are able to develop the unit meaningfully to measure area.

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