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# A LEARNING TRAJECTORY FOR PROBABILITY: A CASE OF GAME-BASED LEARNING

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***Abstract***

This research is aimed to describe a learning trajectory for probability through game-based learning. The research employed design research consisting of three stages: preparing for the experiment, design experiment, and retrospective analysis. A hypothetical learning trajectory (HLT) using Sudoku and Snake-and-ladder games was developed by collecting data through documentation, interviews, and classroom observations. The HLT was implemented in the classroom to investigate students' actual learning trajectory. The results of this research indicate that the games helped students understand the concept of probability. The learning trajectory for probability based on game-based learning is seen from the perspective of four levels of emergent modeling. In the first level – 'situational level' – Sudoku and Ladder-and-Snake games were played by students. The second level is the 'referential level' where the rules of the games were used as a starting point to learn the concept of probability. Communication during game playing stimulated students' knowledge about random events, sample spaces, sample points, and events. At the third level – 'general level' – students used tree and table diagrams to generalize possible outcomes of an experiment and develop an understanding of sample spaces and sample points. Lastly, at the 'formal level' students developed their informal knowledge into formal concepts of probabilities.

**Keywords:** design research, four levels of modelling, game-based learning, learning trajectory, probability

**Abstrak**

Penelitian ini ditujukan untuk mendeskripsikan lintasan belajar (learning trajectory) untuk topik peluang yang dikembangkan dalam suatu pembelajaran berbasis permainan. Penelitian ini menggunakan penelitian desain yang terdiri dari tiga tahap: persiapan eksperimen, desain eksperimen, dan analisis retrospektif. Suatu hypothetical learning trajectory (HLT) yang menggunakan permainan Sudoku dan Ular Tangga dikembangkan dengan menggunakan data dari dokumentasi, wawancara, dan observasi kelas. HLT tersebut diimplementasikan di kelas untuk menyelidiki lintasan belajar siswa yang aktual. Hasil penelitian ini menunjukkan bahwa permainan dapat membantu siswa memahami konsep peluang. Lintasan belajar berdasarkan pembelajaran berbasis permainan ini dilihat dari perspektif empat level pemodelan. Level pertama adalah 'level situasional' dimana Sudoku dan Ular tangga dimainkan oleh siswa. Pada level kedua – 'level referensial' aturan permainan dijadikan sebagai titik awal untuk mempelajari konsep peluang. Komunikasi selama permainan menstimulasi munculnya pemahaman siswa tentang kejadian acak, ruang sampel, titik sampel, dan kejadian. Pada tahap ketiga– 'level general' – siswa menggunakan diagram pohon dan tabel untuk menggeneralisasi luaran yang mungkin dari suatu eksperimen dan mengembangkan pemahaman tentang ruang sampel dan titik sampel. Pada tahap terakhir – 'level formal', siswa mengembangkan pengetahuan informal mereka menjadi konsep formal tentang peluang.

**Kata kunci:** desain riset, empat level pemodelan, pembelajaran berbasis permainan, lintasan belajar, peluang

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Probability is a domain in mathematics that studies the size of the uncertainty of an event that exists in life (Smith, 1998). Understanding probability is important because, according to OECD (2016, p. 74), uncertainty is "a phenomenon at the heart of the mathematical analysis of many problem situations." Furthermore, decision making involving uncertainty has become an integral part of modern society today. Knowledge of uncertainty or probability can empower students to make wise decisions when facing various situations (Kennedy, Tipps & Johnson, 2008).

A good understanding about probability also helps individual to understand the risks and possible benefits of an action and also ensure fairness in everyday lives (Bryant & Nunes, 2012). Considering the importance of probability, many countries place probability as a part of mathematics school curriculum. Moreover, there was a movement that introduced probability earlier at the primary school level (Jun, 2000; Frykholm, 2001). In the Programme for International Student Assessment (PISA), probability is also considered as one of the important mathematical content categories to assess students' mathematical literacy (see OECD, 2009).

The great attention to uncertainty or probability in the curriculum document needs to be supported by effective classroom practices. An effective mathematics teaching should consider 'where students are' and help students to build their mathematical knowledge in their own way (Clements & Sarama, 2009). Helping students build knowledge means it is crucial for a teacher to understand students' thinking for which conceptual analysis and conjectures of students' actions and thoughts are required (Amador & Lamberg, 2013). Teachers' insight into students' understanding serves as an important base for teachers to develop appropriate learning activities including conjectures of students' responses to the learning activities (Simon, 1995). With this respect, several studies consider a so-called hypothetical learning trajectory or learning trajectory. A hypothetical learning trajectory is a means to understand students' thinking and to develop tasks and activities based on conjectures of students' way of thinking. A hypothetical learning trajectory provides conjectures of students' learning route when they learn particular topic (de Beer, Gravemeijer, & van Eijk, 2017; Clements & Sarama in Daro, et al., 2011; Mojica & Confrey, 2009). A hypothetical learning trajectory comprised three main components, i.e. learning goals, learning activities, and hypothesis of students' learning process (Simon, 1995). Learning goals are the first component to indicate the outcomes to be achieved at the end of the lesson. The learning goals are used to formulate possible learning activities as a 'road' to achieve these learning goals. The last component is conjectures or hypothesis of students' learning process. This last component is useful for designing alternative actions or strategies to address students' problems or responses that might occur during the learning activities.

Regarding the teaching of probability, Fischbein (2002) suggested that teachers should not emphasize on teaching procedural skills. Instead, teachers need to give students experiences to understand probabilistic situations through an experiment. When the experiment is contextual, this context can provide a meaningful starting point for learning by students and help them develop their knowledge of mathematics concepts. One of possible contexts for teaching mathematics is in the form of games. Wijaya, Doorman, and Keijzer (2011) used traditional games to teach the topic of linear measurement for second graders. The results of their study showed that the traditional games helped students grasping the notion of measurement of length. Furthermore, the discussion about the games also naturally raised students' need for a standard measurement unit. The transition from non-standard measurement unit to standard measurement unit emerged from students themselves. At the end of the lesson series, students successfully perceived the measurement of length as concept, not merely as a procedure. A study of Perry dan Dockett (2002) also shows how game techniques could effectively support students' development in mathematics. An important key of a game-based learning is how students' informal notion of mathematics concepts is brought to formal understanding of the concepts. In this respect, Gravemeijer's (1994) level of emergent modeling can be seen as an important framework. Gravemeijer proposed four levels of models, i.e. situational, referential, general, and formal. The situational level of a game-based learning starts when students are playing the game. At this level, students' discussion and thinking process are mainly within the context of the games while they are playing the games. In the second level, i.e. the referential level, students no longer play the games, but during their discussion the students make reference to the games, such as the rule of the games. Students' discussion starts to discard the context of the games when students achieve the general level. Lastly, in the formal level students discuss the mathematical properties concerning the topics.

Learning trajectories are important references for teachers to connect student-centered instruction with domain-specific understandings of students' thinking (Nickerson & Whitacre, 2010; Sztajn, Confrey, Wilson, & Edgington, 2012; Wilson, Sztajn, Edgington, & Myers, 2015). Therefore, sufficient and appropriate learning trajectories are crucial to support students' conceptual understanding. In this respect, this study highlights the importance of learning trajectory for learning probability and the potential use of game-based learning. The present study is aimed to better understand how to support students' learning trajectory for probability when students learn mathematics through a game-based learning approach.

## METHOD

### *Research Procedure*

This study employed the design research approach of Gravemeijer and Cobb (2013) because the intention of the present study was to develop a theory about the process of learning a particular concept and the instructional design to support that learning. This design research approach comprised three phases: (1) preparation for the experiment, (2) design experiment, and (3) retrospective analysis (review analysis). In the first phase, initial ideas for a set of instructional activities were explored through reviewing literature and interviewing teacher. The literature review was aimed to see theoretical perspectives. The interview involved only one mathematics teacher from the school where the experiment was conducted. This teacher was selected because she was the only teacher who taught probability during the period of data collection. The intention of the interview was to identify teacher's factual teaching practices in particular about students' difficulties in learning probability and to explore conjectures of students' thinking. Following the idea of Simon and Tzur (2004) and Schneider and Gowan (2013), teacher's understanding of students' learning processes is considered in developing a basis to solve pedagogical problems. The combination of theoretical and factual perspectives served as the basis for designing a hypothetical learning trajectory (HLT). The HLT comprised three elements, i.e. learning objectives, learning activities, and conjectures of students' learning process.

In the second phase – i.e. design experiment phase – the hypothetical learning trajectory was implemented in two stages of experiment: pilot experiment and teaching experiment. The pilot experiment involved 32 eight graders and was set as a bridge between the initial design phase and teaching experiment. The focus of the pilot experiment was to see the feasibility of the initial design in the HLT and to collect data that might be needed to revise the HLT. After the pilot experiment, a teaching experiment involving other 32 eight graders were performed to investigate how the revised HLT helped students learn probability. Students' thoughts were observed from their works and interviews with 18 students. An interview was conducted after every activity and involved three students representing low, medium, and high ability students. The interviews were aimed to explore students' learning opportunity and learning obstacles.

The last phase of this design research was retrospective analysis. In this phase, all data obtained in the design experiment phase were analyzed by focusing on the comparison between the HLT and the actual learning process of students. This analysis also included analyzing possible causes and synthesizing possibilities that can be done to improve the HLT.

The essence of this design research is a cyclic process. The thought experiment in the first phase – which in this study was in the form of initial HLT for probability – was implemented as an instructional experiment, i.e. pilot experiment and teaching experiment. The results of the instructional experiment were analyzed through a retrospective analysis resulting new thought experiment.

***Subjects***

For the design experiment phase, a total of 64 eight graders (14 years old) at a public junior high school located in urban area in Indonesia participated in the study. Thirty-two students participated in the pilot experiment and 32 students participated in the design experiment. In addition to these 64 students, a mathematics teacher was also involved in the study, i.e. in an interview prior to the design process and in implementing the instructional activities.

***Hypothetical Learning Trajectory***

As mentioned earlier, a HLT comprising learning objectives, learning activities, and conjectures of students' learning process was designed during the thought experiment stage (see Figure 1). The learning objectives were formulated by referring to the Indonesian mathematics curriculum and the theoretical structure of the concept of probability because according to Clements, Wilson, and Sarama (2004), existing research serves as a primary means to construct the first draft of learning trajectories. The learning activities consisted of five activities situated in game-based learning which was arranged from simple activity such as deciding the rule of the games to discussing the results of game playing. Sudoku and snake-and-ladder games were chosen for the learning activities these games because the use of rolling die and the rules of the games were relevant to the topic probability, e.g. the rolling die was related to random events. Another consideration for choosing the two games was students' familiarity with the games because these games were famous among students. Familiarity is an important aspect for a meaningful learning. Familiar contexts could change the students' mindset from mathematics as an isolated concept to an integrated part of life (Risdiyanti & Prahmana, 2020). By using the games, they were already familiar with, students could build their understanding on new concepts because as mentioned by Mayer (2002), a meaningful learning occurs when students can connect their existing knowledge and experience to the new knowledge. Lastly, students' learning processes were conjectured on the basis of theoretical prediction and teacher's responses during the interview.

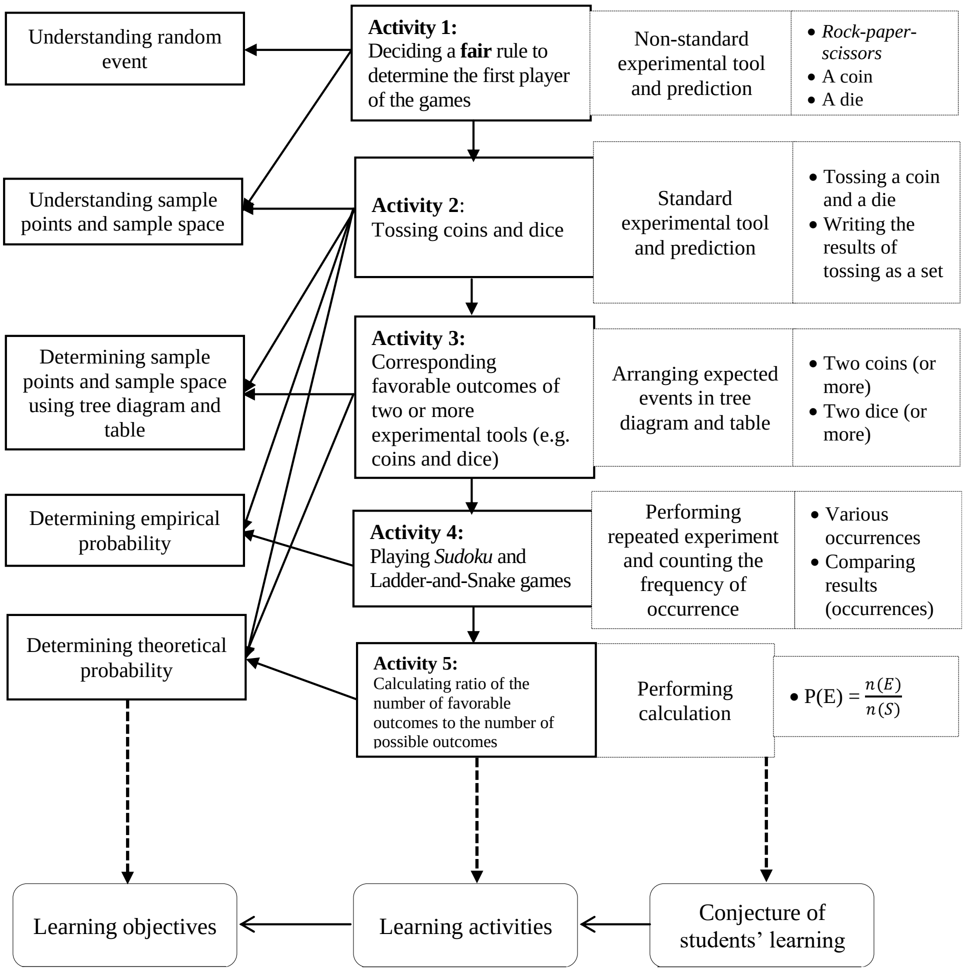



**Figure 1.** A game-based hypothetical learning trajectory for probability

### *Data Collection and Data Analysis*

This study used qualitative data obtained through interview, observation, and documents. Teacher interview was conducted to explore teacher's regular teaching practices and to identify students' difficulties in learning probability and conjectures of students' learning. The interview was performed in a semi-structure way in which three aspects – i.e. teaching practices, students' learning difficulties, and conjectures of students' thinking – were used as a guideline, but teacher's responses were elaborated into more detailed aspects. Interviews with students were also conducted to explore students' learning potential and learning obstacles during the learning activities. With regard to the observation, the researchers focused on students' actual learning processes, which were later, compared to the conjectures in the HLT. Lastly, the documentation was done by collecting students' works.

Following the phases of design research (see Gravemeijer & Cobb, 2013), the data was analyzed retrospectively by using the HLT as the guideline for the analysis. The first step of the analysis focused on the implementation of the learning activities to check whether they were implemented according to the design. In the next step, students' actual learning process as obtained from empirical observations was compared to the learning objectives to see to what extent the learning objectives were achieved and how students gained their understanding of probability. Furthermore, students' actual learning process was compared to the conjectures or predictions of students' learning process in the HLT. Analyzing the interplay between the HLT and empirical observations served as the basis for developing an instructional theory. After the retrospective analysis, the initial HLT was adjusted to obtain a new HLT.

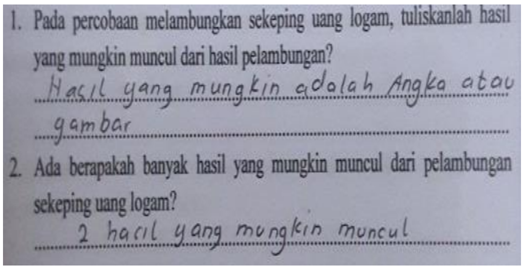
## RESULTS AND DISCUSSION

The results of this study concern the development of students' notion about probability that was gained through a game-based learning, i.e. from Activity 1 to Activity 5. The description of this development is elaborated in the following sub-sections.

### *Activity 1: Deciding A Fair Rule to Determine the First Player of Sudoku and Snake-and-Ladder Game*

The first activity was about deciding a fair rule to determine the first player of games. This activity was aimed to introduce the concept of random event to students. The random event here was connected to the idea that the rule for determining first player is said to be fair if none of the players know in advance who will be the first player. This idea of fair game is close to the concept of random event, i.e. an event or process that cannot be made exact or whose outcome cannot be predicted, e.g., the numbers showed on two rolled dice.

In pairs, the students were asked to discuss rules to fairly determine the first player of the games. The observed classroom activities showed that the students did not have difficulty in Activity 1. They could find fair ways to determine the first player of the games. They said that the first player could be determined by tossing a coin, tossing a die, or using rock-paper-scissors game. At the beginning, it was not clear whether the chosen strategies or rules was simply due to students' familiarity of the games or the students really understood the idea of the 'unpredictability of the outcomes of the strategy'. Therefore, random students were asked about their reasons for choosing particular methods. It was revealed that the students could explain that a fair way meant "not knowing or being uncertain about the result of tossing a coin, tossing a die, or playing rock-paper-scissors, which therefore every player would get the same opportunity for being chosen as the first player." Figure 2 shows an example of students' response to problems about tossing a coin that was provided in the worksheet.


1. Pada percobaan melambungkan sekeping uang logam, tuliskanlah hasil yang mungkin muncul dari hasil pelambungan?
Hasil yang mungkin adalah Angka atau gambar
2. Ada berapakah banyak hasil yang mungkin muncul dari pelambungan sekeping uang logam?
2 hasil yang mungkin muncul

1. What are possible outcomes of tossing a coin?

   *The possible outcomes are 'number' or 'image'*
2. How many possible outcomes of tossing a coin?

   *There are **two** possible outcomes*

**Figure 2.** Example of a student's work

In order to know students' arguments or reasons for their answers to the worksheet, the researcher interviewed three students representing low, medium, and high ability students to get a confirmation. The following excerpt shows an interview with a student.

Researcher : *"... why did Dian use the word 'possibly'?"*

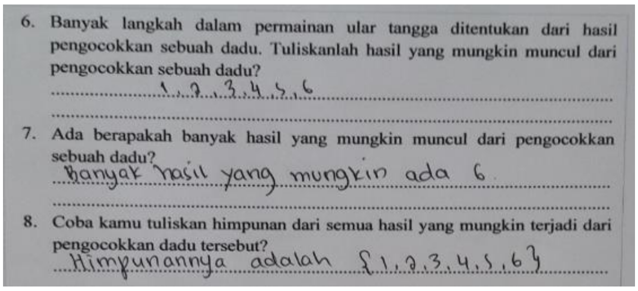
Dian : "*Because a coin has two faces which are of 'number' or 'image', so we cannot know which face will come on top if we toss the coin. The result of tossing a coin could be the 'number' or the 'image'.*"

The abovementioned excerpt indicates the students' notion about random event that was presented as fairness in the games. It was found that the term 'fair way' [to determine the first player of the games] can potentially be used as the starting point to discuss and learn the concept of random event. In this regard, the students perceived that a random event was the situation at which we could not predict the result of tossing a coin, tossing a die, or playing rock-paper-scissors.

***Activity 2: Tossing Coin(s) and Rolling Dice***

Activity 2 was about tossing coins and rolling dice and then recording the results. The learning objective of Activity 2 was to develop students' understanding on the concept of sample point, sample space and events of an experiment. At the beginning of this activity, the students were asked to revisit the concept of random event they developed in Activity 1, e.g. what were possible outcomes of tossing a coin or rolling dice. In Activity 2, the students worked in pairs to work on worksheet that elicited the concept of sample point, sample space and events. To solve problems in the worksheet, the students were provided with dice and coins – which in the next activity were used as tools to play the games – so that they could directly experimented the tossing. The students were asked to toss the coins and roll the dice and then mention all possible outcomes. The students could successfully complete the worksheet with the help of dice and coin. Figure 3 shows an example of students' works.

From Figure 3, it can be seen that the students could determine all possible results of rolling a die which in the snake-and-ladder game represents the number of steps taken by a player. The students could correctly list all possible outcomes from tossing a die, i.e. is 1, 2, 3, 4, 5, and 6. Furthermore, the students could also write down a set containing all possible outcomes of tossing a die. The results of Activity 3 indicate that the students started to grab the idea of sample point and sample space.

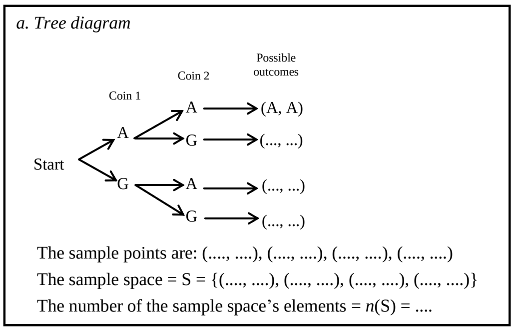

6. Banyak langkah dalam permainan ular tangga ditentukan dari hasil pengocokkan sebuah dadu. Tuliskanlah hasil yang mungkin muncul dari pengocokkan sebuah dadu?
1, 2, 3, 4, 5, 6
7. Ada berapakah banyak hasil yang mungkin muncul dari pengocokkan sebuah dadu?
Banyak hasil yang mungkin ada 6
8. Coba kamu tuliskan himpunan dari semua hasil yang mungkin terjadi dari pengocokkan dadu tersebut?
Himpunannya adalah {1, 2, 3, 4, 5, 6}

6. The number of steps to be done in the Snake-and-ladder game is determined by tossing a die. Write down all possible outcomes of tossing a die.
*1, 2, 3, 4, 5, 6*
7. How many possible outcomes can we get from tossing a die?
*There are 6 possible outcomes*
8. Write down the possible outcomes as a set
*The set is {1, 2, 3, 4, 5, 6}*

**Figure 3.** Tossing a die and a coin to learn sample point and sample space

***Activity 3: Corresponding Favorable Outcomes of Two or More Experimental Tools (e.g. Coins and Dice)***

In Activity 3 the students had not yet played the Sudoku and Snake-and-ladder games. In this activity, the students still continued playing with dice and coins without involving the Sudoku and Snake-and-ladder games. The aim of this activity was to support students learned to determine sample point and sample space by using tree diagram and table. Classroom activities were commenced by recalling sample point, sample space, and random event of tossing dice and coins in the previous meeting. In Activity 1 and Activity 2 the students mostly worked in situational level and referential (see Gravemeijer, 1994), i.e. the situation of games, whereas in Activity 3 the students were brought into the higher level, i.e. general level. In this general level the students were introduced 'formal tools' – i.e. tree diagram and table – to tabulate sample points resulted from tossing dice and coins (Figure 4). Tree diagram and table are tools which were potential to guide students into formal procedure. After tossing a die or coins, students were asked to record their results in tree diagram and table.




*b. Using table*

| | | Coin 1 | |
|---|---|---|---|
| | | A | G |
| Coin 2 | A | (A, A) | (..., ...) |
| | G | (..., ...) | (..., ...) |

The sample points are: (...., ....), (...., ....), (...., ....), (...., ....)
The sample space = S = {(...., ....), (...., ....), (...., ....), (...., ....)}
The number of the sample space's elements = $n$(S) = ....

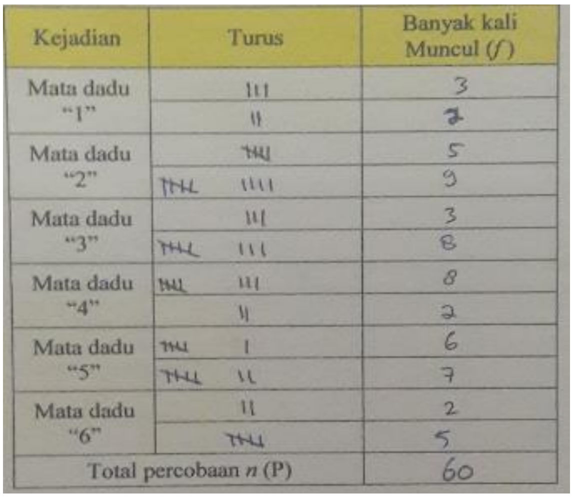
**Figure 4.** Tree diagram and table as formal tools to present sample points

Students did not have difficulties in using tree diagram and table to tabulate the possible outcomes of tossing dice and coins because they have developed the notion about sample points, sample space, and random event from previous activities. After students tossed coins and rolled dice to fill in the table and tree diagrams, students were asked to mention possible outcomes of an event by completing tree diagram and table without using any coin or die, such as rolling three dice. Students' works also indicate that the students have developed their notion about sample points, sample space, and random event into higher and more formal form because they no longer relied on real experiment with coins and dice.

***Activity 4: Playing Sudoku and Ladder-and-Snake Games***

In Activity 4 students played Sudoku and Snake-and-ladder games. While playing the games, the students were asked to pay attention to the ways to determine the first player for the case of Sudoku and to determine the number of steps in Snake-and-ladder. Furthermore, the students were asked to record the game playing in tables (see Figure 5). The intention of Activity 4 was to introduce the concept of empirical probability. Therefore, after filling in the tables the students were asked whether tossing a coin twice would definitely give them both faces of the coin, i.e. face and number. Some students indeed could get the two faces of a coin after tossing the coin twice. Therefore, a next question was posed to narrow down the focus of students to empirical probability, i.e. "if we tossed the coin six times, did we certainly get the 'number' three times?". Similarly, for the case of rolling a die the students were asked whether rolling a die six times would make all numbers from 1 to 6 appear.

The aforementioned guiding questions were posed to help students realize the uncertainty of tossing a coin or rolling a die. As shown in Figure 5, after rolling a die 60 times the students got unbalanced appearance of the faces of the die; for example, number '2' was the most frequently appeared face, whereas number '1' was the least frequently appeared face. In the next step, the students were asked to focus on the ratio of the occurrence of a particular face of the die to the total occurrences. The term 'empirical probability' was introduced to the students to label the ratio they got. The students were guided to construct their understanding about empirical probability to represent the probability of a particular event which is obtained from an experiment.



| Kejadian | Turus | Banyak kali Muncul (*f*) |
|---|---|---|
| Mata dadu "1" | ||| | 3 |
| | || | 2 |
| Mata dadu "2" | |||| | 5 |
| | |||| |||| | 9 |
| Mata dadu "3" | ||| | 3 |
| | |||| ||| | 8 |
| Mata dadu "4" | |||| ||| | 8 |
| | || | 2 |
| Mata dadu "5" | |||| | | 6 |
| | |||| || | 7 |
| Mata dadu "6" | || | 2 |
| | |||| | 5 |
| Total percobaan *n* (P) | | 60 |

| Event | Tally | Frequency (*f*) |
|---|---|---|
| Face "1" | | |
| Face "2" | | |
| Face "3" | | |
| Face "4" | | |
| Face "5" | | |
| Face "6" | | |
| Total number of tossing *n* (P) | | |

**Figure 5.** A table to record the game playing

***Activity 5: Calculating Ratio of the Number of Favorable Outcomes to the Number of Possible Outcomes***

The objective of Activity 5 was to introduce the concept of theoretical probability of a particular event. After the students grasped the idea of empirical probability as the result from an experiment, they were guided towards the idea of theoretical probability. For this purpose, a connection to Activity 4 was made, such as "you could determine the occurrence of face '1' after rolling the die 60 times. The question now is how many times will face '1' appear if we roll the die for 1000 times?" A thousand of rolling was chosen on purpose because it was not easy to roll a die 1000 times manually. Therefore, it triggered the students to estimate the possible number of occurrences of face '1'. The students might narrow down from 1000 times of rolling to 500 times or even smaller number and at the end of the activity the students arrived at predicting the probability for face '1' to occur when the die was rolled once. Worksheet was used to help students grasp the idea of theoretical probability from rolling a die (Figure 6).

| Experiment | Sample space S | $n$(S) | An event $X$ | The sample point of event $X$ | Number of sample point(s) $n$(X) | Theoretical probability $P(X)=\frac{n(X)}{n(S)}$ |
|---|---|---|---|---|---|---|
| Tossing a coin | {A, G} | 2 | Face 'number' | A | 1 | $\frac{1}{2}$ |
| Tossing three coins | ... | ... | ... | (A,A,A), (G,G,G) | ... | ... |
| Tossing two dice | ... | ... | ... | ... | ... | $\frac{1}{18}$ |

**Figure 6.** An example of task in the worksheet for Activity 5

In Activity 5 the students have arrived at Graveimejer's (1994) formal level at which the students mostly worked with mathematical representation and concept. In general, the students did not have difficulties in solving tasks addressing theoretical probability of an event. It was confirmed during an interview with students. The following excerpt is example of conversation between the researcher and a student.

Researcher : "*Nando, why did you choose 'dice's faces showing a total of 3' for this event*" [pointing to a task in the worksheet that had a theoretical probability of 1/18]

Nando : "*Because there two sample points, Mam. These two refers to dice's face showing a total of 3*"

Researcher : "*Where did 2 come from?*"

Nando : "*The theoretical probability is 1/18 and we have two dice with 36 elements of the sample space. It means 18 is a half of 36, so the sample point is 1 multiplied by 2*"

Researcher : "*What are these two events (sample points)?*"

Nando : "*Here they are*" [pointing to (1,2) and (2,1) on the worksheet]

The students' works, observation and the results of observation and interview indicate that the students have gained their understanding about theoretical probability.

### *Learning Trajectory for Probability: An Emergent Modeling from Game-Based Learning*

The result of this study was in the form of students' trajectory in learning probability through a game-based learning. Combining the results from the five activities, the learning trajectory can be focused on four levels of modelling: situational level, referential level, general level, and formal level. In the situational level the students grabbed an early notion about random event, sample points, sample space, and empirical probability directly from the game playing. The notion of random event emerged when the students were discussing the ways to fairly determine the first player of the games, i.e. a fair rule is when none of the players know in advance who will be the first player which in terms of probability it refers to the concept of random event. Coins and dice played a crucial role in this situation because as highlight by Paparistodemou, Noss, and Pratt (2008), because they are popular for children and "incorporate both apparent and unsteerable flavours of fairness" (p. 107). Students' notion about sample space was visible when they were discussing the possible number of steps can be taken by a player in Snake-and-ladder game. These possible number of steps were the results of rolling a die. When discussing the fair game rule and possible number of steps in Snake-and-ladder game, students still positioned themselves in game situation. Therefore, the students were still at situational level.

Students' early notion about the concepts of random event, sample points, sample space was strengthened in the referential level. In the referential level the students were asked to refer to the rules and their communication during the game playing. For example, regarding the ways to determine the first player of the games, the students were asked what 'fair' meant and what ways they used to fairly determine the first player. From the classroom discussion, it was revealed that the term 'fair' has led students to the concept of random event. This result is quite similar to the study of Wijaya et al. (2011) which found how the term 'fair' led students to the concept of standard measurement. Such process is quite close to the idea of 'lateral learning' in which students used their current schemes in new situations or ways to establish new schemes (Steffe, 2004). Another term which stimulated students to grab a mathematics concept was the term 'possible outcomes'. During the classroom discussion, this term strengthened students' conception of sample points and sample space.

After the students had a good understanding of the concepts, in the general level some mathematical tools were introduced to the students. Tree diagram and table were used to present the outcomes of tossing coins and a die. These tools were used to generalize possible outcomes of various events which were not directly experienced or experimented by the students; such as tossing a coin and a die or rolling three dice. Furthermore, the tree diagram and table were used to highlight the concept of sample points and sample space. As revealed by Doorman and Gravemeijer (2009) and Gravemeijer, Bowers, and Stephan (2003), there is a positive interaction between the tools that are used by students and students' acquisition of mathematics concepts. Similarly, Shanty (2016) found that informal tools used by students are gradually developed into a more formal mathematics.

In the formal level students already worked with mathematical concept and representation. In this level, the students determined the empirical probability and theoretical probability of particular events. In this matter, the actual results of Sudoku and Snake-and-ladder games were used to help the students grabbed the idea of empirical probability. In the next step, the games were used to help the students constructed the notion of theoretical probability. The notion of theoretical probability was built upon students' understanding of empirical probability combined with a guiding question to give a kind of cognitive conflict that triggered and help students constructed new understanding.

One of the main ideas of HLT is to develop a set of learning activities based on a teacher's conjectures regarding his or her students' current understanding of a targeted mathematics concept including potential obstacles or challenges experienced by students (Lobato & Walters, 2017). In line with this idea, the results of this study indicate how a game-based learning could support the development of students' conceptual understanding of the concept of probability. Furthermore, the alignment between the learning activities and the progress of students' conceptual understanding can be framed as a process of emergent modelling (see Gravemeijer, 1994) from situational level to formal level. Students' learning trajectory from the situational level to formal level in understanding the concept of probability taught in a game-based learning was presented in Figure 7.

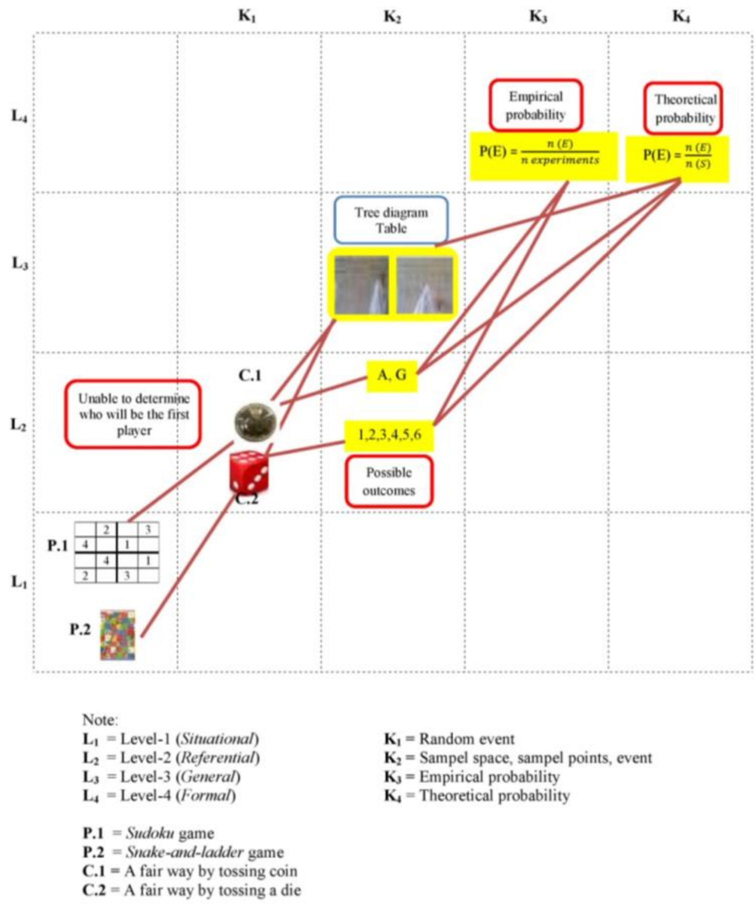



**Figure 7.** Learning trajectory for probability situated in a game-based learning

The present study is aimed to contribute to the development of a local instruction theory for the teaching and learning of probability through a game-based learning. It is revealed that the use of games and their tools can support students develop their understanding of the concept of probability. Students' understanding of probability emerges gradually through four levels of modelling: situational level, referential level, general level, and formal level. The use of 'tools' – both game-based tools (such as coin and dice) and mathematical tools (such as table and diagram) – helped students gain their understanding of probability concept and move from one level to a higher level of modeling. Furthermore, this study reveals that terminologies and interactions used in game playing can stimulate students' notion of mathematics concept. As an example, is the term 'fair' that leads students to the idea of random event. In the situational and referential stages students often used their informal knowledge which underlying mathematical notion. Students' acquisition of concept resulted from the game playing needs to be transformed into more formal level of mathematics. In the general level, mathematical tools – i.e. tree diagram and table – can be used to present students' notion of the mathematics concepts in more formal notation or representation. Finally, in the formal level the students can be guided to see relations between mathematics concepts, e.g. empirical probability and theoretical probability.

## CONCLUSION

The learning trajectory resulted in this study can be considered as an alternative way or a framework of reference for teachers to design a set of learning activities that support the development of students' conceptual understanding of mathematics concept. Nevertheless, generalizing the findings of this study might be done with cautions because different games might give different results. Furthermore, cultural aspects might influence the game playing and the interaction during the games. Therefore, a challenge for further studies is to develop learning trajectories that can work in more general situations. Another future challenge is teachers' competence in developing learning trajectories that fit their own classroom. In this respect, there are important aspects to consider including teachers' knowledge of mathematics, teachers' knowledge of mathematical activities, teachers' knowledge on mathematics teaching, teachers' knowledge of students' learning of a particular topic, and also teachers' ability to hypothesize students' knowledge and learning obstacles.

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