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GENERALIZATION STRATEGY OF LINEAR PATTERNS FROM FIELD-DEPENDENT COGNITIVE STYLE

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Abstract

Linear pattern is the primary material in learning number patterns in junior high schools, but there are still many students who fail to generalize the linear pattern. The students' failure in generalizing the pattern occurred when the students ended to view the problems globally without breaking them into the constructors' components such as the experience of field-dependent type students. For this reason, this study was carried out to explore the thinking process of students who fail and investigate the thinking processes of students who succeed in generalizing linear patterns. The results of this study provide an effective learning strategy solution for field-dependent students in generalizing linear patterns. This study employed a qualitative approach with a case study design to junior high school students. The results indicated that students in the field-dependent cognitive style looked at pattern questions represented in the form of geometric images globally without looking at the structure of the image. Two strategies for generalizing linear patterns used by field-dependent students were examined, namely recursive and different strategies.

Keywords: Generalization, Generalization Strategy, Cognitive Field-Dependent Style, Linear Pattern

Abstrak

Pola linear merupakan materi utama dalam pembelajaran pola bilangan di Sekolah Menengah Pertama, akan tetapi masih banyak siswa yang mengalami kegagalan dalam menggeneralisasi pola linear tersebut. Kegagalan siswa dalam menggeneralisasi pola diduga banyak dilakukan oleh siswa yang cenderung memandang masalah secara global tanpa memecah ke dalam komponen penyusunnya seperti siswa *field-dependent*. Untuk itu dalam penelitian ini bertujuan untuk mempelajari proses berpikir siswa yang gagal dan mempelajari proses berpikir siswa yang berhasil dalam menggeneralisasi pola linier. Hasil penelitian ini memberikan solusi strategi belajar yang efektif untuk siswa *field-dependent* dalam menggeneralisasi pola linier. Penelitian ini adalah penelitian kualitatif dengan pendekatan studi kasus terhadap siswa Sekolah Menengah Pertama. Hasil penelitian menunjukkan bahwa siswa dengan gaya kognitif *field-dependent* memandang soal pola yang direpresentasikan dalam bentuk gambar geometri secara global tanpa melihat struktur gambar. Pada penelitian ini mempelajari dua strategi generalisasi pola linearyang digunakan oleh siswa *field-dependent*, yaitu strategi rekursif dan strategi beda.

Kata kunci: Generalisasi, Strategi Generalisasi, Gaya Kognitif Field-Dependent, Pola Linier

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Pattern generalization is essential in mathematics learning since it can develop new knowledge (Ellis, Lockwood, Tillema, & Moore, 2017). Furthermore, pattern generalization is conceptually related to mathematical structures (Rivera, 2015), and pattern realization is the latest approach to studying algebra (Barbosa, Palhares, & Vale, 2007). The importance of pattern generalization has resulted in the curriculum shifts in Indonesia, teaching pattern generalization to students at the secondary school level (Kemendikbud, 2016). Likewise, the National Council of Teacher of Mathematics recommends the idea of algebraic reasoning through pattern generalization in elementary and secondary schools (NCTM, 2000). Therefore, pattern generalization would be one of the important supporting topics to learn mathematics in Indonesia.

One of the pattern materials is generalization of linear patterns. Generalization is defined as the process of finding similarities in each example or case, so that it applies in general (Brief, 2003; Kaput, 2008; Kaput, Blanton, & Moreno, 2008). While the linear pattern is a pattern that has the first difference that is constant and the terms are formulated with $U_n = an + b$ (Stacey, 1989). Thus, the generalization of linear patterns in this study is the process of finding similarities in each of the terms of a linear pattern, so the formula is used to determine the terms of the linear pattern.

A study carried out by Becker and Rivera (2005) yielded that junior high school students still failed to generalize linear patterns because they tend to start with numerical strategies. These students focused on numerical data which were trapped in recursive relationships (Hourigan & Leavy, 2015). In addition, students failed to generalize linear patterns because they used a trial and error strategy without understanding coefficients in linear patterns (Becker & Rivera, 2005). Furthermore, in many cases students who generalize linear patterns follow patterns recursively (Chua, 2009; Hourigan & Leavy, 2015). The results of Ellis' study (2007) also showed that students who recognize patterns or rules may not be able to generalize the pattern. Students' failures in generalizing linear patterns are due to the strategies students use.

Factors influencing students' failures in generalizing linear patterns are worth exploring. In their study, Lannin, Barker, and Townsend (2006) found that there were three factors influencing the selection and use of pattern generalization strategies, namely task factors, social factors, and cognitive factors. Task factors relate to tasks that are based on problem situations (Lannin, et al. 2006). For instance, students tend to use recursive strategies when solving generalization problems whose independent variables are implicitly stated. Social factors are social interactions when students are involved in the task of simultaneous generalization influenced by other students or teachers (Lannin, et al. 2006). Lastly, cognitive factors can be in the form of knowledge possessed by students (Lannin, et al. 2006) or cognitive factors can also be cognitive styles of students. Cognitive style is the tendency of individuals to understand, think, and store information (Hadfield & Maddux, 1988).

Recent research only focused on generalization strategies used by students, such as strategies to find differences, namely generalization strategies that focus on the differences between terms in number patterns (Montenegro, Costa, & Lopes, 2018; Becker & Rivera, 2005; Stacey, 1989), strategies for finding patterns, that is, generalization strategies focusing on the pattern of the formation of terms from a number pattern (Ellis, 2007), quantity relationship strategy, namely the strategy of generalizing patterns involving the relationship of input values with output values (Ellis, 2011), trial and error strategies (Becker & Rivera, 2005), linear pattern strategies, namely generalization strategies using linear pattern formulas (Stacey, 1989), visual strategies, namely visual grouping strategies, and visual growth strategies (Becker & Rivera, 2005). Even though students often used recursive strategy, namely the strategy to find the next n^{th} term using the previous n^{th} term (Becker & Rivera, 2005; Hourigan & Leavy, 2015; Lannin, et al. 2006). Through this recursive strategy, students often experienced failure in generalizing patterns, so identification of the strategy was required to know the layout of the error of the students in generalizing patterns.

Previous studies partly focused on strategies employed by students in generalizing patterns. This means that there is still a space to conduct research of pattern generalizations involving strategies to generalize cognitive patterns and factors. Lannin, et al. (2006) contended that when students work on tasks with different structures (i.e. tasks with increasing and decreasing structures), they have a tendency to use the same strategy. The results also portrayed that students who consciously use recursive strategies will change to use more effective strategies. For this reason, Lannin, et al. (2006) proposed for further research of pattern generalization involving cognitive factors. Based on this fact, the present study seeks to uncover effective strategies employed by students in generalizing linear patterns in terms of cognitive factors.

There are two kinds of cognitive factors, field-dependent and field-independent styles. Cognitive style is defined as the tendency to see the problem globally, while the cognitive style of field-independent is defined as the tendency to see the problem into constructor components (Onyekuru, 2015; Tinajero & Paramo, 1998; Loranger, et al. 1984; Coventry, 1989). Several studies captured that cognitive style influences students in learning. The results of the study Pithers (2002) indicated that the field-dependent and field-independent cognitive style approaches have implications on the effectiveness of individual and group learning. The results of the study by Karamaerouz, Abdi, and Laei (2013) found that field-dependent students are passive and dependent, while field-independent students are active and independent. The results of Al-Salameh's study (2011) indicated that individuals with field-dependent cognitive styles cannot handle objects that are perceived separately from the surrounding elements, while field-independent students can handle objects separately from the surrounding elements. This study concluded that cognitive style influences how a person learns, including solving problems of generalizing linear patterns.

This study focuses on field-dependent cognitive styles. The main reason for choosing this cognitive styles is due to the characteristic possessed by field-dependent students who view complex situations globally without identifying key elements of these complex situations (Coventry, 1989; Loranger, Gosselin, & Kaley, 1984; Onyekuru, 2015; Tinajero & Paramo, 1998). The second reason is because students with field-dependent cognitive styles experience more mathematical anxiety (Hadfield & Maddux, 1988); less active in classroom learning (Loranger, et al. 1984); lacking in accuracy (Ling & Salvendy, 2009); more errors in algebraic thinking (Agoestanto, Sukestiyarno, Isnarto, Rochmad, & Lestari, 2019); better for verbal information than acting analytically (Karamaerouz, et al. 2013). This study explores how students with characteristics looked at the situation globally when solving problems of generalization. In addition, recommendations from previous studies Lannin, et al. (2006) to conduct further research of cognitive factors in the selection and use of linear pattern generalization strategies were implemented.

This study provides more understanding that cognitive styles also influence the selection and use of strategies for generalizing linear patterns. The results of this study are also useful for teachers in giving strategies to generalize appropriate linear patterns to students with field-dependent cognitive styles. In addition, it also contributes to solving problems that field-dependent students have in learning to generalize linear patterns, especially mathematical anxiety.

METHOD

This research was conducted in two state secondary schools with "A" accreditation. This research implemented randomized sampling consisting of 40 students of grade 8th who has learned the materials about the patterns of numbers. A descriptive qualitative approach was employed in this study. Forty junior high schools were recruited to complete the Group Embedded Figure Test (GEFT) instrument. From this test, six students were found to have field-dependent characteristics, eight students have field-independent characteristics, and twenty-six students have the same characteristics between field-dependent and field-independent. From this data, six students who had field-dependent characteristics were invited as the participants. The data were analyzed by grouping the generalization strategies employed by the participants. Interviews were used to validate the descriptive results of the generalization strategies.

The first phase was to determine the research participants, namely students with field-dependent characteristic. Researchers used a copy of Group Embedded Figure Test (GEFT) developed by Witkin, Oltman, Raskin, and Karp (in Mykytyn, JR., 1989) to determine participant students with field-dependent characteristic. GEFT is a perceptual test that requires the students to find a picture that was seen before in more complex pictures. GEFT consists of 7 simple images and 18 complex pictures tested in 20 minutes. A copy of the instrument Group Embedded Figure Test (GEFT) was translated into Indonesian and given to forty students in grade eight of a secondary school. When the students finished working on a copy of the instrument Group Embedded Figure Test, the result was computed to classify the students into the field-dependent, field-independent, and ambiverts (neither field-dependent nor field-independent). This test consisted of three sessions, the first session with simple questions contained seven test items, the second session consisted of nine medium-difficult test items and the third session consisted of nine difficult test items. Witkin (in Morrison, 1988) reported that seven the first points did not have a discriminatory value with a population, or they were exercise session. Nine items in the second section and nine items in the third part of each were scored one for the correct answer and score zero for incorrect answers (Morrison, 1988). So, the students' minimum score was zero, and the maximum score was 18.

Categorization of the field-dependent, field-independent, and ambiverts in this research adopted the procedures of Maghsudi (2007). The reason researchers adopted Maghsudi' procedure (2007) was that Witkin, Oltman, Raskin, and Karp did not specify a clear score to categorize the individual of field dependent and field independent by their performance in the GEFT (Onyekuru,2015). Maghsudi's procedure (2007) was also adopted by Onyekuru's research (2015) and also the research of Ayvaz, Gündüz, Durmuş, and Dündar (2016) who utilized Mean and Standard Deviation. Maghsudi (2007) used Mean $\pm$ Standard Deviation as a limit point (see Figure 1) where the Mean and Standard deviation was determined from the students' score after they completed the Group Embedded Figure Test instrument (GEFT).

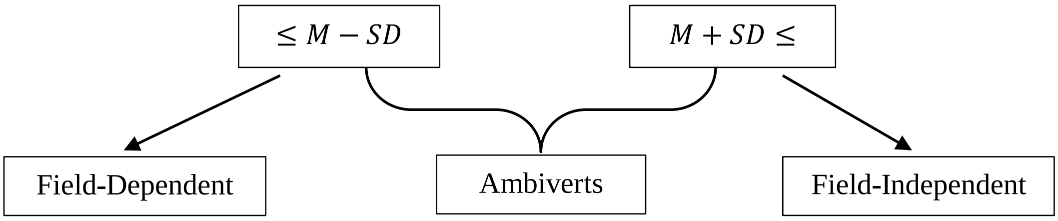


Figure 1. Category Cognitive by Implementing Style (M: Mean and SD: Standard Deviation)

From the Figure 1, it is clear that if the students obtained score $\leq \mathrm{M} - \mathrm{SD}$, they were in the category of field-dependent, but if the students got a score $\geq \mathrm{M} + SD$, then they were included in the field-independent category, and the rest were ambiverts (Maghsudi, 2007). Therefore, the students got less than or equal to the average minus Standard Deviation were classified into the field-dependent category while whose score were greater or equal to the average plus Standard Deviation were classified into the field-independent category (Onyekuru, 2015). The rest of the students were in the category of ambiverts. The students of the field-dependent category were selected as the participants of the research.

The second stage was to analyze generalization strategies used by the participants in the completion of the second instrument, namely the problem of the linear pattern (see Figure 2). The instrument generalization linear pattern is the test that requires the students to find a formula of the term nth term from the pattern of numbers that have a constant first difference. The second instrument was used to find out strategies for generalizing linear patterns used by the participants. This instrument consisted of three questions with different representations (see Figure 2). The first question used a representation of a number sequence. This question was used to find out how field-dependent students found input values to generalize linear patterns. Questions number 2 and number 3 were used to find out whether students with field-dependent cognitive styles viewed images globally or broke down the components of the image. This problem could be used to find out the pattern generalization strategies of students in a field-dependent cognitive style, whether students in a field-dependent cognitive style used visual strategies or numerical strategies, or changing the image to a number sequence.

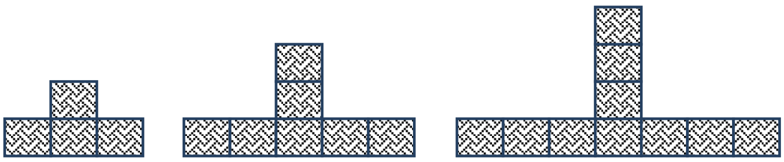
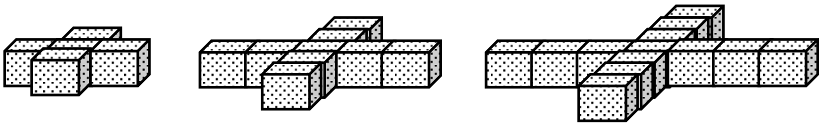
Questions of linear pattern using number representation	1. Number sequence is presented as follows: 6, 10, 14, 18, … Decide: a. Number of the 5th until the 10th term! Write down how you find the formula! b. General formula for term *n*! Write down your method! c. The 57th term! Write down your method!
Questions of linear pattern using plane figures	2. Pay attention to this linear patter:  Decide: a. How many squares appear in the 4th until 10th pattern! Write down your method! b. General formula to decide the number of squares in pattern *n*! Write down your method! c. How many squares in the 71th pattern! Write down your method!
Questions of linear pattern using solid figures	3. Pay attention to this linear pattern:  Decide: a. How many cubes appear in the 4th until the 10th pattern! Write down your method! b. General formula to decide the number of cubes in pattern *n*! Write down your method! c. How many cubes in the 83rd pattern! Write down your method!

Figure 2. Pattern Generalization Instrument

The second data obtained from the results of field-dependent students' work in solving the problem of generalizing patterns was analyzed descriptively to describe the strategies of generalizing linear patterns used by the students with field-dependent cognitive styles. The qualitative analysis for the students' generalization strategy consisted of three steps. The first step was to analyze the views of students about the given Mathematics questions. The results proved that the characteristics of the field-dependent students tended to view the complicated situation globally without identifying the elements of the constructors (Onyekuru, 2015; Tinajero & Paramo, 1998; Loranger, et al. 1984; Coventry, 1989).

The second step was to analyze the process of generalization that the field-dependent students did. Further, the analysis of the process of generalization in this research used the taxonomy of generalizations that consists of relating, searching, and extending (Ellis, 2007). The relating action that how the students form the relationship between the situations in the questions with the other situation in the act of searching was analyzed. Then, whether the students perform repetitive actions, find similarities, or find patterns was also analyzed. Meanwhile, the extending activities related to how the students extended the similarity or a more general pattern was analyzed later (Ellis, 2007). The third step was to analyze the results of how the pattern generalization that students wrote the formula of the n^{th} term (Ellis, 2007). Then, from the results of the analysis using the steps in Figure 2, it was obtained a descriptive process of the students field-dependent in generalizing the pattern began from paying attention to the questions globally. Then, they generalized the patterns and produced the formula generalizations of the n^{th} term.

RESULTS AND DISCUSSION

From the Group Embedded Figure Test (GEFT) filled out by the participants, scores and frequencies are presented in Table 1.

Table 1. GEFT Score and Frequency

Score	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Frequency	1	0	0	0	2	1	2	3	3	2	1	4	3	6	1	3	5	2	1

Table 1 shows the average and standard deviation of the student scores were determined using the formula (1).

$$\bar{x} = \frac{\sum_{i=1}^{n}(x_i \times f_i)}{n} \quad (1)$$

The average student score is 11.175, followed by determining the standard deviation (4.29) using the formula (2).

$$S = \sqrt{\frac{n\sum x_i^2 - (\sum x_i)^2}{n(n-1)}} \quad (2)$$

From this analysis, the average range was also determined (between the lower limit and the upper limit). The lower limit was obtained from the mean less standard deviation (M-SD), that is 11.175-4.29 = 6.885 and the upper limit was determined from the average added standard deviation (M + SD), which is 11.175 + 4.29 = 15.465.

From the average range, three student score categories were obtained, consisting of six students who received the GEFT score below the average range ($x_i \leq 6.885$), twenty-six students who obtained GEFT score between the average range ($6.885 < x_i < 15.465$), and eight students who achieved GEFT score above the average range ($x_i \geq 15.465$). From the student score category, it was found that there were six students having field-dependent characteristics (because they got a GEFT score below the average range), twenty-six students had the same characteristics between field-dependent and field-independent (because they got GEFT scores between ranges average), and eight students had field-independent characteristics (because they get a GEFT score above the average range). Consequently, the six students were opted as the research participants.

From the linear pattern questions answered by the participants, it was found that the field-dependent subjects employed two strategies to generalize linear patterns; these are recursive and difference strategies. This data were presented in Table 2.

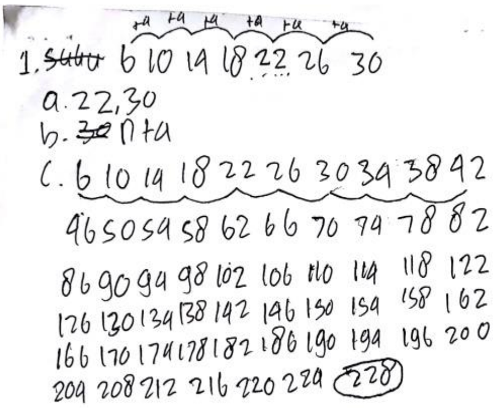
Table 2. Generalization Strategy Based on Question Number

No	**Generalization Strategy**		**Failed**
	Recursive	**Difference**	
1	1 students	2 students	3 students
2	2 students	2 students	2 students
3	1 students	2 students	3 students

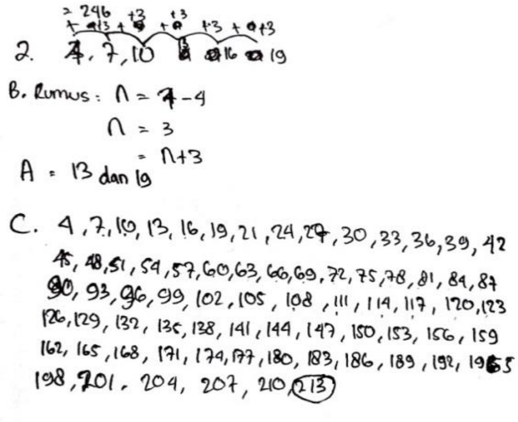
The results of the analysis on the strategy used by the participants and interviews to explore students' thinking process of the strategy are presented below.

Recursive Strategy

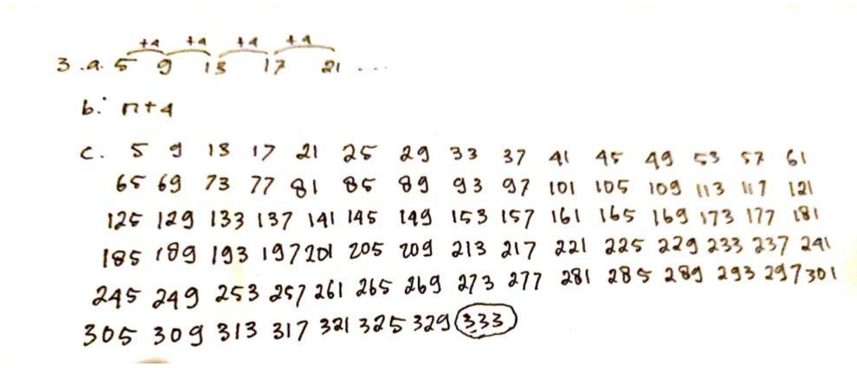
The first strategy used by field-dependent subjects is a recursive strategy. Recursive strategy is term-to-term reasoning (Lannin, et al. 2006) or in other words recursive strategies describe relationships that occur in situations between sequent values of independent variables. The results of this study indicate that participants use a recursive strategy by writing all of their terms to the terms in question. The results of the participants completing generalization questions using recursive strategies can be viewed in Figure 3. These results indicate that the participant's answer is correct, while the way to write the generalization formula is incorrect.



(a) Answer of Question 1



(b) Answer of Question 2



(c) Answer of Question 3

Figure 3. Solving Problem Using Recursive Strategy

From Figure 3 (a), it can be seen that in solving number generalization problems, participants focus on differences. Participants use differences to complete close generalizations, namely determining the 5th term, 6th term, and 7th term. In addition, participants write the results of the wrong generalization, which is number 1 is $U_n = n + 4$. The interview below was conducted to understand how participants obtained the formula shown in Figure 3.

Researcher	:	*in the generalization formula* $U_n = n + 4$ *(what is n? and 4?)*
Participant 1	:	*n is 6 Sir and 4 is the different, because* $6 + 4 = 10$.
Researcher	:	*Why did you write* $U_n = n + 4$*?*
Participant 1	:	*Because I could not find the generalization formula.*
Researcher	:	*Why did you write all terms until the 57th terms?!*
Participant 1	:	*Because I didn't know the formula, so I wrote it all.*
Researcher	:	*How did you know this formula?!*
Participant 1	:	*I did it by myself, Sir!*

From the interviews, it is unveiled that the field independent subjects do not view the input value, but the output value by saying n is 6. The participants cannot find the generalization formula. As the result, the participants write the formula to determine the second term with the first term plus difference. To determine the third term, the participants write the second term which is added differently, and to determine the fourth term, the participants write the third term that is added differently, and so on until the 57th term. Furthermore, because participants write all the terms until the questioned term, this strategy is called recursive strategy, which is to write all the terms until the term is asked.

In the question of plane figure (Figure 3 (b)) and in the question of solid figure (Figure 3 (c)), it can be seen that the field-dependent subjects in solving generalization problems in the form of visual images change the image into number sequence pattern. It means that participants view visual images globally and do not view the components that make up images that can be used as a basis for generalizing patterns. Then participants also write the formula for generalizing the wrong pattern for question number 2 with the formula $U_n = n + 3$. After finding this wrong generalization formula, participants write all of their terms until the 71st terms. Participants also write the formula for generalizing the wrong pattern for question number 3 with the formula $U_n = n + 4$. After finding this wrong generalization formula, participants write all of their terms to the 83rd term.

Based on the tasks and interviews with participants, the recursive strategy process of field-dependent subjects that causes failure of pattern generalization can be seen in Figure 4.

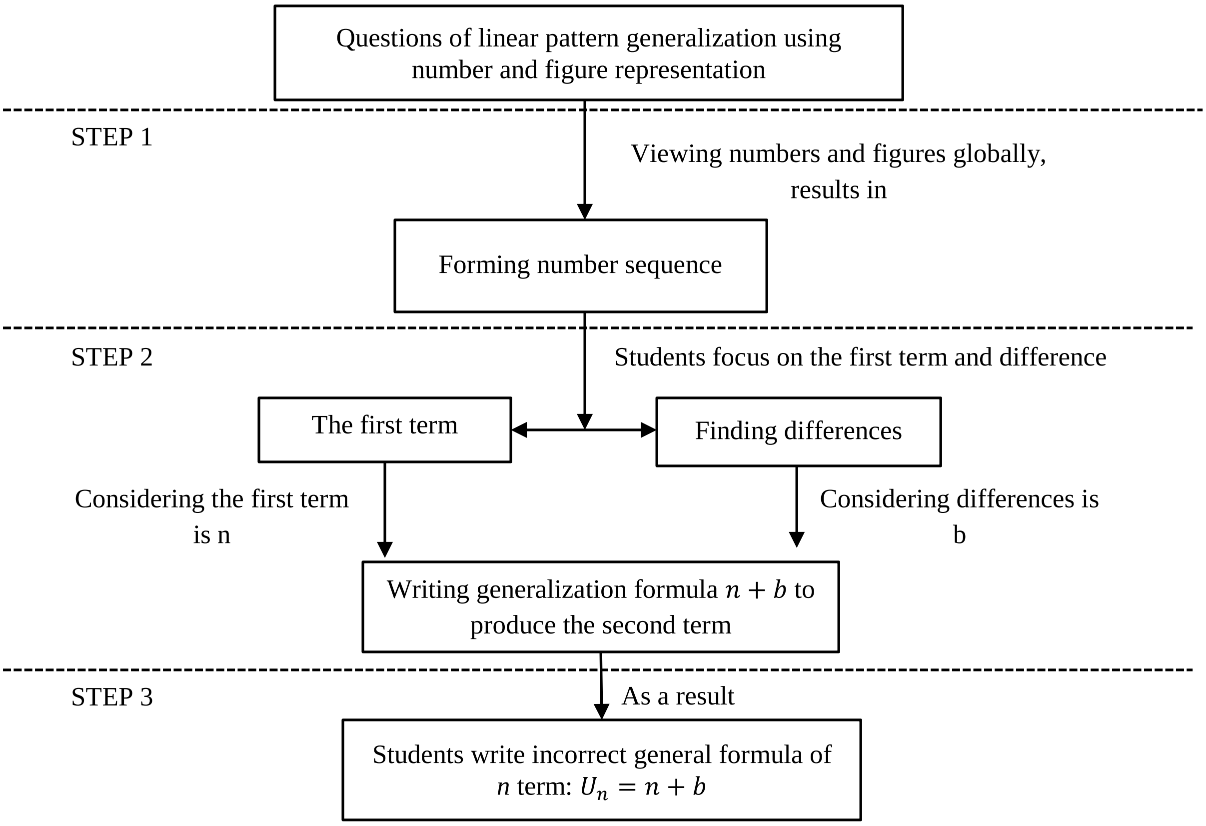


Figure 4. The Thinking Process of Incorrect Recursive Strategy

From Figure 4, there are three stages of the recursive strategy process carried out by field-dependent subjects. The first step undertaken by the field-dependent subjects looks at the question of generalization globally and is converted into a row of numbers. This is consistent with the characteristics of globally oriented field-dependent students and prefers external information structures (Onyekuru, 2015). Individuals with field-dependent cognitive styles tend to be global in analyzing learning situations and have difficulty breaking information into isolated parts (Onyekuru, 2015). Field-dependent subjects are those who have greater difficulty releasing part of the context (Tinajero & Paramo, 1998). Field-dependent subjects react to complex situations globally without identifying key elements and tend to be controlled by situations rather than themselves that make up the situation (Coventry, 1989; Loranger, et al. 1984). Field-dependent students are less able to think analytically (Karamaerouz, et al. 2013). Field-dependent students are not good at choosing numbers, pictures in detail (Sözcü, İpek, & Kınay, 2016).

In the second step, it was obtained that the participants connected with the situation of the question during the linear pattern learning at school. These actions resulted in the participants focusing on searching the variance from the sequence numbers pattern. The participants expanded the activities to search for a variance to find the solution for the nth term. The dependent field subjects were influenced by the characteristics of the environment, especially by their teachers. This means that field-dependent subjects are influenced by social factors, such as the teacher. It is depicted from an interview carried out to the participant: "Where did you get this method?". The participant answers "I got this method from the teacher". This is consistent with the theory that field-dependent students have higher social skills than field-independent students, in which field-dependent students prefer group learning, interact with teachers and peers (Rayner & Riding, 1997). Field-dependent students tend to be friendly, such as working with other people, influenced by others, being adaptive at solving problems related to social interaction, influenced by criticism, preferring jobs based on interaction with others, disliking tasks that require analysis (Witkin, Moore, Goodenough, & Cox, 1977), and field-dependent students tend to have sensitivity to others (Holmes, Liden, & Shin, 2013).

In the third step, participants form a generalization formula using the first and different terms. Participants assume the first term is n, so participants write the formula for generalizing $U_n = n + b$. This can be seen from the results of student interviews which state that n is 6 of the formula n + 4, so participants contend 6 + 4 = 10. It is written when participants find a term obtained from the second term plus 4, and so on. As a result of this formula, participants include in a recursive relationship, which uses the rule "add 4". This is in line with the opinion of Hourigan and Leavy (2015) arguing that if students focus on numerical data, students are at risk for recursive relationships. Recursively participants write all their terms to the questioned term (this is seen in the results of participants who write all of their terms up to the 57th term). Hourigan and Leavy (2015) said that this recursive rule approach cannot be said as a generalization strategy.

Different Strategy

Generalization strategies that are also used by field-dependent subjects are different strategies. Different is defined by the result of the reduction in the *n-th* term by the term (n-1). Mathematically, the difference is written in the formula $b = U_n - U_{(n-1)}$. Stacey (1989) contended that different strategies are assumed to be repeated additions. The different strategies in this study are defined as generalization strategies using the results of the reduction in the *n-th* term by the (n-1) term. The results of participants completing the generalization problem using a different strategy can be seen in Figure 5.

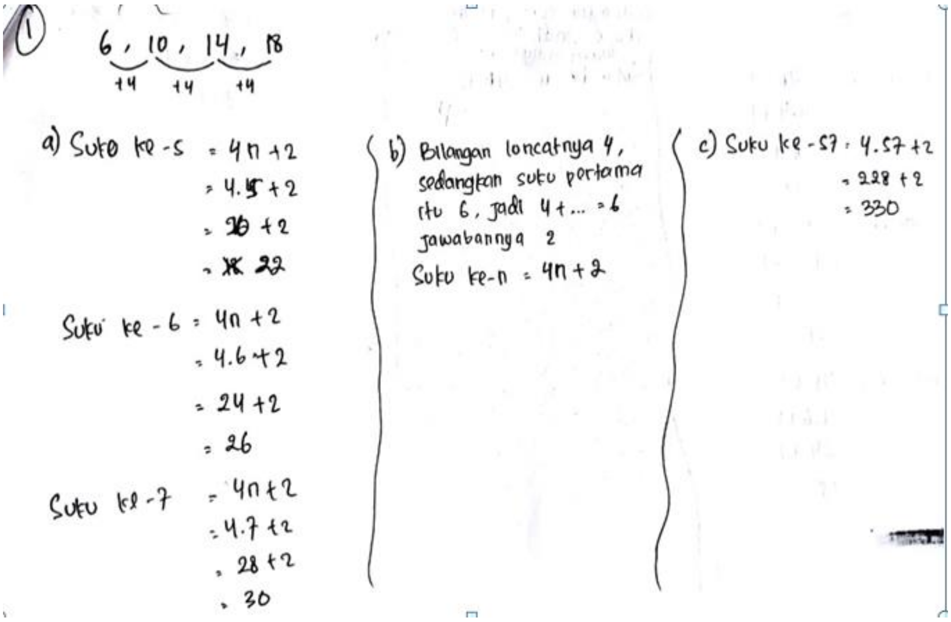


Figure 5. Generalization Done by Participants Using Difference Strategy

From Figure 5, participants have completed problem number 1 by finding a difference. First, participants look for differences from the number sequence. Second, participants use differences to find the general formula for the sequence. Participants argued that difference is a jump number (problem number 1 has a difference of 4, so participants proceed to number 4). Afterwards, participants connect the difference with the first term, which is 6. Different relations with the first term written by participants in the form of $4 + \cdots = 6$ (participants write the answer is 2). By combining jump numbers and jump number relationships with the first term, participants write the first generalization formula, namely $U_n = 4n + 2$. Participants complete questions number 2, 3, and 4 in the same way when solving question number 1. These steps are confirmed to participants through the interview as follows.

Researcher	:	*For what purposes did you find a difference?*
Participant 2	:	*To decide the generalization formula, Sir*
Researcher	:	*What did you think difference as jump number!*
Participant 2	:	*Yes Sir, because it has equal difference between the terms, so jump to 4!*
Researcher	:	*Where did you obtain the generalization formula* $U_n = 4n + 2$
Participant 2	:	*4n was obtained from jump number, Sir. If jump one time, so it is* $4(1) = 4$, *two times is* $4(2) = 8$, *and so on.* *2 was obtained from 4 added 2 equals 6. So, I wrote the formula as follows:* $U_n = 4n + 2$
Researcher	:	*Why did you complete number 2, 3, and 4 using the same way?*
Participant 2	:	*Yes, Sir. Because this is the best way*
Researcher	:	*How did you get this way?!*
Participant 2	;	*I am thinking by myself, Sir..*

Based on the results of the interview above, it was found that participants determine the difference to find the generalization formula. Participants interpret differences as jump numbers, and write jump numbers in the form of 4n. Afterwards, participants look for relationships 4 with 6 (4 + ... = 6), namely plus 2. Participants write their generalization formula in the form of $U_n = 4n + 2$. Participants solve the generalization questions number 2 and number 3 in the same way, namely finding differences. The results of the interview indicate that participants share how to find differences that can be done and it is easier to solve the problem of generalizing linear patterns. This strategy is obtained from the participants themselves.

The process of finding strategy differs from field-dependent subjects can be seen in Figure 6. From Figure 6 it is found that there are 3 steps in field-dependent subjects completing generalizations of linear patterns using strategies to find differences. The first step taken by field-dependent subjects is to look at the problem globally and fertilize it into a row of numbers. This stage is the same as students who use recursive strategies. This means that individuals with cognitive style have similarities in information processing, where field-dependent individuals process information globally (Coventry, 1989; Loranger, et al. 1984; Onyekuru, 2015; Tinajero & Paramo, 1998). In addition field-dependent students find it difficult to break information into isolated parts (Onyekuru, 2015).

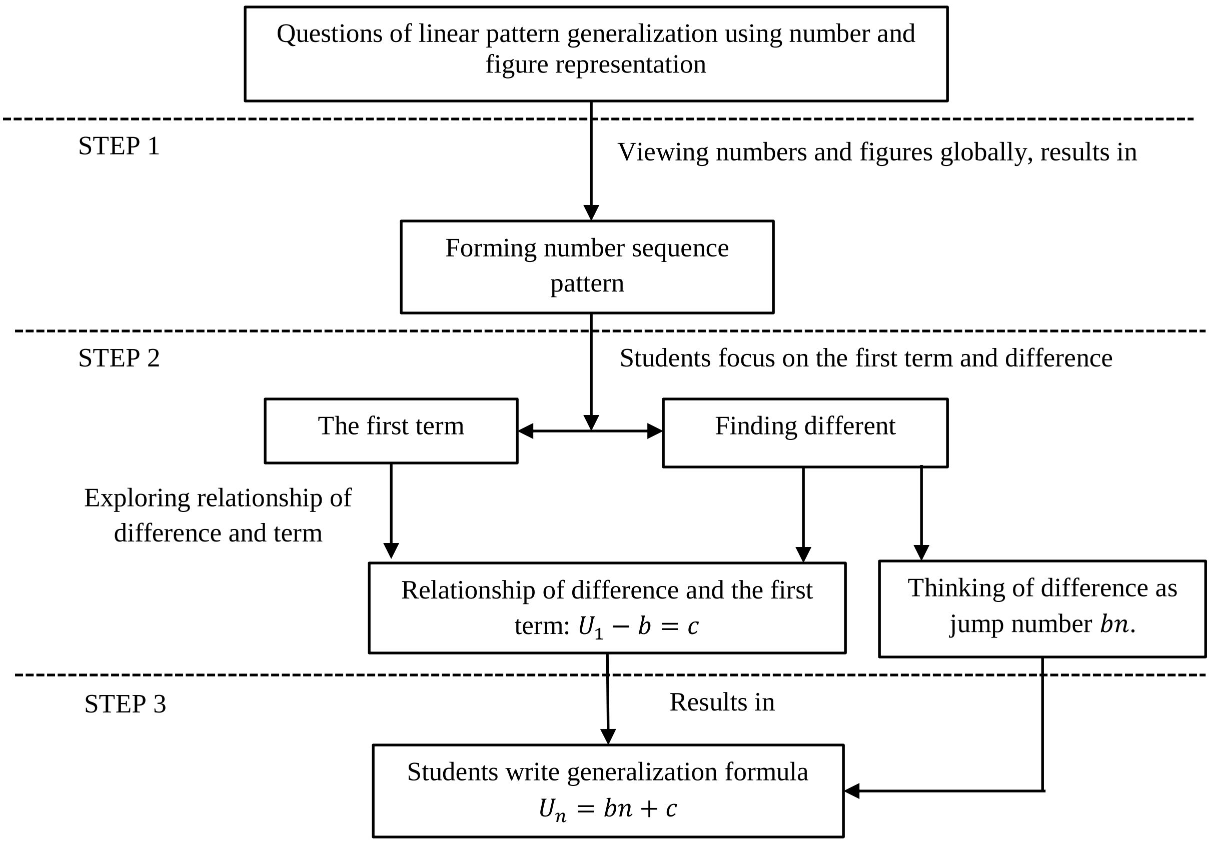


Figure 6. The Thinking Process of the Presence of Difference Strategy

The second step, this second stage has similarities and differences with participants who use recursive strategies. Its similarity was to connect with the current situation of teaching in schools. In addition, the similarity of the recursive strategy with the strategy of finding differences at this stage is that participants focus on differences and the first term. The difference in recursive strategy with the strategy of finding differences at this stage is the way to kick different and the first term. Participants with recursive strategies view the first term plus different to get the second term and view the second term plus the difference to get the third term, and so on. Meanwhile, participants with different find strategies view differences as jump numbers written with *bn* and then look for different relationships with the first term, namely $U_1 - b = c$. Then the participants expanded the relationship between the jump numbers bn with c to obtain more general rules. So, the participants successfully wrote the generalization linier formula, namely $U_n = bn + c$.. This means that field-dependent subjects can see the structure of information that is explicitly visible (Tinajero & Paramo, 1998), which are the first and different terms. At this stage, it is also expected that learning for field-dependent subjects sees the relationship of the visible information structure (i.e. the first and different terms). Participants are also asked to find the relationship of the first term with different (6 with 4, obtained 4 (1) + 2 = 6), the relationship of the second term with different (10 with 4, obtained 4 (2) + 2 = 10), the third term relationship with different (14 with 4, obtained 4 (3) + 2 = 14), so that the relationship is generally obtained between the terms with different, namely 4n + 2.

In the third step, there is a tangible difference between a recursive strategy and a strategy for finding differences. The recursive strategy is done by writing all the terms until questioned term, while the strategy of finding differences succeeded in generalizing linear patterns by writing the results of generalizing linear patterns, namely $U_n = 4n + 2$. Afterwards, participants use this generalization formula to find the next terms. For instance, participants find the 57th term with this formula, namely $U_{57} = 4(57) + 2 = 230$. The strategy of finding this difference is ultimately successful in making generalizations. Thus, strategies to find differences include strategies for generalizing linear patterns.

Based on the resolution of the problem of linear pattern generalization, it was found that there were two strategies used by field-dependent subjects in generalizing patterns, namely recursive and difference strategies. The recursive strategy starts with subjects changing the image pattern into a row of numbers. Second, subjects look for differences from the number sequence. Third, subjects use the difference and the first term to determine the second term, use the difference and the second term to determine the third term, and so on which results in the subjects writing all the terms to the terms that is questioned. While the strategy of finding differences starts with students changing the image pattern into a row of numbers. Second, subjects look for differences from the number sequence. Third, subjects interpret differences as jump numbers (written bn) and look for the first term relationship with different ($U_1 - b = c$). Subjects who use the find difference strategy succeed in generalizing a linear pattern by writing the general form of the nth term generalization, namely $U_n = bn + c$.

The critical findings of this research are that the participants considered the first term is n, and the vary is b, which led them to the failure in generalizing the term of a linear pattern, so that the participants wrote an error generalization formula, $U_n = n + b$. As a result, recursively, the participants rendered all the term terms of the pattern of numbers until the term questioned term. This failure is resolved by using a strategy of difference, which the students look different as the jump numbers written in "bn" and seek the relationship of varies with the first term, $U_1 - b = c$. So, they could obtain an appropriate generalization of the linear pattern, $U_n = bn + c$.

CONCLUSION

The results of this study contribute to generalization learning of linear patterns for students with field-dependent categories. The limitation of the present study was that the researcher selected 40 students of grade 8 from two state secondary schools as the participants. Nevertheless, the findings of the present research have significant implications for the mathematics teachers in teaching linear patterns, especially in resolving the problem of recursive strategy that caused the students to fail to generalize linear pattern. From the results of this study, it is recommended that teachers teach linear patterns to field-dependent students by changing the pattern of images into number patterns since field-dependent students prefer to generalize geometric patterns by changing into number patterns rather than finding image structures that can be generalized. For this reason, it is also necessary that field-dependent students recognize the relationship between different terms until a general formula from the nth term is found. Further, future studies can employ a larger sample to obtain strategies used by the field-dependent students and the errors they make to generalize the linear pattern. In addition, further studies are worth-conducting to examine generalization strategies of students who have field-independent cognitive styles. By examining the generalization strategies of students in a field-dependent cognitive style, it impacts for learning betterment of field-independent students.

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