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ELEMENTARY PRESERVICE TEACHERS' KNOWLEDGE, PERCEPTIONS AND ATTITUDES TOWARDS FRACTIONS: A MIXED-ANALYSIS

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Abstract

Previous research has shown knowledge, perceptions, and attitudes are essential factors during mathematics classroom instruction. The current study examined the effects of a 3-week fraction instructional unit using concrete models, problem-solving, and problem-posing to improve elementary preservice teachers' knowledge, perceptions and attitudes towards fractions. A quasi-experiment design was implemented to gather data via closed-ended, open-ended, and essay tasks from a convenience sampling of 71 female elementary preservice teachers during pre- and post-assessments. The study discovered that the select preservice teachers were weak in the content knowledge specifically on unit-whole, part-whole, equivalent area, arithmetic operations, and ordering fractional values. In contrast, the incorporation of concrete models, problem-solving and problem-posing was effective in improving the preservice teachers' level of pedagogical content knowledge, perceptions and attitudes towards fractions. Implications of the results and suggestions are discussed.

Keywords: Elementary School, Problem Posing, Teacher Preparation Program, Preservice Teachers, Mixed Methods

Abstrak

Penelitian sebelumnya menunjukkan bahwa pengetahuan, persepsi, dan sikap merupakan faktor penting dalam pembelajaran matematika. Penelitian saat ini meneliti tentang pengaruh dari pembelajaran pecahan selama kurun waktu 3 minggu menggunakan model konkret, pemecahan masalah, dan *problem-posing* untuk meningkatkan pengetahuan, persepsi, dan sikap calon guru sekolah dasar terhadap materi pecahan. Sebuah desain eksperimen semu diimplementasikan untuk mengumpulkan data melalui permasalahan dalam bentuk soal *closed-ended*, *open-ended*, dan uraian dari pengambilan sampel yang representative dari 71 calon guru wanita sekolah dasar selama pra dan pasca penilaian. Studi ini menemukan bahwa calon guru terpilih lemah pada konten materi pecahan, khususnya pada *unit-whole*, *part-whole*, area yang sama, operasi aritmatika, dan nilai pecahan berurutan. Sebaliknya, penggabungan model konkret, pemecahan masalah, dan *problem-posing* adalah efektif dalam meningkatkan level pengetahuan konten pedagogis calon guru, persepsi, dan sikap terhadap materi pecahan. Implikasi hasil dan saran lebih lanjut dibahas dalam tulisan ini.

Kata kunci: Sekolah Dasar, *Problem Posing*, Program Persiapan Guru, Calon Guru, Metode Campuran

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Knowing numbers and operations is a cornerstone of mathematics education in the school curriculum (National Council of Teacher of Mathematics [NCTM], 2000). Young students should have acquired a conceptual understanding of number systems, their structures, and properties during classroom instruction (Lamon, 1999). Fractions are considered central concepts for school mathematics but have conventionally been challenging and cumbersome for teachers to deliver as well as for students to learn (Barnett-Clarke, Fisher, Marks, & Ross, 2010; Lin, 2010). An incomplete understanding of fractions can eventually affect individuals' learning in subsequent mathematical areas such as algebra, trigonometry, and calculus and related disciplines (Barnett-Clarke et al., 2010).

Numerous previous studies have emphasized the impact of diverse classroom instruction (e.g., traditional, concrete models, web-based, one-on-one) on student learning of fractions (Lin, 2010; Newton, 2008; Osana & Royea, 2011). However, there has been scant research measuring the potential benefit of an instructional practice that integrates concrete models for developing preservice teachers' conceptual understanding of fractions (Sarama & Clements, 2009). In this study, we focused on measuring the integration of Mathematics TEKS Connection module with concrete model instructional practices in facilitating preservice teacher knowledge construction, perceptions and attitudes towards fractions during the teacher preparation programs.

A body of literature has documented that a majority of in-service teachers and preservice teachers have limited profound knowledge for teaching mathematics (Ball, 1993; Hill, 2010; Ma, 1999; Newton, 2008; Timmerman, 2004). In addition, White, Way, Perry and Southwell (2005) and Llinares (2002) stated preservice teachers' belief or perceptions is reflected in their action or attitudes during teaching and learning that influence the classroom instructional practices. Despite this evidence, little is known about the nature of knowledge for teaching fractions, perceptions and attitudes that teachers should have nurtured during teacher preparation, the place where teachers should have acquired their teaching repertoire.

Teacher Knowledge

Ball (1993) and Ma (1999) stated that teachers' content knowledge could be a possible aspect affecting classroom instruction. Meanwhile, Shulman (1986) noted content knowledge alone is not sufficient; pedagogical content knowledge is also significant for making the learning of mathematical concepts understandable. Shulman (1986) pointed out that content knowledge is solely an amount of mathematics knowledge that one should have for teaching a particular concept. It includes the conceptual and procedural understanding of specific mathematical ideas (Shulman, 1986). Many researchers argue the importance of teacher knowledge in making teaching and learning of mathematical content meaningful. When teachers do not possess in-depth knowledge of a particular concept (Ball, Thames, & Phelps, 2008) and do not know how to represent the idea and to make it comprehensible and understandable (Shulman, 1986), they often fail to deliver the concept for students' understanding (Barnett-Clark et al., 2010). In contrast, pedagogical content knowledge includes the way a teacher represents fractions to facilitate student learning by using appropriate models, analogies, illustrations, examples, explanations, and demonstrations (Shulman, 1986). Also, teachers must be aware of students' knowledge of fractions and know the strategies for reorganizing students' understanding appropriately (Shulman, 1986).

Perceptions and Attitudes

Previous research showed preservice teachers entered teacher preparation programs with preexisting views grounded mainly on their background and positive or negative experiences in learning mathematics (Timmerman, 2004; White et al., 2005). These preexisting perceptions are resistant to change and difficult to break (Kane, Sandretto, & Heath, 2002). White et al. (2005) claimed an intervention is needed to stop a cycle of negativity perceptions and attitudes towards mathematics especially for preservice teachers who in turn influence the formation of student attitudes. According to White et al., negative perceptions could create negative teaching ways, that can successively affect students' beliefs, attitudes and learning outcomes. Thus, teacher educators should perceive preservice teacher perceptions and attitudes expressly for utilizing the potential ways for knowledge construction and stimulating learning.

Multiple Tasks and Concrete Models

Appropriate instructional materials and mathematical tasks in classrooms are indeed crucial for helping students grasp abstract concepts for knowledge construction (Cramer & Wyberg, 2009; Sarama & Clements, 2009). The incorporation of problem-solving and problem-posing tasks can help teachers to enhance students' understanding of fraction with the assistance of concrete models such as fraction strips, and paper-folding. For instance, when students are asked to generate a problem for this division of fractions, $\frac{3}{4} \div \frac{1}{8} = n$ (Sowder, Philipp, Armstrong, & Schappelle, 1998), they can use a concrete model to discover fraction size (Empson, 2002) and the relationships between the $\frac{1}{4}$ and the $\frac{1}{8}$. The teacher can examine students' understanding by asking students to explain the meaning of $\frac{3}{4} \div \frac{1}{8}$ as illustrated in Figure 1. Based on the representational model of Figure 1, students should be able to make connections to their prior knowledge about fractional numbers and find the answer to the problem.

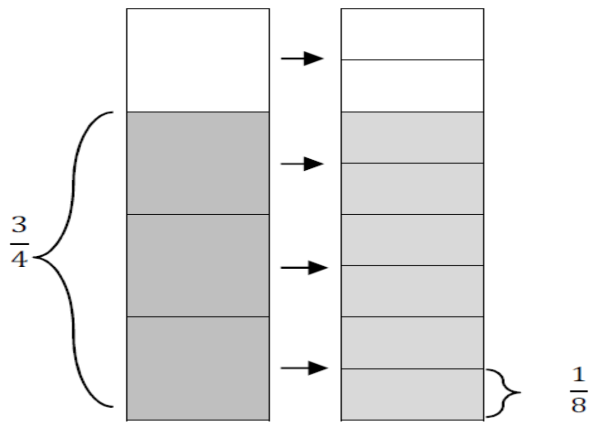

Figure 1. Models for Division of Fraction

Teachers may assist students' learning during the process of operating the models. A student must be able to grasp the concepts and ponder the underlying association on six sets of $\frac{1}{8}$ in $\frac{3}{4}$. Based on the necessary knowledge, the student could think of a fraction situation that fits the given equation and then teachers would ask the students to share their situations with others in the class.

METHOD

In utilizing both quantitative and qualitative data and rigorous methodological research techniques, we adapted close-ended and open-ended problem-solving and problem-posing tasks to shed light on the research question: What was the effect of the instructional unit of developing conceptual understanding of fractions with concrete models on the levels of elementary preservice teachers' content knowledge, pedagogical content knowledge, perceptions and attitudes towards learning and teaching fractions?

The pragmatist approach (Onwuegbuzie, Johnson, & Collins, 2009) drove the study that involved administering two pre- and post-assessments containing close-ended items and open-ended questions. Pragmatist researchers believe that there are many useful ways of seeking knowledge, including mixing quantitative and qualitative data and methods (Onwuegbuzie et al., 2009). A quasi-experimental research method was utilized for gathering both quantitative and qualitative data that focused on a one-group pretest-posttest design (Shadish, Cook, & Campbell, 2002). The qualitative component of the study was based on the constructivist-naturalistic paradigm (Lincoln & Guba, 1985). We attempted to assess the preservice teachers' understanding and attitudes about fractions by examining the open-ended responses and seek "individual and collective reconstructions that may unite around consensus" (Onwuegbuzie et al., 2009, p. 122). It was hoped that we could assess the treatment integrity (e.g., fidelity score) of the fraction instruction using both quantitative and qualitative instruments (Collins, Onwuegbuzie, & Sutton, 2006) to reduce any implementation biases (Onwuegbuzie, 2003). Besides, mixing quantitative and qualitative data can provide significant enhancement of the findings for generalizability purposes (Collins et al., 2006).

The study employed the use of a convenience sampling technique to select the participants who were willing to be part of the study (Collins, Onwuegbuzie & Jiao, 2007). The instructor, Dr X who is an experienced professor, has been teaching elementary methods courses for more than ten years at one of the public universities in Texas. The method course was compulsory for the completion of the undergraduate degree with certification in the EC-Grade 6 Core Subjects/Generalist program. The participants were 71 female preservice teachers who were pursuing a Texas teacher certification in elementary school classrooms. They were from two class sections of the experienced professor in the Fall semester. The course comprised of one hour and 15 minutes of 16-week face-to-face meetings together with an online module through the Blackboard Learning System (eLearning) of the university. Concerning ethical considerations, permission was granted to perform the study over the Institutional

Review Board (IRB) of the university and an information sheet was provided to the participants informing them that the study would run as regular class session. The times for both class sections were back to back and the professor taught the same content and used similar instructional strategies.

Close-ended items from the Learning Mathematics for Teaching [LMT] (2008) were utilized for examining content knowledge and pedagogical content knowledge. Only 11 close-ended were selected based on content-related validity to fractional concepts (Measure A) under investigation. In addition, we adopted five close-ended items on geometry (Measure B) to examine the non-equivalent dependent variable for improving the design. This instrument was made available to participants using an online survey tool, Qualtrics, and was piloted to preservice teachers in the other class sections of the methods course. The internal consistency scores (Cronbach's alpha) were .74 for fractions and .73 for geometry using the Statistical Program for Social Science (SPSS) software version 21.0 (IBM Corp., 2013).

Additionally, nine open-ended items were selected from Sowder et al., (1998), five questions were used to measure content knowledge and four were utilized to assess pedagogical content for teaching fractions. Two professors reviewed these open-ended questions to determine the content-related validity such as the relevancy and appropriateness of the tasks (Collins et al., 2006). The open-ended instrument was piloted and no changes were made to the instrument and the administration time of the present study.

The participants received three weeks of classroom instruction (each week for 4 hours) on basics fraction concepts including addition, subtraction, multiplication, and division. The classroom instruction was comprehensively concentrated on concrete models that helped preservice teachers discover and construct basics mathematics knowledge based on the constructivist theory. Four to five preservice teachers were grouped into a small station (table) and concrete models such as tangrams, fraction bars, fraction circles, fraction towers, fraction strips, and pattern blocks were made available in every class session. During class instruction, Dr. X adopted a meaningful teaching approach through demonstrations and problem-posing activities that involved preservice teachers' use of modeling to develop conceptual knowledge of fractions. Preservice teachers were given various fraction problems to create a scenario for and model with manipulatives or drawings with their partners. Problems were such as as $\frac{1}{2}+\frac{1}{4}$, $\frac{2}{3}-\frac{1}{4}$, $\frac{1}{2}\times\frac{3}{4}$,$\frac{3}{4}\times\frac{1}{2}$,$4\div\frac{1}{2}$, and $\frac{1}{2}\div 4$.

In addition to classroom instruction, preservice teachers engaged and participated actively in online modules, which utilized multiple instructional strategies including web-based activities, videos, and readings. The modules were developed and field-tested that followed the Mathematics Texas Essential Knowledge and Skills (TEKS) Connection [MTC Project]. It was a part of the Texas Math Initiative and represented a partnership between the Texas Education Agency (TEA) and the university. With three different bands (K-4, 5-8, 9-12), it covered some specific and vital mathematics concepts (e.g., place value, fractions) for each grade level. The modules were made available for future teachers use from http://mtc.tamu.edu/home.htm?intro-pre.htm.

The fraction assessments were administered sequentially online during class periods in October 2011. A week before the professor began the instruction on fractions, the preservice teachers were asked to access the Qualtrics survey tool and respond to 16 multiple-choice items by choosing their best options. Then, during one class period, a graduate assistant administered the open-ended tasks wherein participants were required to write their responses on an answer sheet. The fraction instruction took place during three weeks wherein unstructured observations were made and field notes were taken to capture all potentially relevant phenomena. The graduate assistant was an observer-as-participant (Johnson & Christensen, 2012) wherein the primary role was to collect related information (e.g., setting, interactions, and subtle factors) concerning fraction instruction (Erlandson, Harris, Skipper, & Allen, 1993). A week after Dr. X completed the fraction instruction, the preservice teachers were required to login to the Qualtrics instrument and answer the same items as on the pre-assessment. Similarly, the graduate assistant administered the same open-ended assessment to the participants.

The close-ended items were scored based on their correctness, one point for a exact answer or zero point for an inappropriate one. Data were stored in the SPSS software (SPSS Inc., 2007) and points for items were totaled up for content knowledge (i.e., Item 1, 3, 4, 6, 7, 10, and 11) and pedagogical content knowledge (Item 2, 5, 8, and 9). Next, the open-ended items were transformed quantitatively where each pre- and the post-written response was assessed and coded into a numerical value (Teddlie & Tashakkori, 2006) based on the degree of accomplishment, zero (lowest) - four points (highest). For inter-rater agreement percentage, two independent raters graded seven identical scripts and attained an 83.3% agreement. The raters discussed to resolve any disagreement.

Based on the data collection, a few statistical analyses were run and effect sizes for pre-post contrast were calculated. Then, we analyzed the written responses and observation data through a constant comparison analysis (Glaser, 1965) for a more in-depth and better understanding of the elementary preservice teachers' strategies (Lincoln & Guba, 1985) for solving fractional tasks. The essay parts were unitized using the QDA Miner 3.2 (Provalis Research, 2009) following the classical content analysis method (Onwuegbuzie & Teddlie, 2003). The written essays were examined, underlined into chunks, and unitized into smaller significant parts (codes). The codes that were frequently appear were counted to symbolize essential concepts of preservice teachers' perceptions and attitudes towards fractions.

RESULTS AND DISCUSSION

Based on the quantitative results and qualitative findings, we hoped to make external (statistical) generalizations regarding the level of fractional understanding to the entire population from where the participants were conveniently drawn. In the present study, the differences from pretest to posttest scores on the problem-solving and problem-posing tasks were utilized to observe the effect of the fraction instruction that focused on concrete models. These employed mixed data analyses to measure the change in the levels of elementary preservice teachers' content knowledge, pedagogical content knowledge, perceptions and attitudes regarding learning and teaching fractions.

Analysis of Quantitative Data

This quasi-experimental design utilized a one-group pretest-posttest design to assess the cause-and-effect relationship of fraction instruction (treatment) and preservice teachers' knowledge and belief about fractions. For reducing threats to validity of the experimental design, we adapted six geometry items as the nonequivalent dependent variable (measure B) in addition to 11 fraction items for measure A. The results showed the preservice teachers' scores of fraction knowledge (measure A) changed statistically significantly, $t(59) = 3.50$, $p = .001$ with a mean gain effect size of .32 from pretest to posttest possibly because of the treatment of fraction instruction. No statistically significant change for geometry knowledge (measure B) was noted from pretest ($M = 5.13$) to posttest ($M = 5.05$). According to Shadish et al. (2002), this indication is useful for ruling out the possibility of a cause-and-effect relationship in a research study.

The results of the close-ended assessment included 60 completed responses from the select preservice teachers. Eleven answer scripts were eliminated due to an invalid/incomplete pretest or posttest. The t-test value indicates that the mean score for fraction knowledge on the posttest was significantly greater than the mean score on the pretest. Specifically, the results from the close-ended items revealed statistically significant differences in means scores for both preservice teachers' content knowledge [$t(59) = 2.14$, $p = .037$] and pedagogical content knowledge [$t(59) = 2.87$, $p = .006$]. The standardized mean gain effect sizes were 0.19 and 0.38 respectively. The results indicate that the preservice teachers' content knowledge and pedagogical content knowledge improved during instruction as measured by using close-ended items.

A review of the item analysis showed that preservice teachers scored a lower percentage for content knowledge specifically on Items 4, 6, 7 10, and 11 with less than 40% answering correctly on both pretest and posttest. Also, we found that preservice teachers had the most difficulty on Item 4 with only 10% correct on the pretest and posttest. They were not able to interpret and to analyze different situations given in the question that related to the unit-whole of a fraction. The results supported Lamon's (1999) argument that most of the classroom instruction failed to present the unit-whole fraction that contributes to students' misunderstanding of the concept. However, the majority of preservice teachers were able to answer correctly the part of Items 2, 5, and 8 that involved their knowledge of problem posing. Besides, the most significant differences between the percentage of scores on the pretest and posttest were found on Items 6, 2 (item b only), 8 (all sub-items), and 9.

Analysis of Qualitative Data

As mentioned in the previous section, the qualitative data were mainly collected from preservice teachers' responses to the open-ended assessment consisting of five fraction content knowledge items and four fraction pedagogical content knowledge items. Based on 71 completed responses, each item was assessed and transformed into a numerical value (0-4 points) that can be analyzed statistically; then the subtotals were calculated for content knowledge and pedagogical content knowledge. Overall, the Wilcoxon signed-rank test revealed that a 3-week instruction did not elicit a significant improvement statistically in the level of elementary preservice teachers' content knowledge [Z = -2.25, p = .024, r =.21]. Indeed, the median content knowledge score was 2.0 both pre- and post-assessment. The statistical analyses of each item under content knowledge were utilized and the Bonferroni adjustment with $\alpha = .05/5 = .01$ was applied in controlling for the familywise error rate (Jaccard & Guilamo-Ramos, 2002). Even though scores mostly were higher on the post-assessment, no statistically significant difference was evident in respect to solving arithmetic operations of fractions (Item 2), recognizing a fractional part-whole (Item 3), when finding an equivalent area (Item 4), and ordering fractional values (Item 5). Instead, the study revealed an impressive result where the post-assessment score was statistically significantly lower than pre-assessment [Z = -3.54, p < .001] for identifying a unit whole (Item 1) with effect size, r = .30. In both assessments, about 58% to 77% preservice teachers did not conceptually understand unit wholes—they were not able to recognize that the shaded part of two pizzas eaten was $\frac{5}{8}$ (item 1). Instead, they believed that the two pizzas were a separated unit whole, supported Lamon's (1999) and Ball's (1993) argument that many individuals tended to refer to a single item (e.g., a single pizza) as a unit (one).

Similarly, the Wilcoxon test analyses were performed for multiple comparisons among items under pedagogical content knowledge for adjusted α values of .05/4 = .0125. The results indicated that the select participants gained much input from the 3-week fraction instruction as measured by their pedagogical content knowledge with all items showing statistically significant differences between the assessment results ($p \leq .001$). Specifically, the preservice teachers indicated statistically significantly greater improvement when recognizing the accurate representation, $2 \times \frac{3}{4}$ for Item 2 with Z = 4.33 and an effect size, r = .36. When they were asked, "A recipe calls for $\frac{3}{4}$ the cup of flour. How much flour is needed if the recipe is doubled?" during the pre-assessment, many mentioned $2 \times \frac{3}{4}$ and $\frac{3}{4} \times 2$ were the same representations and accurate. They stated that because of the commutative property, multiplication is reciprocal thus resulting in the same final answer. Nevertheless, after the instruction, they were able to differentiate that both representations were not identical.

Preservice teachers attained statistically significantly higher scores on the post-assessment for Item 1, 3, and 4 with similar effect size, r = .27. Through a closer examination of the written responses, rich information was revealed about preservice teachers' pedagogical content knowledge on different concepts. For instance, for Item 1 on problem posing, when creating a scenario problem on pre-assessment to match the division fraction $\frac{3}{4} \div \frac{1}{8} = n$, many confused it with $\frac{3}{4} \div 8 = n$. Here are the examples of the fraction scenarios created from the same preservice teacher in the study:

There are 24 students on the class roll. Monday at school, only ¾ of the class was present. The teacher needed to divide the class into eight groups. How many students are in each group? (Pre-assessment)
How many 1/8 ft long strips of ribbon can be cut from a ribbon that is ¾ ft long? (Post-assessment)

The responses were similar to the previous results (Item 2) that showed many preservice teachers in the present study could not differentiate the underlying fractional concepts between two arithmetic operations. To build preservice teachers' understanding of fractions, Dr. X allocated extra time presenting and using various concrete examples for creating scenarios during class instruction. Preservice teachers spent time practicing and generating their scenario problems based on the given fraction operation. As a result, they were able to pose meaningful and accurate scenarios during post-assessment.

For Item 3 and 4 that represented classroom situations for pedagogical analysis, the majority of preservice teachers were having some difficulties in analyzing the contexts and in providing brief descriptions supporting their arguments. For example, only a small number of preservice teachers could identify a missing unit from the children's responses in Item 3. For this reason, preservice teachers had difficulties deducing the size of chunks the children considered in the case of cola. The participants were unable to analyze the children's thinking and were difficult to make inferences based on the children's unlabeled responses. However, other participants considered students' thinking, understanding and misunderstanding when responding to this open-ended item and focused on the number that children provided in much detail. They attempted to predict children's thinking with some drawings, assuming a case with 12 or 24 colas, and working backward to solve the problem. Item 4 focused on the conversation among three children who were comparing two fractions $\frac{4}{4}$ and $\frac{4}{8}$ with their representation. Initially, preservice teachers' responses failed to include the vital concept of unit-whole and the role of a common unit. However, during post-assessment, they were able to point out that the children struggled to recognize which one was a more substantial value using the fraction model, failing to understand that the size of rectangles must be the same (equal unit-whole) before comparing the fractions. For instance, a preservice teacher responded:

Student 2 had the right idea that if student one had made the whole rectangles the same instead of the fractional part of a whole, he would have seen that 4/4≠4/8. However, student 3 is correct in that four parts out of 4 take up the entire rectangle while four parts of 8 take up half of a rectangle, which shows two different fractions. The students definitely could have used fraction bars to make the whole shapes even, but students 2 and 3 seemed to have understood without seeing the representation. They knew that the whole was represented with four parts for one fraction and 8 for the other. They knew *the parts of the whole were being compared.*

Observation during Fraction Instruction

From the classroom observation, most of the preservice teachers were not experiencing problems creating scenarios involving the addition and subtraction of fractions. They were able to pose scenarios and check each other for appropriate modeling and terminologies. However, when it came to multiplication and division, preservice teachers demonstrated the most considerable anxiety because they were not able to pose scenarios that fit the algorithmic operations. They struggled to understand the concept clearly, thus were not able to come up with appropriate word problems. For instance, in one of the class sessions preservice teachers were demonstrated and illustrated scenario problems for two fraction multiplications $2 \times \frac{1}{3}$ and $\frac{1}{3} \times 2$. They were guided in creating meaningful scenarios to differentiate the underlying concepts between these arithmetic operations that can produce the same answer, $\frac{2}{3}$ but have different meanings. Many were not capable of grasping the idea for the first time and showed their frustration by expressing their anxiety and anxiousness to their group members. Then, they asked Dr. X to show other examples that would hopefully help them build their conceptual understanding. After several explanations and practices with concrete models, most of them were able to create some meaningful scenarios involving multiplication and division even though some still did not fully understand the notion. We believed more time and effort would be necessary for them to build and understand these new concepts.

Overall, we found the majority of preservice teachers enjoyed and gained much knowledge from each session of their fraction instruction and the online modules. In class, they actively participated in exploring and experimenting with various manipulatives with group members or partners to build their fractional understanding. One preservice teacher stated, "I believe that manipulatives are the best learning tool for children because they are concrete objects that they can work out the math problems with". The response supports Piaget's (1964) argument that hands-on materials or concrete models can help younger children build mental sense and abstract ideas in mathematics.

Perceptions and Attitudes towards Fractions

After the semester end, sixty-six preservice teachers submitted short written responses that described their perceptions and attitudes towards learning and teaching fractions based on their involvements in the method course. Each written response was coded into meaningful units of data in the form of phrases, sentences, or paragraphs and then these data were grouped into similar emergent themes (Johnson & Christensen, 2012). Eighteen themes were developed from the coding process and were classified into three meta-themes that related to their positive, negative, or unchanged attitudes towards fractions. We categorized preservice teachers under positive attitudes meta-theme when they showed optimistic phrases, statements and words from the written essays. Otherwise, the negative responses noted all preservice teachers' unfavorable endorsement towards learning and teaching fractions.

The results from the coding process of the essays revealed that 62 of 66 selected preservice teachers showed some positive remarks about fractions based on their experiences from participating in the activities during the method course. Nine themes with 377 units of data were formed and considered from the responses demonstrating the change in the perceptions and attitudes. Among all the themes, the course material was frequently coded (100 units of data) indicating this might be the most critical concept noted by the majority of preservice teachers. They believed the incorporation of concrete manipulatives, online modules and videos, daily examples, and scenario problems during class instruction helped their learning about fractions. They discovered meaningful experiences in exploring and practicing different approaches to learning and teaching fractions from the method course. Also, most preservice teachers indicated that they felt comfortable with fractions (71 units of data) and their understanding of fractional concepts had increased (57 units of data) as compared to the beginning of the semester. However, they firmly believed that more time, practice, and research (39 units of data) were needed to understand fractional concepts profoundly and to be able to teach fractional concepts to young children effectively. For example, one preservice teacher stated:

> *I feel that my attitude towards fractions has changed for the better. Dr. X did a great job explaining fractions to the class in ways I could understand. She allowed us to use different manipulatives that helped in understanding fractions rather than just using a pencil and paper. Although I am no 100% confident about teaching fractions I feel so much better about it and I know that with a little more practice I will be able to teach others what I learned about fractions in Dr. X's class.*

Simultaneously, preservice teachers believed the instructor (23 units of data), Dr. X, facilitated the learning of fractional concepts through various teaching aids and scenario problems. Dr. X focused on building the conceptual understanding of fractions with hands-on activities that emphasized the 'why' instead of the 'how' behind addition, subtraction, multiplication, and division of fractions. Based on preservice teachers' past experiences, many argued that they were taught fractions with rules and memorization (40 units of data) in elementary and middle school and the learning process was extremely confusing. Also, they mentioned the 'rote learning' of fractions through a series of steps and formulas (i.e., algorithms) without an understanding of what was going on. A participant stated, "I have known how to get the correct answer when working with fractions, but until this class I didn't understand how I got the answer or why it was correct". It was an unlearning and relearning process for the majority of preservice teachers during the method course with Dr. X.

In addition, creating and practicing different scenarios (e.g., problem-solving problems, word problems) involving fractions with concrete models helped preservice teachers understand the concepts better. After the course, they could "see" how portions make sense in their daily lives and would be interested in incorporating the same teaching method into their future classroom (21 units of data). For instance, one preservice teacher discussed:

When we started with creating word problems I thought no big deal. However, it became difficult to word a problem to show if it was $\frac{3}{4} \times \frac{1}{3}$ versus $\frac{1}{3} \times \frac{3}{4}$, because the modeling was completely different even though the answer will be the same. We practiced a lot on fractions, and also though I do not feel like I am an expert in them, I feel confident enough to teach them. I may have to consult my notes from time to time, but isn't that what those are for? I would use my notes mainly in planning out my lessons.

Contrary to the many positive responses, some preservice teachers had included an opposite perception throughout their essay. We found four themes that revealed their negative attitudes about the learning and teaching of fractions. Most preservice teachers still felt anxious (64 units of data) about fractions and intimidated by mathematics (14 units of data) as a subject in general. They were nervous about teaching fractions to young children (39 units of data) because the resistance towards learning fractions had been entrenched for years and it was not going to disappear overnight demonstrating their low self-esteem and confidence towards fractions. Therefore, they were still conscious and not sure how to explain the concepts in class. For them, it was difficult and challenging to fully understand fractions especially when creating scenarios for multiplication and division with correct terminologies. Also, we found several preservice teachers were overwhelmed with the methods class (14 units of data) but were able to rationalize the situation, thus tried to motivate themselves and be more positive towards the teaching and learning of fractions. Comment for a preservice teacher:

My view of fractions has changed slightly. Before this class, I did not even think about how difficult fractions would be to teach. I got nervous when I heard that it is one of the most challenging subjects to teach. When we started learning about them, I did find that fractions are tough for me to understand. Therefore, I was fearful of teaching them. I think the main thing I am concerned about is the ordering of fractions. For some reason, that gives me the most difficulty. I do believe that after this course, it has gotten a little easier to understand because of the hands-on activities that we did daily. However, I think I will always have fear when teaching fractions.

The analysis of essay questions revealed two preservice teachers had entirely negative attitudes after instruction. They realized how complex working with fractions can be and hated fractions because they did not feel adequate in their teaching abilities even after taking a method course. Also, we noticed a preservice teacher claimed that she did not see any changes in her attitudes towards fractions after the instruction. She had been struggling with fractions and how to understand the concept before and after the methods course. She mentioned “It is hard for me to learn in a group setting. I am much better at learning things one on one when I cannot comprehend them".

The external generalization of the present study is limited to all elementary preservice teachers from a particular university where participants are conveniently drawn and are based on the results taken from the validated measures. However, we emphatically trust the outcomes because we incorporated both quantitative and qualitative research based on a pragmatist approach providing a perfect combination yielding "complementary strengths and nonoverlapping weaknesses" (Johnson & Turner, 2003, p. 299). Notably, the use of both closed and open-ended items uncovered several key findings from multiple perspectives that add to the existing literature on the preservice teachers' level of knowledge and attitudes towards fractions.

Results of the present study advance current understanding of the effect from a 3-week fraction instruction of a mathematics method course in several valuable ways. First, it provides useful insights into the potential benefits of integrating concrete models, problem-solving, and problem-posing activities on building prospective teachers' profound knowledge of teaching fractions. Second, the study was distinctive because it represents the first study using a mixed-methods research design to more deeply understand the complex phenomena of selected preservice teachers' content knowledge and pedagogical content knowledge of fractions, and attitudes towards fractions.

Seventy-one preservice teachers participated in the study where they complete pre- and post-assessments on content knowledge and pedagogical content knowledge of fractions. Also, they submitted an essay about attitudes towards fractions after the 3-week instructional unit. Importantly, the study showed that the incorporation of problem-solving, problem posing, and hands-on activities have the potential to assist instructors in making fraction instruction more meaningful and enjoyable thus eventually helping learners to develop conceptual knowledge of fractions (Thompson, 1994). Some other significant outcomes and issues need to be addressed aside from the inconclusive results as measured by the closed and open assessment tasks.

Related to the select preservice teachers' content knowledge of fractions, results from the descriptive statistics revealed low percentages of preservice teachers achieved the full points for most of the items on both pre-and post-assessments. The results might indicate the participants were weak on the particular fraction concepts being tested. Specifically, we noticed the select preservice teachers were not aware that the unit-whole may include more than one item or may consist of items packaged as one known as a composite unit. The behavior was consistently evident when they were identifying the unit-whole of Item 4 (closed-ended) and Item 1 (open-ended). On pre- and post-assessments, almost 90% of preservice teachers were not able to notice that children were considering a different set of unit-whole on Item 4 (close-ended). Similarly, the picture for Item 1 (open-ended) showed two pizzas with the same or different kinds, which is called a one two-unit and not two one-unit pizzas. However, many preservice teachers assumed a single object (i.e., a single pizza) as a unit (one) and answered $1\frac{1}{4}$ pizza instead of $\frac{5}{8}$ pizza. Lamon (1999) argued that the concept of the unit has frequently been neglected in classroom discussions. Majority teachers and textbook authors emphasized using the same unit-whole such as one cake in which making students assumed a unit was always a single object (Lamon, 1999). Lamon argued many classroom instructions failed to introduce the vital concept of unit-whole of fractions to students.

CONCLUSION

While this study revealed the preservice teachers' lack of content knowledge of fractions, the results showed compelling evidence of their understanding of teaching fractions supported previous research by Ball (1993), Hill (2010), Ma (1999), Newton (2008), and Timmerman (2004). It is noteworthy to see that they demonstrated statistically significant improvement in their level of pedagogical content knowledge even though they received only a three-week fraction instruction unit mainly focusing on problem posing and hands-on activities. We believe the close interaction with the instructor and quality of instruction during the mathematics method course possibly helped preservice teachers develop knowledge about pedagogy and nurture their fraction teaching repertoire (White et al., 2005). Our data warrant further investigation into the relationship among the duration of instruction, the role of instructor, quality of instruction, and level of fractional knowledge they gained. Therefore, a more thorough examination is critical to determine the factors that might affect the level of knowledge before and after instruction.

In the post-assessment, many preservice teachers successfully generated and identified meaningful and accurate fraction situations that matched the given arithmetic operations. When reviewing their fraction essays, they mentioned how problem posing and hands-on activities had an impact on their learning and understanding of fractions. The majority of preservice teachers showed positive attitudes towards fractions after receiving the instruction and believed it was an eye-opening session. Still, some expressed their concerns about additional practice and the time needed for developing confidence in teaching fractions.

Our results revealed that the select preservice teachers were able to reflect on students' work and to suggest basic teaching approaches for helping students' misunderstanding of fraction concepts. However, the responses showed they had limited practical experiences teaching fractions to children. Because the participants would be elementary school teachers later, more exposure working directly with students in a real classroom setting is needed. The experience responding to elementary students would develop their pedagogical skills and improve preservice teachers' understanding of educational components (Llinares, 2002; White et al., 2005).

Taken together, teacher education services are a place for teachers to build expertise, teach skills, values, and understanding in order to become effective teachers in mathematics (Llinares, 2002). Nonetheless, the prior knowledge and behaviors of service teachers have a significant impact on what and how they learn during their teacher training programs (Llinares, 2002). Teacher educators are responsible for providing potential teachers with the in-depth knowledge and experience required to effectively teach the basic concepts of fractions.

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REFERENCES

Ball, D. L. (1993). Halves, pieces, and twoths: Constructing and using representational contexts in teaching fractions. In Carpenter, T. P., Fennema, E., Romberg, T. A. (Eds.). *Rational numbers: An integration of research* (pp.157-195). Hillsdale, NJ: Erlbaum.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407. http://dx.doi.org/10.1177/0022487108324554.

Barnett-Clarke, C., Fisher, W., Marks, R., & Ross, S. (2010). *Developing essential understanding of rational number: Grades 3-5*. Reston, VA: The National Council of Teachers of Mathematics.

Collins, K. M. T., Onwuegbuzie, A. J., & Jiao, Q. G. (2007). A mixed methods investigation of mixed methods sampling designs in social and health science research. *Journal of Mixed Methods Research, 1*(3), 267-294. http://dx.doi.org/10.1177/1558689807299526.

Collins, K. M. T., Onwuegbuzie, A. J., & Sutton, I. L. (2006). A model incorporating the rationale and purpose for conducting mixed methods research in special education and beyond. *Learning Disabilities: A Contemporary Journal*, *4*(1), 67-100.

Cramer, K., & Wyberg, T. (2009). Efficacy of different concrete models for teaching the part-whole construct for fractions. *Mathematical Thinking and Learning*, *11*(4), 226-257. http://dx.doi.org/10.1080/10986060903246479.

Erlandson, D. A., Harris, E. L., Skipper, B. L & Allen, S. D. (1993). *Doing naturalistic inquiry: A guide to methods.* Newbury Park, CA: Sage.

Empson, S. B. (2002). *Organizing diversity in early fraction thinking.* In B. Litwiller & G. Bright (Eds.), Making sense of fractions, ratios and proportions: 2002 Yearbook *(pp. 29-40).* Reston, VA: National Council of Teachers of Mathematics.

Glaser, B. (1965). The constant comparative method of qualitative analysis. *Social Problems, 12*(4), 436-445. http://dx.doi.org/10.2307/798843.

Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education. 41*(5), 513-545.

IBM Corp. (2013). *IBM SPSS Statistics for Windows, Version 21.0* [Computer software]. Armonk, NY: IBM Corp.).

Jaccard, J., & Guilamo–Ramos, V. (2002). Analysis of variance frameworks in clinical child and adolescent psychology: Issues and recommendations. *Journal of Clinical Child and Adolescent Psychology*, *31*(1), 130 –146. http://dx.doi.org/10.1207/S15374424JCCP3101_15.

Johnson, B., & Christensen, L. (2012). *Educational research: Quantitative, qualitative, and mixed approaches* (4th ed.). Boston, MA: Sage.

Johnson, R. B., & Turner, L. A. (2003). Data collection strategies in mixed methods research. In A. Tashakkori, & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 297-319). Thousand Oaks, CA: Sage.

Kane, R., Sandretto, S., & Heath, C. (2002). Telling half the story: A critical review of research on the teaching beliefs and practices of university academics. *Review of Educational Research*, *72*(2), 177–228. http://dx.doi.org/10.3102/00346543072002177.

Lamon, S. J. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers.* Mahwah, NJ: Erlbaum.

Learning Mathematics for Teaching. (2008). *Mathematical knowledge for teaching measures: Rational number*. Ann Arbor, MI: Author.

Lin, C. (2010). Web-based instruction on preservice teachers' knowledge of fraction operations. *School Science and Mathematics, 110*(2), 59-70. http://dx.doi.org/10.1111/j.1949-8594.2009.00010.x.

Lincoln, Y., & Guba, E. (1985). *Naturalistic inquiry*. Newbury Park, CA: Sage.

Llinares, S. (2002). Participation and reification in learning to teach. The role of knowledge and beliefs. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 195–210). Dordrecht, The Netherlands: Kluwer.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Journal, 45*(4), 1080-1110. http://dx.doi.org/10.3102/0002831208320851.

Onwuegbuzie, A. J. (2003). Expanding the framework of internal and external validity in quantitative research. *Research in the Schools*, *10*(1), 71-90.

Onwuegbuzie, A. J., Johnson, R. B., & Collins, K. M. T. (2009). A call for mixed analysis: A philosophical framework for combining qualitative and quantitative. *International Journal of Multiple Research Approaches*, *3*(2), 114-139. http://dx.doi.org/10.3102/000283120832085110.5172/mra.3.2.114.

Onwuegbuzie, A. J., & Teddlie, C. (2003). A framework for analyzing data in mixed methods research. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 351–383). Thousand Oaks, CA: Sage.

Osana, H. P., & Royea, D. A. (2011). Obstacles and challenges in preservice teachers' explorations with fractions: A view from a small-scale intervention study. *Journal of Mathematical Behavior*, *30*(4), 333-352. http://dx.doi.org/10.1016/j.jmathb.2011.07.001.

Piaget, J. (1964). Part I: Cognitive development in children: Piaget development and learning. *Journal of Research in Science Teaching*, *2*(3), 176-186. http://dx.doi.org/10.1002/tea.3660020306

Provalis Research. (2009). *QDA Miner 3.2. User's guide*. Montreal, QC, Canada: Author.

Sarama, J., & Clements, D. H. (2009). "Concrete" computer manipulatives in mathematics education. *Child Development Perspectives*, *3*(3), 145-150. http://dx.doi.org/10.1111/j.1750-8606.2009.00095.x.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA: Houghton Mifflin.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14. http://dx.doi.org/10.3102/0013189X015002004.

Sowder, J. T., Philipp, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction: A research monograph*. New York, NY: State University of New York.

Teddlie, C., & Tashakkori, A. (2006). A general typology of research designs featuring mixed methods. *Research in the Schools*, *13*(1), 12-28.

Thompson, P. W. (1994). Concrete materials and teaching for mathematical understanding. *Arithmetic Teacher, 41*(9), 556-558.

Timmerman, M. (2004). The influences of three interventions on prospective elementary teachers' beliefs about the knowledge base needed for teaching mathematics. *School Science and Mathematics, 104*(8), 369-382. http://dx.doi.org/10.1111/j.1949-8594.2004.tb18003.x.

White, A. L., Way, J., Perry, B., & Southwell, B. (2005). Mathematical attitudes, beliefs and achievement in primary pre-service mathematics teacher education. *Mathematics Teacher Education and Development*, *7*, 33-52.