ISSN 2087-8885
E-ISSN 2407-0610

Journal on Mathematics Education
Volume 11, No. 1, January 2020, pp. 95-110

SEMIOTIC REASONING EMERGES IN CONSTRUCTING PROPERTIES OF A RECTANGLE: A STUDY OF ADVERSITY QUOTIENT

Christine Wulandari Suryaningrum[1,2], Purwanto[1], Subanji[1], Hery Susanto[1], Yoga Dwi Windy Kusuma Ningtyas[2], Muhammad Irfan[1,3]

[1]Universitas Negeri Malang, Malang, Indonesia
[2]Universitas Muhammadiyah Jember, Jember, Indonesia
[3]Universitas Sarjanawiyata Tamansiswa, Yogyakarta, Indonesia
Email: christine.wulandari.1603119@student.um.ac.id

Abstract

Semiotics is simply defined as the sign-using to represent a mathematical concept in a problem-solving. Semiotic reasoning of constructing concept is a process of drawing a conclusion based on object, representamen (sign), and interpretant. This paper aims to describe the phases of semiotic reasoning of elementary students in constructing the properties of a rectangle. The participants of the present qualitative study are three elementary students classified into three levels of Adversity Quotient (AQ): quitter/AQ low, champer/AQ medium, and climber/AQ high. The results show three participants identify object by observing objects around them. In creating sign stage, they made the same sign that was a rectangular image. However, in three last stages, namely interpret sign, find out properties of sign, and discover properties of a rectangle, they made different ways. The quitter found two characteristics of rectangular objects then derived it to be a rectangle's properties. The champer found four characteristics of the objects then it was derived to be two properties of a rectangle. By contrast, Climber found six characteristics of the sign and derived all of these to be four properties of a rectangle. In addition, Climber could determine the properties of a rectangle correctly.

Keywords: Reasoning, Semiotic, Semiotic Reasoning, Construction Concept, Adversity Quotient (AQ)

Abstrak

Semiotik didefinisikan sebagai penggunaan tanda untuk mewakili konsep matematika dalam menyelesaikan masalah. Penalaran semiotik dalam mengonstruksi konsep adalah proses penarikan kesimpulan berdasarkan objek, tanda dan interpretasi. Makalah ini bertujuan untuk menggambarkan tahap penalaran semiotik siswa sekolah dasar dalam membangun sifat-sifat persegi panjang. Subjek penelitian kualitatif ini adalah tiga siswa sekolah dasar yang dikelompokkan menjadi tiga tingkat yaitu Adversity Quotient (AQ): quitter/AQ rendah, champer/AQ sedang, dan climber/AQ tinggi. Hasil penelitian menunjukkan tiga subyek mengidentifikasi objek dengan mengamati objek di sekitar mereka. Dalam tahapan membuat tanda, mereka membuat tanda yang sama yaitu gambar persegi panjang. Namun, dalam tiga tahap terakhir, yaitu menginterpretasikan tanda, menemukan sifat tanda, dan menemukan sifat persegi panjang, mereka membuat cara yang berbeda. Quitter menemukan dua karakteristik objek persegi kemudian menganggapnya sebagai sifat persegi panjang. Champer menemukan empat karakteristik objek kemudian diturunkan menjadi dua sifat persegi panjang. Namun, Climber menemukan enam karakteristik dari tanda dan menjadikan semua ini sebagai empat sifat persegi panjang. Selain itu, Climber dapat menentukan sifat-sifat persegi panjang dengan benar.

Kata kunci: Penalaran, Semiotik, Penalaran Semiotik, Konstruksi Konsep, *Adversity Quotient* (AQ)

How to Cite: Suryaningrum, C.W., Purwanto, Subanji, Susanto, H., Ningtyas, Y.D.W.K., & Irfan, M. (2020). Semiotic reasoning emerges in constructing properties of a rectangle: A study of adversity quotient. *Journal on Mathematics Education, 11*(1), 95-110. http://doi.org/10.22342/jme.11.1.9766.95-110.

Reasoning is the ability of students to analyze a problem, solve a problem, draw conclusions and express their ideas logically. Panchal (2013) stated that reasoning has been determined as a process of attainment conclusions based on relevant facts and sources. The process requires high-level thinking skills. Rapanta (2018) also argues that reasoning is the process of drawing a conclusion based on the

premises, the facts of the investigation results. The reasoning related to symbols is called semiotic reasoning.

Semiotics in mathematics is defined as the use of symbols to represent mathematical concepts in problem-solving (Radford, et al. 2008; Ostler, 2011). Peirce (1931) and Turkcan (2013) believe that semiotics is identical to the logical concepts focusing on the knowledge of the process of human thought. Peirce (1931) also offers an idea related to signs, the use of logic and metaphysics, and a more complete theoretical context for culture called social semiotics. Someone thinks through signs which lead one to communicate with each other and gives any meaning to their environment (West, 2015). Moreover, Peirce asserts a theory of signs focusing on triad dimensions or trichotomy systems. Furthermore, Peirce classifies his theory into three aspects, namely sign, object and interpretant (Yang & Hsu, 2015; Metro-Roland, 2009; Presmeg, 2016).

In semiotics, signs are given meanings from observers to objects to achieve interpretation (Parcell & Parcell, 2009). Furthermore, Deledalle (2013) states that the relationships between sign and its object can be classified into three categories: icons, index, and symbols. An icon refers to the sign which has the same characteristics as the territory object (Miller, 2015). An index stands for the sign that represents a considered object which is the effect of causative relation. A symbol refers to the objects by mutual agreement or law. Peirce (1931) introduces three types of common icons, namely images, diagrams, and metaphors. An image stands for the properties of the icon as indicated by their own simple nature. A diagram is an icon that represents a relational nature. A metaphor refers to the icon representing their original semiotic nature and its certain semiotic properties in an unusual way (Uslucan, 2010).

In recent years, several researchers have studied semiotics, namely Campos (2010), Bjuland (2012), Ng & Sinclair (2015), Turgut (2017). Campos (2010) discussed Peirce's philosophy in mathematics education to foster the development of student semiotic abilities by doing such activities as imagination, concentration, and generalizations to do mathematical investigations on diagrams. Furthermore, Bjuland (2012) elaborated the semiotic sources used by a teacher of sixth-grade students whose students complete their mathematical assignments that provide written text and two inscriptions, namely images and diagrams. On the other hands, Ng & Sinclair (2015) investigated students' learning of reflectional symmetry in a dynamic geometry environment. Students' reasoning for linear transformation when using Dynamic Geometry System (DGS) from the perspective of semiotic mediation (Turgut, 2014). However, the study of semiotic reasoning in constructing the concept of a rectangle has never been done by other researchers.

Semiotic reasoning in constructing concept is a process of deducing a conclusion based on the object, sign (representamen), and interpretant. A concept, sign, and object can be unstoppably interpreted. Every interpretation can add new knowledge. A sign can be configured as verbal, visual, gestural, and musical. A sign is a representation of an object (Eco, 1976). Furthermore, a sign can represent anything that represents an object (Inna, 2013; Stjernfelt, 2015). An interpretant refers to a notion or notation to represent an object (Sarbo & Yang, 2015). The sign is only a sign whether it

considers a subject as a sign while the interpretant exists if and only if there is a sign. On the other hand, nothing can stands for a sign if there is no interpretant. This relation is an indispensable element of Pierce's semiotic triad. A sign can emerge an interpretant which is another sign that is similar to that which is in someone's mind (Inna, 2013). Therefore, each sign can turn out into an object or an interpretant of other signs (Parcell & Parcell, 2009). Mathematical concept which its representation use a lot of signs is geometry.

Geometry is a fundamental concept since geometry can be applied to solve problems in daily life (Ahamad, et al. 2018). It is in line with the opinion Usiskin (1982) revealing that several reasons for teaching geometry, namely geometrical concept can associate mathematics with daily life activities, visualize abstract mathematical ideas by drawing, and exemplify mathematical systems by using representations. One of geometrical concept taught in elementary school is plane figure (2 dimensional shape) as it can improve students' reasoning abilities. This statement is in accordance with the opinion of Fujita and Jones (2007) stating that learning geometry material especially rectangular plane figure can help students in developing their ability to prove and give reason deductively. In addition, learning materials related to plane figure is widely used in daily life. As Hardiarti's research results (2017) prove that the concept of a rectangular plane figure has been found in the structure of the Muaro Jambi temple. This research will be focused on rectangular plane figure.

Constructing a concept is a particular way in which students attempt to understand available information and make connections using the entire cognitive structures to build concept. A student, in his/her own way, learns concepts specifically related to their prerequisite concepts. In constructing the concept of a rectangle, students are involved in observation, investigation, interpretation and drawing a conclusion. One of embedded factors for students in learning mathematics is Adversity Quotient (AQ). AQ is the persistence of a person when dealing with obstacles to obtain success. AQ is expected to contribute a strong motivation of a person to solve encountered problems so that AQ can support a person in achieving success (Stoltz, 2004; Syah, 2010). Stoltz grouped individuals based on their fighting power into three, namely quitter, camper, and climber. Moreover, he stated that people who give up easily are called quitters, people who feel satisfied with certain achievements are called camper, and someone who continues to want to achieve success is called a climber.

This study highlight the semiotic reasoning in constructing properties of rectangle based on students' AQ level. The related studies are still very limited. This study is important because students should understand the mathematical concepts for understanding the mathematical concept. Concepts can be embedded in students' memories (long term memory) if students have a strong motivation to learn.

METHOD

The methodological underpinnings of this study were established through a qualitative approach and explorative descriptive type. The researcher investigated the students' semiotic reasoning in

constructing the concept of a rectangle. The term semiotic reasoning, in this study, means the process of drawing conclusions based on identified objects, making signs and interpretations to interpret the students' made-signs. Constructing a concept is a students' way to understand available information and connect their previous concepts to form a new concept.

The participants were three elementary school students who met the criteria: 1) Fourth-grade students aged 10-11 years, 2) 28 students participated in learning rectangular topics by solving items about rectangular. Students determined rectangular objects based on their experience and their knowledge in determining the properties of rectangular objects, 3) Based on these facts, students were classified into three types who have different levels of Adversity Quotient (AQ), namely one student with quitter/AQ low, one student with champer/AQ medium, and 1 student with climber/AQ high, 4) can communicate their reasoning well (verbal or written), and 5) students carry out semiotic reasoning in constructing the concept of a rectangle.

The data of this study were compiled from video recordings of each lesson, students' notes, and interview results. The videotaping depicted students' activities in constructing the concept of a rectangle. Students' notes described students' identification of objects, students' representamens based on the objects, and students' interpretants in constructing the concept of a rectangle. The interview was employed to explore the students' semiotic reasoning in constructing the concept of a rectangle and complete the unclear data from videos and students' notes.

The learning process carried out by the teacher on 2D shapes. During the study, the researcher focused on following the lesson unit of rectangular shape for 8 meetings. In each lesson, the students' activities in constructing a rectangle concept were videotaped. The researcher, then, selected three students with different levels of abilities based on the results of the video recordings, student's notes, and teacher's consideration. The participants were chosen to triangulate data whether the data were saturated (Creswell, 2015).

The collected data was analyzed to describe students' semiotic reasoning in constructing the concept of a rectangle. The analysis was carried out through 3 steps: 1) selecting appropriate data and excluding the unused one; 2) presenting data by grouping the data based on Pierce's semiotic triad namely object, representamen, and interpretant (Eco, 1976); and 3) drawing conclusion based on the research findings.

RESULTS AND DISCUSSION

The present study observed the fourth-grade learning process on two dimensional (2D) shapes. The subtopic of that material was triangle, square, and rectangle. The researcher attended the lessons as much as 8 meetings. Of the 8 lessons, there were 3 lessons – the 1st, 3rd, and 6th lesson – where the learning process focused on how students find the characteristics of rectangular shape. In the first lesson, the students observed the surrounding objects that were related to 2D shapes.

In this lesson, students collected all figures of 2D shapes. A student (says N) asked the teacher (G):

N: Does this 2D shapes, Ma'am? (see Figure 1)
G: Try to draw a picture on your book!
N: N rushed to sketch the picture on his book (see Figure 2)
G: what is the shape?
N: hemmmmmm ... I don't know ma'am (with a confused face)

Figure 1. N Collected Object

By looking at the object (Figure 1), the student N sketched the object as seen in Figure 2.

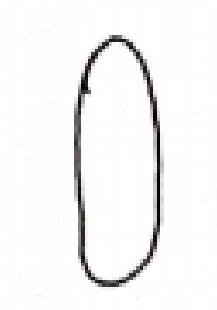


Figure 2. N Sketched Object

The teacher asked all students in the class to pay attention to the N-made figure. The following is the conversation between the teacher (G) and some students (says S) to find out the concept of 2D.

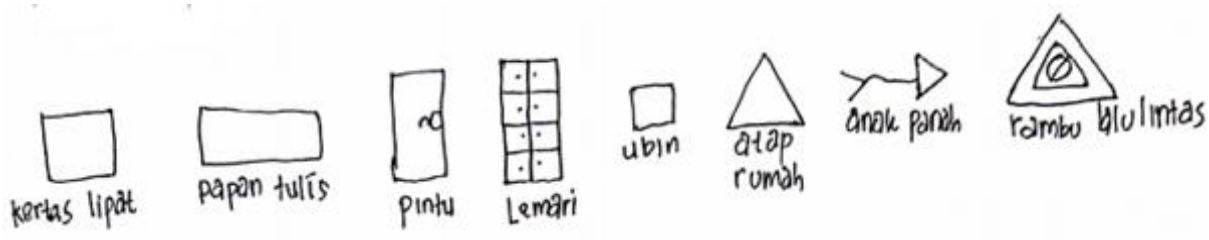
G: Is this picture of 2D?
S : ... (none of the students answered the teacher question)
G: Can we fill this shape with something?
S: No Ma'am .... (all students answered the question loudly)
G: If we cannot insert an object into a figure, then what is that called?
S : 2D shape Ma'am ... (one student answered doubtfully)
G: That's right ...
G: If we cannot insert an object into a figure, then we called 2D shape/figure (the teacher attempted to confirm students' understanding)
G: The 2D shape can be bounded by straight lines or curved lines. The N-made figure is a 2D shape that is bounded by a curved line
S: So is the circle, Ma'am... (one student asked the teacher)
G: That's right ...
G: Do you understand N?
N: Yes, I understand, Ma'am ...
M: Do others understand?
S : Yes, we understand, Ma'am ... (students answered simultaneously)

The teacher, then, invited students to classify the 2D shapes by its boundary lines, namely straight lines and curved lines (see Figures 3 and 4).

Translation:
plane figure

Figure 3. 2D Shape with Its Curved Lines

Figure 3 shows the 2d shape drawn by the students as they classified it into an object with curved lines. Meanwhile, Figure 4 shows the shapes which are surrounded by straight lines.



Translation:

Folding paper Whiteboard Door Cupboard Tile Roof Arrow Traffic signs

Figure 4. 2D Shapes with Its Straight Lines

In the third lesson, the teacher asked students to observe objects bounded by 4 sides. In this activity, the students began to determine the properties of rectangles. The following is the conversation between the teacher and students.

G: What kind of objects are bounded by 4 sides?
S: Tables, doors, blackboards, tiles, windows, creative boards, folding paper
G: yes ... all answers are right
S: Ma'am ... but the form of folding papers and the tiles are square (what he meant is the four sides are similar), but the form of table, door, blackboard, window, and board are not square (the right and left side are similar, the top and bottom side are similar)
G: yes, you are right ... now, please all of you, separate all objects who four sides with its shape have formed square and not
S: students rushed to group them

In the sixth lesson, the teacher invited students to observe the objects that have four sides but the shape was not a square, and identify its characteristics (each object is at least two characteristics). The following were the answered of three participants.

First Participant

The first participant (S1) is a quitter who has a low Adversity Quotient. In determining the properties of the rectangle, the first step taken by S1 was to collect rectangular objects, such as

Blackboard, pencil case, and door (**identify object**). After the objects were collected, the S1drew pictures (**create signs/representamens**) of the founded object (see Figure 5). From these pictures, S1 discovered the properties of the objects (**interpret sign**), then S1 could mentioned the properties of each object, namely blackboard has two equal sides and have four equal angles, the characteristics of a pencil case are the same as a blackboard, the characteristics of the door are the same as the blackboard (**find out properties of sign**). From each of the properties of the objects, S1 **determined the properties of a rectangle**, namely (1) the length of two sides are equal, (2) the measure of each angle is 90^0. Figure 5 shows that the results of S1 in constructing the properties of rectangles.

Translation:

Whiteboard	Pencil case	Door
Characteristics:	1. It has 2 equal sides	1. It has 2 equal sides
1. It has 2 equal sides	2. It has 4 angles with equal measure	2. It has 2 angles with equal measure
2. It has 2 angles with equal measure		

Figure 5. S1 Notes in Constructing the Properties of a Rectangle

The interview results between researcher (P) and first participant (S1) in constructing the properties of rectangles are as follows.

P: what activities do you do to construct the properties of a rectangle?
S1: I collect rectangular objects
P: What objects do you collect?
S1: blackboard, pencil case, and door
P: How do you know that the objects are the rectangular shape?
S1: It is because the shape is like this (while pointing at figure 5)
P: after collecting rectangular objects, what do you do then?
S1: I draw pictures of each object
P: You draw a picture? What for?
S1: To find the characteristics of the object
P: What are the characteristics?
S1: the characteristics of the blackboard are had two equal sides (this side and this side, this side and this side, while pointing at Figure 5), have four equal angles (this angle, this, this and this, while pointing at Figure 5), the characteristics of a pencil case are

the same as a blackboard, the characteristics of the door are the same as the blackboard
Q: What characteristics of the object for?
S1: to find the characteristics of a rectangle (in a soft voice)
Q: What are the characteristics of a rectangle?
S1: a rectangle has two equal sides (this side and this side, this side, and this side, while pointing to a figure of a rectangular object) and an angle of 90^0 (this angle, this, this and this, pointing to the figure of one rectangular object).
P: In the properties of objects, you don't say the measure of the angle, 90^0, but in properties of the rectangle you say the measure of the angle, 90^0. Why can it be like that?
S1: yes ... there are 4 angles, the measure of the angle is 90^0

Considering the aforementioned interview results, S1 identified the same characteristics of three rectangular objects. S1 derived the characteristics of the three objects to determine the properties of rectangles: (1) two sides are equal, (2) the measure of each angle is 90^0. The scheme of S1 in determining the properties of rectangle depicts as Figure 6.

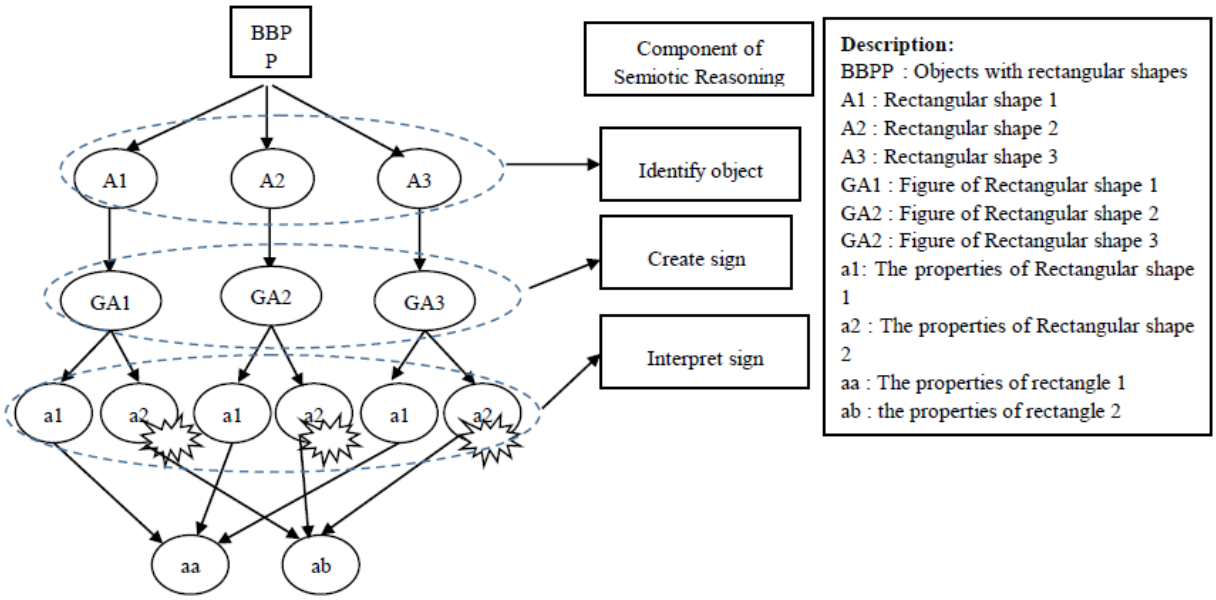
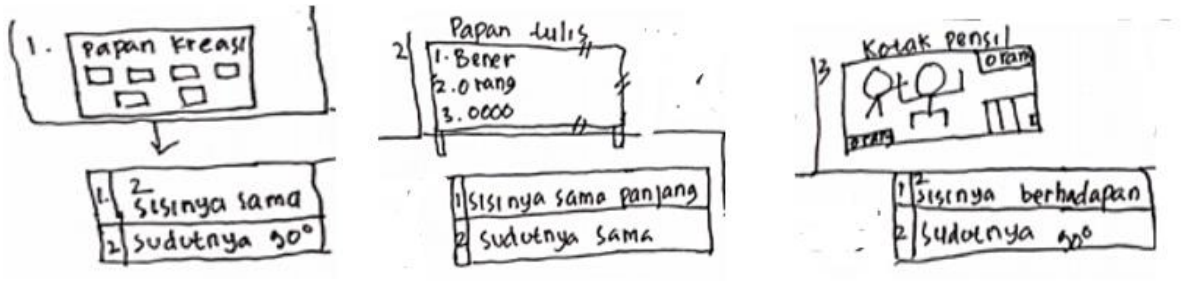


Figure 6. S1 Stages in Determining the Properties of Rectangle

Second Participant

The second participant (S2) is a participant with a medium level of AQ, champer. The first step taken by S2 in determining the properties of a rectangle was the same as S1 steps, namely collecting rectangular objects, such as the creative board, blackboard, and pencil case (**identify objects**). Based on Pierce's semiotic, the activity was called the identifying object. After collecting the objects, S2 drew a figure (**made a sign**) of the determined object (see Figure 7). From these figures, S2 identified the characteristics of the objects (**interpret sign**), namely the creative board has two equal sides and has angles which its measurements are 90^0, the blackboard has two equal sides and similar angle, the pencil case has two sides and the measure of its angle is 90^0 (**find out properties of sign**). From each property of the three objects, then, S2 **constructed the properties of a rectangle**, namely two sides are equal, the measure of its angle is 90^0. However, the properties of rectangles determined by S2 differed from

S1 identification. Figure 7 shows that S2 steps in constructing the properties of rectangles.



Translation:

A Creative Board	A whiteboard	A pencil case
1. It has two equal sides	1. It has equal sides	1. Its sides are opposite
2. Its angles are 90^0	2. The measure of its angles are equal	2. Its angles are 90^0

Figure 7. S2 Notes in Constructing the Properties of Rectangle

The interview results between researcher (P) and second participant (S2) in constructing the properties of rectangles described as follows.

P: what activities do you do to find the properties of rectangle?
S2: I am looking for a rectangular object
P: What objects do you collect?
S2: creative board, blackboard, and pencil case.
Q: What do you do after collecting rectangular objects?
S2: I draw a picture
Q: What are the pictures for?
S: to find out the properties of the figure
Q: What properties did you find from the figure?
S: the creative board has two equal sides, 90^0 angles,
the blackboard has the equal two sides, the similar angle
the pencil case has two sides, the measure of the angle is 90^0
Q: After discovering the properties of the figure, what other activities did you do?
S2: determine the properties of a rectangle
Q: What are the properties of rectangles?
S2: the properties of a rectangle are two equal sides in length, the angle is 90^0
Q: You already mentioned the number of characteristics of a rectangular object is six, but then you mention the properties of a rectangle, only two characteristics. Why?
S2: hemmmmm ... (with a confused face), two opposite sides has equal length and the measure of the angles is equal to 90^0

Based on the results of the interview above, S2 found six characteristics of three rectangular objects. As S2 determined the properties of a rectangle, S2 only identified two properties of a rectangle: (1) two sides are equal, (2) the measure of the angle is 90^0. The properties of the rectangle were derived

from several characteristics of 2D shapes. The S2 stages in determining the properties of rectangles can be seen in the following scheme.

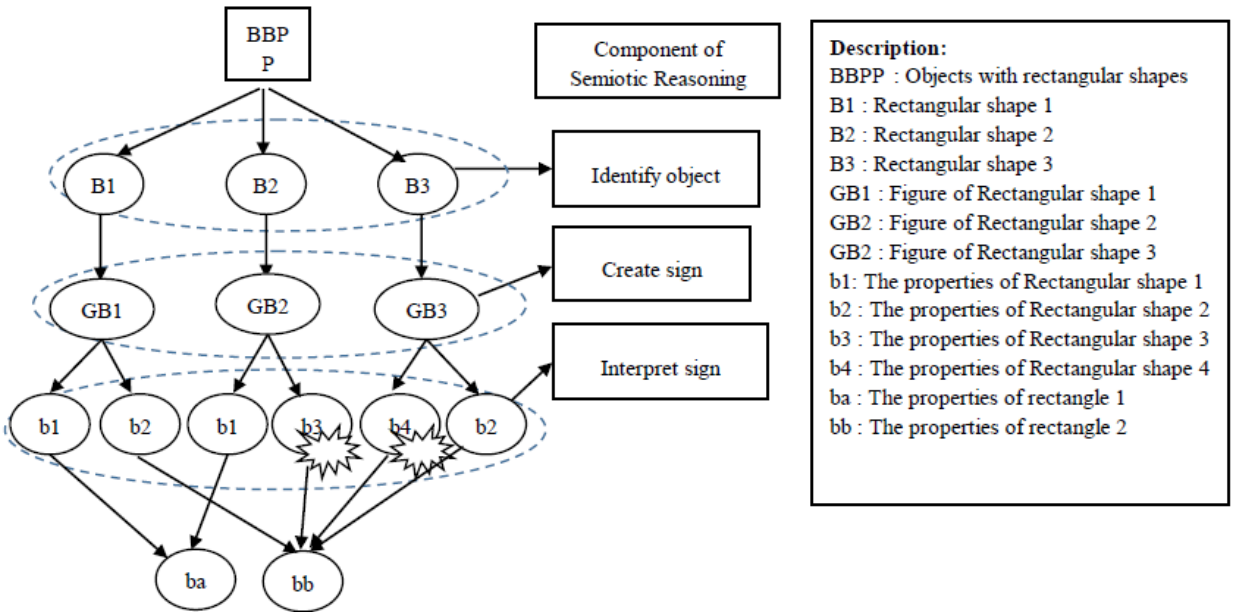
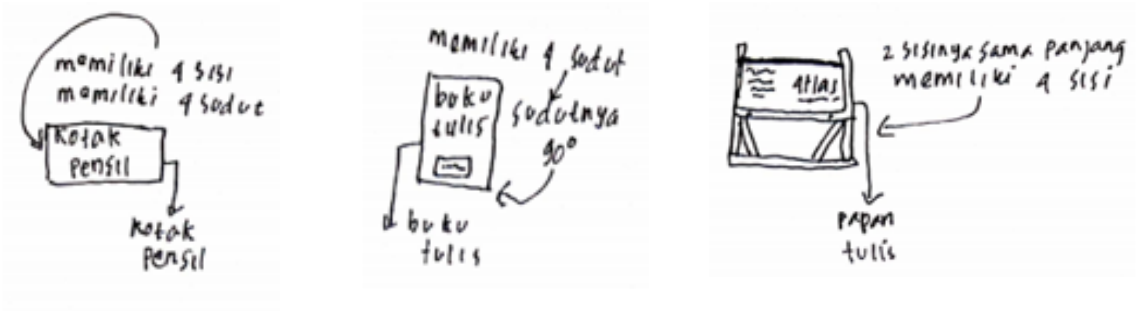


Figure 8. S2 Stages in Determining the Properties of Rectangle

Third Participant

The third participant, called S3, is a student with a high level of AQ, climber. The steps of S3 in determining the properties of rectangles are the same as S1 and S2 did, namely collecting rectangular objects including pencil boxes, notebooks, and blackboards (**identify objects**). After collecting, S3 also made an image (**making a sign**) of the object that had been found (see Figure 9). From these images, S3 discovered the characteristics of objects (**interpret sign**). From each of the properties of the three objects, S3 found that the pencil box has four sides and four angles, the notebook has four 90^0 angles, blackboard has four sides and two sides are equal in length (**find out properties of sign**). Based on these findings, S3 **constructed the properties of rectangles**, but the properties of the rectangles found by S3 differ from the properties of rectangles found by S1 and S2. The properties of rectangles founded by S3 are the measure of each angle is 90^0, has four sides, has four angles, its two sides have equal length (Figure 9).



Translation:

It has 4 sides	It has 4 angles	It has 2 equal sides
It has 4 angles	Its angles are 90^0	It has 4 sides
A pencil case	A book	A whiteboard

Figure 9. S3 Notes in Constructing the Properties of Rectangle

The following dialogs are the results of interview between researcher (P) and third participant (S3) in constructing the properties of rectangles.

Q: *How do you find the properties of a rectangle?*
S3: *I am looking for a rectangular object*
P: *What objects do you collect?*
S3: *pencil box, notebook, and blackboard*
Q: *What do you do after finding rectangular objects?*
S3: *I made pictures of these objects*
P: *what are you making pictures for?*
S3: *to look for the properties of that object*
Q: *What properties do you find from each of these objects?*
S3: *pencil box, has four sides and four corners*
notebook, has four 90^0 angles and angles
blackboard, has four sides and two sides are equal in length
Q: *After knowing the properties of that objects, what do you do?*
S3: *I write the properties of a rectangle which its angle is 90^0 angles, has four sides, has four angles, and two equal sides*

Based on the results of the interview above, S3 found five characteristics of three rectangular objects. When S3 determined the properties of a rectangle, only two traits can be identified, but in the interview process S3 was aware that an error occurred while finding the properties of a rectangle. Therefore, S3 found the properties of a rectangle as (1) the measure of each angle is 90^0, (2) has four sides, (3) has four angles, (4) the two sides are equal. The properties of the rectangle were taken from all the characteristics of 2D objects. The steps of S3 in determining the properties of rectangles can be seen in the following scheme.

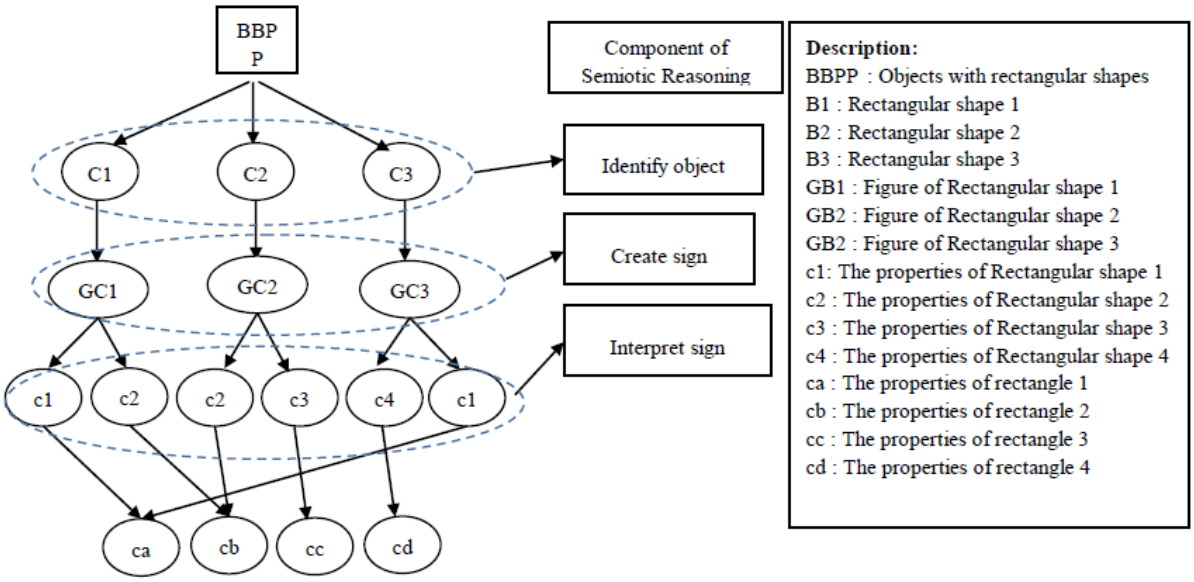


Figure 10. S3 Stages in Determining the Properties of Rectangle

First activity of discovering the properties of rectangle is identifying rectangular objects. The activity was carried out by three participants by observing objects around them. These activities, in Peirce semiotics are called identifying objects. Objects are the result of observing signs from external students (Schreiber, 2013).

Based on the phases of the three research participants, it was found that there were differences in the objects found. It can be seen from the objects collected by the three participants. The objects found by the subject of research are influenced by the interpretation of the participants in observing objects around the subject. It is in accordance with the opinion of Brier (2015) and Priss (2016) which states that an object is something that represents the interpretant produced. Interpretation is a response to an object through the interpretation of signs.

In creating a sign, the three participants made the same sign that was a rectangular image. Pictures made, in the semiotic theory of Peirce is called a sign. It is in line with the opinion of Arzarello and Sabena (2011) which states that images are signs that represent their original properties with their own simple characteristics. The simple characteristics means that these properties are relevant to their original nature. The relationship of images to other properties does not play a role in developing iconic representational relationship (Kralemann & Lattmann, 2013; Hendroanto, et al. 2018).

The participants carried out the activity of interpreting the sign by identifying the properties of each object found in the object identification activity. The interpretation of the participants to the sign made is different. For instance, in determining the properties of blackboard, S1 mentioned its characteristics such as having two equal sides and having four equal angles. S2 mentioned the characteristics of the blackboard, namely having two equal sides and having 90^0 angles of each measure. Whereas S3 mentioned the characteristics of the blackboard were having two equal sides and having

four sides. It can be seen that one's interpretation of an object can vary. A person can have different interpretations related to the image. It depends on how the person interpreted the image (Sáenz-Ludlow & Kadunz, 2016; Godino, et al. 2011).

In determining the properties of a rectangle, there are differences between S1, S2 and S3 findings. S1 took from one of the characteristics of a rectangular object and found a new properties interpreted by S1 in interpreting one of the characteristics of a square object. S1's interpretation of the sign did not change when S1 found rectangular properties. It is in line with the opinion of Kralemann and Lattmann (2013) and Brier (2015) stating that each sign can act as an object or as an interpreter of other signs that are part of a single set of interpretations which a person has regarding certain specific matters. Meanwhile, the properties of the rectangle discovered by S2 were obtained from one of the characteristics of a rectangular object and there were two characteristics of a rectangular object which merged into one properties of rectangle. S3 derived all the characteristics of rectangular objects in determining the properties of rectangles. The interpretation of research subjects to the sign varies. The differences are influenced by the experience and knowledge possessed by the subject. It is consistent with the opinion of Schreiber (2013) who states the interpretation of each individual based on their previous experience and background. Each sign can be interpreted differently by other subjects. Moreover, the interpretation of each sign can trace the subject's judgment (Kralemann & Lattmann 2013; Ali, 2016).

CONCLUSION

The steps of semiotic reasoning conducted by the three participants (quitter, campers, and climber) of this research in determining the properties of rectangles are identifying objects, is the activity of collecting objects that are relevant to a rectangle; creating a sign, is an activity of making pictures based on identified objects; interpreting the sign, is an activity to interpret the sign based on the relationship between objects; determining the characteristics of the sign; and determining the properties of a rectangle by deriving the characteristics of a rectangular object. The results show that the three participants identify object by observing objects around them. In creating sign stage, three participants made the same sign that was a rectangular image. However, in three last stages, namely interpret sign, find out properties of sign, and discover properties of the rectangle, they did different ways.

In interpreting the sign, quitter found only two characteristics, namely the objects have four sides and four angles. Based on these characteristics, the quitter determined that a rectangle has properties such as it has four sides and four angles. On the other hands, the properties are not specific since the properties also belong to other rectangular objects. The champer, in interpreting the sign, found four characteristics of the sign namely the objects have equal sides, the measure of its angle is 90^0, the opposite sides and angles are the same. However, the properties of a rectangle which is derived from the properties of the sign by the champer is only two, that are, its two sides are equal and the measure

of its angle is 90^0. Two properties of the sign merge into one rectangular property. Consequently, the properties of rectangle found by the quitter are not specific, because these properties are also possessed by right trapezoid.

In contrast, Climber, in interpreting the sign, found six characteristics of the sign that are the objects have four sides, four angles, 90^0 angles and two equal sides. Of the four sign characteristics, the Climber derived all of these properties so that the subject can find the properties of a rectangle, namely a rectangle has four sides, has four angles, 90^0 angles, and two opposite sides are equal. In the activity of finding the properties of a rectangle, Climber determined the properties of a rectangle correctly.

ACKNOWLEDGMENTS

Our gratitude goes to the Principal of one of the private elementary schools in the district of Jember Indonesia who has allowed researchers to retrieve data at their school. Furthermore, we also send our humble gratitude to the fourth grade teachers who helped us in conducting the research, so that we could obtain our research. We would also like to thank the six observers who helped us in taking the video, participated in the observation so that the data we needed could be recorded properly.

REFERENCES

Ahamad, S.N.S.H., Li, H.C., Shahrill, M., & Prahmana, R.C.I. (2018). Implementation of problem-based learning in geometry lessons. *Journal of Physics: Conference Series*, *943*(1), 012008. https://doi.org/10.1088/1742-6596/943/1/012008.

Ali, R.H., & Aslaadi, S. (2016) A cognitive semiotic study of students ' reading a textless image versus a verbal image. *Advances in Laguage and Literary Studies, 7*(5), 1-13. http://dx.doi.org/10.7575/aiac.alls.v.7n.5p.1.

Arzarello, F., & Sabena, C. (2011). Semiotic and theoretic control in argumentation and proof activities. *Educational Studies in Mathematics*, *77*(2-3), 189-206. https://doi.org/10.1007/s10649-010-9280-3.

Bjuland, R. (2012) The mediating role of a teacher's use of semiotic resources in pupils' early algebraic reasoning. *ZDM, 44*(5), 665-675. https://doi.org/10.1007/s11858-012-0421-2.

Brier, S. (2015). Cybersemiotics and the reasoning powers of the universe : philosophy of information in a semiotic- systemic transdisciplinary approach. *Green Letters Studies in Ecocriticism*, *19*(3), 280-292. https://doi.org/10.1080/14688417.2015.1070684.

Campos, D.G. (2010). Peirce's philosophy of mathematical education: Fostering reasoning abilities for mathematical inquiry. *Studies in Philosophy and Education*, *29*(5), 421-439. https://doi.org/10.1007/s11217-010-9188-5.

Creswell, J.W. (2015). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Educational Research (Vol. 4). https://doi.org/10.1017/CBO9781107415324.004.

Deledalle, G. (2013). Peirce and semiotic–An introduction. *KODIKAS/CODE. Ars Semeiotica*, *36*(3-4), 185-191.

Eco, U. (1976). *A Theory of Semiotics*. Bloomington: Indiana University.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Edication*, *9*(1), 3-20. https://doi.org/10.1080/1479800008520167.

Godino, J.D., Font, V., Wilhelmi, M.R., & Lurduy, O. (2011). Why is the learning of elementary arithmetic concepts difficult? Semiotic tools for understanding the nature of mathematical objects. *Educational Studies in Mathematics*, *77*(2-3), 247-265. https://doi.org/10.1007/s10649-010-9278-x.

Hardiarti, S. (2017). Ethnomatematics: The application of quadrilateral plane figure in muaro Jambi temple [in Bahasa]. *Aksioma*, *8*(2), 99-110. https://media.neliti.com/media/publications/217393-none.pdf.

Hendroanto, A., van Galen, F., van Eerde, D., Prahmana, R.C.I., Setyawan, F., & Istiandaru, A. (2018). Photography activities for developing students' spatial orientation and spatial visualization. *Journal of Physics: Conference Series*, *943*(1), 012029. https://doi.org/10.1088/1742-6596/943/1/012029.

Kralemann, B., & Lattmann, C. (2013). Models as icons: Modeling models in the semiotic framework of Peirce' s theory of signs. *Synthese*, *190*(16), 3397-3420. https://doi.org/10.1007/s.

Metro-Roland, M. (2009). Interpreting meaning : An application of Peircean semiotics to tourism. *Tourism Geographies*, *11*(2), 270-279. https://doi.org/10.1080/14616680902827225.

Miller, J. (2015). Young indigenous students' engagement with growing pattern tasks: A semiotic perspective. *Proceeding of the 38th Annual Conference of the Mathematic Education Reseacrh Group of Australasia*, 421-428. https://files.eric.ed.gov/fulltext/ED572490.pdf.

Ng, O.L., & Sinclair, N. (2015). Young children reasoning about symmetry in a dynamic geometry environment. *ZDM*, *47*(3), 421-434. https://doi.org/10.1007/s11858-014-0660-5.

Ostler, E. (2011). Teaching adaptive and strategic reasoning through. *International Journal of Mathematics Science Education*, *4*(2), 16-26. https://pdfs.semanticscholar.org/4673/f9f3c2892b621b05b1ce4ddff02d9a5c2757.pdf.

Panchal, C. (2013). A study of abstract reasoning of the students of standard IX of Ahmedabad city. *International Journal for Research in Education*, *2*(3), 30-34. http://www.raijmr.com/ijre/wp-content/uploads/2017/11/IJRE_2013_vol02_issue_03_14.pdf.

Parcell, W.C., & Parcell, L.M. (2009). Evaluating and communicating geologic reasoning with semiotics and certainty estimation. *Journal of Geoscience Education*, *57*(5), 379-389. https://doi.org/10.5408/1.3544288.

Peirce, C.S. (1931). *The Collected Papers of Charles Sanders Peirce*. Cambridge: Harvard University Press.

Presmeg, N. (2016). Semiotics in theory and practice in mathematics education. *ICME-13*. https://doi.org/10.1007/978-3-319-31370-2.

Priss, U. (2016). A semiotic-conceptual analysis of conceptual learning. In *International Conference on Conceptual Structures* (pp. 122-136). Cham: Springer.

Radford, L., & Schubring, G. (2008). *Semiotics in Mathematics Education*. Rotterdam: Sense Publishers.

Sáenz-Ludlow, A., & Kadunz, G. (2016). *Semiotics as a Tool for Learning Mathematics.* Rotterdam: Sense Publishers.

Sarbo, J.J., & Yang, J.H. (2015). A semiotic approach to critical reasoning. In *International Conference on Informatics and Semiotics in Organisations* (pp. 10-19). Cham: Springer. http://www.cs.ru.nl/~janos/project-site/iciso2015-final.pdf.

Schreiber, C. (2013). Semiotic processes in chat-based problem-solving situations. *Educational Studies in Mathematics*, *82*(1), 51-73. https://doi.org/10.1007/s10649-012-9417-7.

Semetsky, I. (2013). *The Edusemiotics of Images.* Rotterdam: Sense Publishers.

Sendera, H., Yakin, M., & Totu, A. (2014). The semiotic perspectives of peirce and saussure: A brief comparative study. *Procedia-Social and Behavioral Sciences*, *155*(October), 4–8. https://doi.org/10.1016/j.sbspro.2014.10.247.

Stjernfelt, F. (2015). Dicisigns Peirce's semiotic doctrine of propositions. *Synthese: An International Journal for Epistemology, Methodology and Philosophy of Science*, *192*(4), 1019-1054. https://doi.org/10.1007/s11229-014-0406-5.

Stoltz, P.G. (2004). *Adversity Quotient: Reverse the Threats into Opportunities Fifth Edition* [in Bahasa]. Jakarta: Grasindo.

Syah, M. (2010). *Learning Psychology* [in Bahasa]. Jakarta: Rajagrafindo Persada.

Turgut, M. (2017) Students' reasoning on linear transformations in a DGS: A semiotic perspective. *CERME 10* (pp. 01946324). Dublin, Ireland. https://hal.archives-ouvertes.fr/hal-01946324.

Türkcan, B. (2013). Semiotic approach to the analysis of children's drawings. *Educational Science: Theory & Practice*, *13*(1), 600-607. https://files.eric.ed.gov/fulltext/EJ1016743.pdf.

Usiskin. (1982). *Van Hiele Levels and Achievement in Secondary School Geometri*. Chicago: The University of Chicago.

Uslucan, H.H. (2004). Charles Sanders Peirce and the semiotic foundation of self and reason. *Mind, Culture, and activity*, *11*(2), 96-108. https://doi.org/10.1207/s15327884mca1102_2.

West, D.E. (2015). Embodied experience and the semiosis of abductive reasoning. *Southern Semiotic Review*, *5*(1), 53-59.

Yang, C., & Hsu, T. (2015). Applying semiotic theories to graphic design education: An empirical study on poster design teaching. *International Education Studies*, *8*(12), 117-129. https://doi.org/10.5539/ies.v8n12p117.