

Development of Derivative Understanding Task Instruments to Explore Student Commognition

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Abstract

Instruments are tool that used to collect research data. The instrument consists of two types, namely the main and supporting instruments. In this paper, we develop the supporting instruments which are used in qualitative research such as commognitive perspective. The instrument development aims to explore and reveal students' cognition in understanding derivative tasks that are valid and reliable. It means the instrument is needed in order to explore cognition and communication in an inseparable manner according to the theory used in this research. The two supporting instruments that developed in this study are the mathematical ability test (MAT) and the derivatives understanding task (TMT). Moreover, the developed MAT instrument is accompanied by source questions, grids and indicators. The MAT consists of 10 questions, and this was tested empirically in the category of valid and high reliability. Furthermore, TMT is developed as a reference for exploring student commognition. The TMT consists of 14 questions. The preparation and development of the instruments in this study are based on relevant theories and supported by empirical data. At the expert review step, validation is carried out in terms of content, construct and language by experts. Each step is tested for readability, then suggestions and comments are provided for improvement. The final results obtained show that the two supporting instruments (MAT and TMT) are feasible to use in exploring student commognition because these bring up keywords, visual mediators, endorsed narratives, and routines, as a commognition characterher.

Keywords: Commognition, Development, Instrument, Derivative

Abstrak

Instrumen adalah alat yang digunakan untuk mengumpulkan data penelitian. Instrumen terdiri dari dua jenis, yaitu instrumen utama dan pendukung. Dalam tulisan ini, kami mengembangkan instrumen pendukung yang digunakan dalam penelitian kualitatif seperti perspektif komognitif. Pengembangan instrument bertujuan untuk menggali dan mengungkapkan komognisi mahasiswa dalam memahami tugas turunan yang valid dan realibel. Hal ini berarti diperlukan instrument yang dapat menggali kognisi dan komunikasi secara sejalan yang tidak terpisahkan sesuai teori yang digunakan dalam penelitian ini. Dua instrumen pendukung yang dikembangkan dalam penelitian ini adalah tes kemampuan matematika (MAT) dan tugas pemahaman turunan (TMT). Selain itu, instrumen MAT yang dikembangkan dilengkapi dengan pertanyaan sumber, kisi-kisi dan indikator. MAT terdiri dari 10 soal, dan diuji secara empiris dengan kategori valid dan reliabilitas tinggi. Selanjutnya, TMT dikembangkan sebagai acuan untuk menggali komognisi mahasiswa. TMT terdiri dari 14 soal. Penyusunan dan pengembangan instrumen dalam penelitian ini didasarkan pada teori yang relevan dan didukung oleh data empiris. Pada tahap expert review, dilakukan validasi dari segi isi, konstruk dan bahasa oleh para ahli. Setiap langkah diuji keterbacaannya, kemudian saran dan komentar diberikan untuk perbaikan. Hasil akhir yang diperoleh menunjukkan bahwa kedua instrumen pendukung (MAT dan TMT) layak digunakan dalam menggali kognisi siswa karena memunculkan *keywords*, *visual mediator*, *endorsed narrative*, dan *routines* sebagai karakter komognisi.

Kata kunci: Komognisi, Pengembangan, Instrumen, Turunan

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INTRODUCTION

Mathematics plays a role in advancing various scientific disciplines. Broadly speaking, mathematics is divided into four disciplines, namely arithmetic, algebra, geometry and analysis (Bell, 1978). In order to solve a mathematics problems, everyone has different ability. In this study, we develop standardized test to explore student ability such as commognition aspect. According to the Big Indonesian Dictionary, the word "ability" means being able to do something (Ministry of Education, 2007). Robbins & Judge (2007) said "ability refers to an individual's capacity to perform the various tasks in a job", which means that ability is based on an individual's capacity to complete various tasks in a job. Furthermore, mathematical abilities in this study are the ability of mathematics education students in solving standardized questions (standardized tests).

Cognition is an individual understanding that is intrapersonal. In addition, cognition is supported by the relationship of knowledge and causal process which one of that is communication (Adams, 2014). Communication is a crucial element in learning (Wichelt, 2009). By communicating, lecturers can recognize their student ability. Moreover, they can measure how understand their student about the material that being studied. Finally, they can provide an assistance for student who have some difficulties (Pourdavood & Wachira, 2015).

According to Hiebert & Carpenter (1992), understanding is a mathematical idea or procedure or fact that understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representation. On the other word, an idea or procedure in mathematics that can be understood by students if a person's mental is a part of a network that is interconnected with its internal representation. Understanding in the context of mathematics learning is related to the teaching and learning process of students in the classroom. One of the main goals of learning mathematics is students will have a good level of understanding of mathematical knowledge. Anderson and Krathwolh (2001) stated that the seven cognitive processes in the category of understanding including interpreting, exemplifying, classifying, summarizing, inferring, comparing dan explaining. Based on these seven cognitive processes, researchers developed instruments to explore aspects of student commognition.

Commognition Theory

Sfard's (2008; 2020) perspective, which known as the commognitive framework, used the term cognition as a combination of words that cannot be separated, namely cognition and communication. The characteristic that stands out in commognition is discourse. The word discourse used in this study denotes "any specific instance of communication, whether diachronic or synchronic, either with others or with oneself, whether predominantly verbal or with the help of other symbolic systems" (Sfard & Kieran, 2001). Discourse shows human activities which some experts define as "languages in action" (Brown & Yule, 1983). The emphasis on mathematics as discourse is defined as a particular form of

communication.

In the commognition framework, learning mathematics is defined as a change in students' discourse that can be seen when they have communication (Sfard, 2007; Tabach & Nachlieli, 2016). Furthermore, Ben-Zvi and Sfard (2007), Sfard (2008, 2020) and Nardi et al. (2014) said that the mathematical discourse is characterized by keywords, visual mediators, endorsed narratives, and routines. Moreover, mathematics is a discourse because mathematics is a particular well-defined form of communication (Sfard, 2007; 2008; 2012; 2017). This is in line with World Economic Forum USA competencies that are needed and must be possessed by every individual in the 21st century, one of which is communication.

Derivatives are difficult material for some students because there is still a lack of understanding of the terms that support the material that have been studied previously, for example "function", "limit", and "difference quotient" (Park, 2015; Gallego-Sánchez et al., 2022; Wille, 2020). Several previous studies using derivative instruments were conducted by Park (2013) on the derivative as a function; Berger (2013) about a function at an undefined point and determines whether a vertical asymptote corresponds to that point. Furthermore, 7 cognitive processes are created made as research instruments with the derivation material so that they are called the derivatives understanding task (TMT).

Development of Supporting Instruments

According to the National Research Council Committee on Scientific Principles for Educational Research, instrument development in a study must pay attention to the relevance of the instrument to certain research questions (Taber, 2018). The research instruments consist of: (1) the researcher himself as the main instrument that plans, implements and analyzes, (2) the supporting instruments in the form of cognitive style tests, tests of mathematical abilities, gender questionnaires and the task of understanding derivatives. The researcher as the main instrument plays a role in controlling the entire process from data collection to data analysis (Lincoln & Guba, 1985; Merriam, 1988). As the main instrument, the researcher acts as an observer, participant, insider and outsider because in order to dig up information from the subject the researches cannot be replaced by other people. Researcher is as observers and participants of the activities observed in obtaining data. The roles as insiders and outsiders are in analyzing and interpreting data. As an insider, the researcher maps the complex relationship between the things said and the actions taken by the subject, while as an outsider the discourse being studied.

The supporting instruments used in this study are: (1) The GEFT (Group Embedded Figures Test) cognitive style test instrument developed by Witkin et al. (1971) using pictures. The GEFT instrument to determine subjects who have field-independent (FI) and field-dependent (FD) Cognitive styles uses the specified criteria. The criteria used in the selection of subjects is the criteria according to (Kepner & Neimark, 1984). Moreover, subjects who can answer correctly 0-9 are classified as field-dependent

and 10-18 are classified as field-independent. The GEFT test instrument and its implementation instructions are obtained from Hanne Huygelier, a research unit for experimental psychology, Leuven Belgium via email, (2) gender questionnaire was measured using a tool called BSRI (Bem Sex Role Inventory), (3) Mathematical Ability Test (MAT) and, (4) Derivative Understanding Task (TMT). Researchers can collect data through instruments or tests (Creswel, 2014). Furthermore, the development of instruments that can be used to see student cognition is carried out. A study requires an instrument which is used to measure something observed such as natural and social phenomena (Sugiyono, 2010). This paper will discuss the development of MAT and TMT instruments.

The MAT instrument is obtained from questions in the Calculus book (Varberg, Purcell, & Ridgon, 2010) and the 2018 SBMPTN exam questions, the Science and Technology Basic Ability Test (TKD Saintek), where the questions have been modified into a description of 10 questions. The items used in this test are taken from practice questions in the Calculus book by Varberg et al. (2010) (5 items) and from the 2018 SBMPTN exam questions, the Science and Technology Basic Ability Test (TKD Science) (5 items).

The test questions given are in the form of descriptions with material on Derivatives, Composition of Functions, Geometry, Algebra, Probability, and Numbers. The reason for choosing the form of the description is to determine the mathematical ability of the subject, because with the description the problem solving process will be seen. The MAT questions and their sources can be seen in Table 1.

Table 1. Source of MAT questions

Question Numbers	Source	Problems
1	Problem 1.6 no 32 Calculus	Function continue
2	Problem 2.3 no.3 Calculus.	Derivative
3	Problem 0.6 no.2 Calculus	Function composition
4	Problem 3.3 number 12	Derivative
5	Problem 10 SBMPTN 2018	TKD Science
6	Problem 3 SBMPTN 2018	TKD Science
7	Problem 0.3 Calculus	Coordinate system
8	Problem 14 SBMPTN 2018	TKD Science
9	Problem 7 SBMPTN 2018	TKD Science
10	Problem 5 SBMPTN 2018	TKD Science

In order to find out which MAT used in this study is valid and reliable, a rational test is carried out through trial tests. The validity is achieved when there is a match between the test part (items) and the test as a whole. Furthermore, the reliability test is carried out both rationally and empirically by taking into account the value of Cronbach's Alpha. Cronbach's alpha value is statistically presented as evidence at instrument quality (Taber, 2018).

The difficulty index, symbolized by p (proportion), is shown to determine the case and the difficulty of a given question. The difficulty index of item is considered by three criteria. These criteria

are divided by interval to indicate high difficulty ($p < 0.25$), moderate difficulty ($0.25 - 0.75$) and low difficulty ($p > 0.75$). Furthermore, a discrimination index is also considered, namely the ability of an item to be able to distinguish between students who master the material or not. The discrimination index of 0.40 and above is considered an excellent item, 0.30–0.39 quite good, 0.20–0.29 an item is marginal (i.e. can be improved), and 0.19 or less is a poor item that can be rejected or improved (Ratumanan & Lauran, 2011; Taib & Yusuf, 2014).

Furthermore, the TMT instrument was developed using the seven cognitive processes in the category of understanding including (1) interpreting occurs when a student can change the form of representation to another; (2) exemplifying, namely giving specific examples of general knowledge; (3) classifying occurs when a student is given a stimulus, for example several examples of questions, the student is able to classify them; (4) summarizing is able to abstract/generalize the main features of a statement; (5) inferring, presenting information or providing a collection of information, based on the information provided, conclusions can be drawn; (6) comparing, namely determining the similarities or differences between two things in a problem; (7) explaining, building a causal model or a particular system, then using that relationship (Anderson & Krathwohl, 2001). The TMT instrument is later used to explore student commognition.

METHODS

The MAT questions were tested on mathematics education students with a letter of approval which was ratified number 138 of 2020. The questions were given to 23 students. The time allotted to work on the question is 2 hours. In accordance with the specified time, student answer sheets are collected and then corrected based on the score determined for each question. To find out the MAT instruments used are in the valid and reliable categories, an analysis is carried out. Furthermore, readability test was carried out. The two criterias for the MAT instrument is said to be valid are (1) the question sentence does not cause multiple interpretations; (2) sentence questions using simple language that are easily understood by students. If the MAT item is considered invalid, then the item is aborted and replaced and then discussed with the research team.

Furthermore, the development of the TMT which consists of 14 questions and grouped into two parts and hereinafter referred to as TMT 1 and TMT 2. Each TMT consists of 7 questions with the same content for each corresponding item. The Derivative Understanding Task (TMT) in this study is designed to obtain the derivate understanding data, as a basis for exploring student commognition. The task of understanding derivatives is in the form of descriptive questions compiled by researchers and consulted to the experts team. The TMT was validated by three experts in mathematics education before being used. Furthermore, it is given to students who become research subjects.

RESULTS AND DISCUSSION

Development of Mathematics Ability Test (MAT)

The mathematical ability test (MAT) which has been compiled and developed to collect data on students' mathematical ability scores is then used to select the research subjects. The score data obtained by students are categorized into three groups, namely high, medium, and low mathematical abilities. In this study, a medium MAT score is used with the scoring limit range is 0.00 – 100.00. The development of this instrument is accompanied by scoring guidelines. The MAT instrument of the MAT question grid can be seen in [Table 2](#).

Table 2. Mathematical ability test grid

Subject	Indicators	Level	Problem Number
Continuity	Equation graph sketch Investigate continuity	Application (C3)	1
Derivative	Specifying the first derivative using definition.	Application (C3)	2
	Using the Second Derivative Test.	(C3)	4
Geometry	Determine the distance from the point to the line.	Analysis (C4)	6
	Finding the radius of a circle from a system of three circles.	(C4)	7
	Determine the tangent on the curve.		8
Function composition	Determine the value of the composition of two functions	Evaluation (C5)	3
Algebra	Finding the sum of the first terms of a geometric series	Analysis (C4)	10
Probability	Determining many permutations	Analysis (C4)	9
Number	Determine the value of the parameter form of the exponential function	Analysis (C4)	5

Based on [Table 2](#), the mathematical ability test is then made where consist of 10 items, with 7 learning materials. The given MAT questions can be seen in [Table 3](#).

Table 3. MAT problems

No	Problems
1	Given the following function $f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 1 \\ 2 - x & \text{if } x > 1 \end{cases}$
	a. Sketch the graph of the function f
	b. Find the continuity of the function f at the point $x = 1$.
2	Given $(t) = t^2 - t$. Use the definition of a derivative to determine the value of $f'(1)$
3	For $f(x) = \frac{x}{x-1}$ and $g(x) = \sqrt{1+x^2}$, determine each value of the following:
	a. $(f \circ g)(\sqrt{8})$ b. $(g \circ f)(0)$

No	Problems
4	Determine the maximum value of the function $f(x) = -\frac{1}{3}x^3 + x$ using the second derivative test.
5	If the tangent to the curve $y = \frac{1}{4}x^2 - 1$ at point $P(a, b)$ with $a < 0$ intersects the y-axis at point $Q(0, -2)$, find the value of $a + b$.
6	Given a cube ABCD.EFGH with side length $2\sqrt{2}$. Let P in the middle of \overline{AB} and point Q in the middle \overline{BC} . Determine the distance between point H and \overline{PQ} .
7	If the circle $x^2 + y^2 + 2Ax + By = 0$ and $x^2 + y^2 + Ax + 2By = 0$ have radius 5 and 10, respectively, find the radius of the circle $x^2 + y^2 + Ax + By = 0$.
8	It is known that the curves $y = 2^{3x^2+cx-1}$ and $y = 4^{x^2-\frac{c}{2}}$ are tangent. Determine the value of $c^2 + c$.
9	Ari and Ira are two members of a group of 9 people. Determine how many ways to make a straight line, provided that Ari and Ira are not side by side.
10	Given a geometric sequence (u_n) , where $u_3 + u_4 = 4(u_1 + u_2)$ and $u_1 u_4 = 4u_2$. Find the sum of the first 4 possible terms of the sequence.

After the instrument has been arranged the next step is to test the readability of the MAT. The readability test is carried out to determine the readability level of words, phrases, and sentences for each item of the MAT. The purpose of the readability test is to determine the effectiveness of the sentences used in each item. As a result errors that may occur due to the incompatibility of the research objectives with the understanding of the prospective research subjects can be minimized. The readability test can be seen in [Table 4](#).

Table 4. The Readability test of the MAT Instrument

Question	No	Student Answers
Does the question sentence not cause multiple interpretations?	1	Yes, because each question is clear what you want to ask and does not cause double interpretation
Are the wording questions simple and easy to understand?	2	No, the question sentence has a clear interpretation
	1	Yes, I think the use of language is easy to understand
	2	Yes, but the question sentence no.9 is not effective. It is better if the first sentence reads "Ari and Ira are members of a group consisting of 9"

In this readability test, the draft is given to the two Mathematics Education students and this is carried out before being tested in the field. After the readability test is carried out, the sentences in the MAT that are not clear then be corrected so that it can be understood by students. Based on the results of the readability test on the MAT, the student stated that they understand with the given sentence. As a result, it can be stated that TKM can be used in selecting research subjects.

Instrument validation is then carried out by paying attention to aspects of (1) construction, (2) content, and (3) items. A measuring instrument has a high construct validity if the results of the measuring instrument are in accordance with the characteristics or indicators of the behavior being measured. In this study, construct validity is carried out by ensuring that the results of the student's MAT actually measured their mathematical ability, not other variables. This is based on the MAT questions which adopted from the Calculus book (Varberg et al., 2010) and the 2018 SBMPTN exam questions, Science and Technology Basic Ability Test (TKD science). Validation relates to the extent to which an instrument measures the level of mastery of the content of a particular material that should be mastered. In order to the research instrument will have good content, it is essential to study and create a question grid.

Furthermore, item validity is carried out by empirical trials. Scores are processed using Bivariate Pearson analysis through SPSS software. The results of the item validity test of the MAT using the Pearson Bivariate is obtained with its the correlation value is given in Table 5.

Table 5. Correlation criteria between item scores and total scores

Problem	<i>r</i> Value	Sig value	Category
P1	0.77	0.000	Valid
P2	0.45	0.031	Valid
P3	0.48	0.022	Valid
P4	0.65	0.001	Valid
P5	0.56	0.006	Valid
P6	0.47	0.024	Valid
P7	0.56	0.005	Valid
P8	0.58	0.003	Valid
P9	0.59	0.003	Valid
P10	0.71	0.000	Valid

Based on Table 5, all items can be said to be valid because they have a calculated *r* value greater than *r* table 0.43 or a sig value is less than 0.05. This means that the MAT questions have a significant correlation with the total score so that these can be declared valid. Furthermore, measuring the reliability of the instrument is conducted. The instrument reliability can be interpreted that the instrument can be trusted to be used as a data collection tool. Instruments that can be trusted and which are reliable will produce data that can be trusted too. To find the reliability coefficient of the description test instrument, we use the Alpha formula. The Cronbach's Alpha value as given in Table 6 is 0.770. This indicates that the questions given to students have a high level of reliability.

Table 6. Reliability statistics

Cronbach's Alpha	N of Items
0.770	10

In this study, the high reliability instrument criteria is used because it lies in the interval of (0.60 - 0.80). After the validity and reliability of the test is known, the next step is to determine the difficulty level of the problem by using the problem item difficulty index criteria which is interpreted based on three criteria low difficulty, moderate difficulty and high difficulty (Ratumanan & Laurant, 2011; Taib & Yusuf, 2014). Table 7 shows the difficulty level of each item that obtain by apply SPSS level of the items.

Table 7. Problem item difficulty level criteria

Problem Item Number	Mean	Difficulty Level Criteria of Problem Item
P1	2.00	Low difficulty
P2	2.39	Low difficulty
P3	0.70	Moderate Difficulty
P4	1.61	Low difficulty
P5	1.30	Low difficulty
P6	0.57	Moderate Difficulty
P7	0.57	Moderate Difficulty
P8	0.35	Moderate Difficulty
P9	0.61	Moderate Difficulty
P10	0.61	Moderate Difficulty

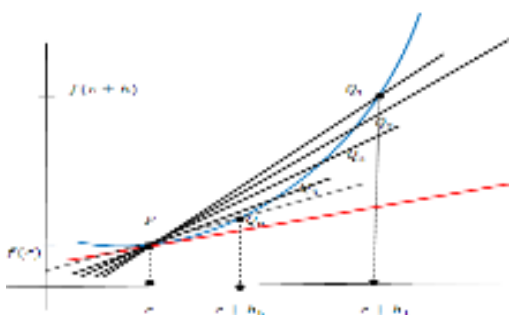
From Table 7, it is found that there were 4 items with low levels (P1, P2, P4 and P5). The difficulty level of questions with moderate criteria is given by P3, P6, P7, P8, P9 and P10. In order to determine the discriminating power, we use the Pearson correction value (rcount). The discriminating power index (D) is given to measures how well an item differentiates students' ability levels. Item discriminating power index is interval -1.00 to +1.00. The higher the D score of an item is obtained the better the item to differ student with ability from students with low ability (Hopkins & Antes, 1999). The questions given to students have very good discriminating power. The summary of the calculated r value is given in Table 5. The result show that this question given to students have very good discriminating power.

Based on the Correlation Criteria between Item Scores and Total Scores (Table 5) all the valid question items are obtained. From the results of Reliability Statistics calculations (Table 6), it can be concluded that the reliability of the questions is high. The distinguishing power index is then conducted and we obtained that all the items are very good. Furthermore, it can be concluded that TKM can be used to select research subjects. Questions that have a poor level of difficulty or differentiating power will result the questions not being able to measure students' true abilities. On the other hand, if a question has a good level of difficulty and differentiating power then the better the question will be in measuring students' abilities (Dewi, Hariastuti, & Utami, 2018).

Development of Derivative Understanding Task (TMT) Instruments

The derivatives understanding task (TMT) consists of 14 questions which are grouped into two part. Moreover, the two part are referred to as TMT 1 and TMT 2 respectively. TMT was developed based on the Semester Learning Plan (RPS) in the Derivative Calculus course which is used as a reference for knowing the commognition of Mathematics Education students Each TMT consists of 7 questions with the same content and composition in each corresponding item. The contents of the seven questions are as follows: (1) interpreting, changing the form of one representation to another; changing the form of images to the form of words, changing the form of formulas to the form of words, (2) exemplifying, making examples of functions that have derivatives and being able to give examples of functions that do not have derivatives, (3) classifying, with the stimulus providing examples of functions that have derivatives and functions that have no derivatives, students classify into functions that have derivatives and do not, (4) summarizing, abstracting/generalizing the main features of functions that have derivatives and functions that do not have derivatives, (5) inferring, presenting information or providing a set information, based on the information provided draw conclusions. For example, "If f has a derivative in c , then f is continuous in c ", (6) comparing, finding correspondence, namely determining the similarities and differences between two things in a problem, and (7) explaining, building a causal model of a system, for example by certain functions are known, using the relationship of the derived function. The TMT instruments can be seen in [Table 8](#).

Table 8. The TMT 1 and TMT 2 instruments

No	Problems 1	Problems 2
1	<p>Look at the picture below</p>  <p>PQ_1, Q_2, \dots, Q_n the points of intersection of the secant line with the curve. Express the definition of the derivative of f at $x = c$ based on the information above in your own words.</p>	<p>Given the following derivative formula</p> $(i). f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$ $(ii). f'(c) = \lim_{x \rightarrow 0} \frac{f(x) - f(c)}{x - c}$ <p>Express the definition of the derivative based on the above formula in your own words!</p>
2	<p>Give two examples of functions that have a derivative at one point</p>	<p>Give two examples of functions that have no derivative at one point</p>
3	<p>Pay attention to the following functions</p> <ol style="list-style-type: none"> $f(x) = x$, $di x = 0$ $f(x) = [x]$, $di x = 0$, $f(x) = x - 1$, $di x = 1$ 	<p>Pay attention to the following functions</p> <ol style="list-style-type: none"> $f(x) = x^2$, $di x = 0$ $f(x) = x x$, $di x = 0$ $f(x) = x^2 - 1$, $di x = 0$

No	Problems 1	Problems 2
4	<p>Which of the above functions has a derivative at a point?</p> <p>Some functions with formula information and their properties are given below:</p> <p>a. $f(x) = x$, continue, have no derivation at $x = 0$.</p> <p>b. $f(x) = [x]$, not continue, have no derivation at $x = 0$.</p> <p>c. $f(x) = x - 1$, continue, have no derivation at $x = 1$.</p> <p>a. $f(x) = x^2$, continue, have derivative at $x = 0$</p> <p>What is the characteristic of a function that has a derivative?</p>	<p>Which of the above functions has a derivative at a point?</p> <p>Some functions with formula information and their properties are given below:</p> <p>a. $f(x) = x x$, continue, have derivative at $x = 0$.</p> <p>b. $f(x) = x^2 - 1$, continue, have derivative at $x = 0$.</p> <p>c. $f(x) = \frac{x}{ x }$, not continue, have no derivation at $x = 0$.</p> <p>What is the characteristic of a function that has a derivative?</p>
5	<p>Given the formula</p> $f(x) = \begin{cases} \frac{1}{3}x^2, & x < 3 \\ 3x, & x \geq 3 \end{cases}$ <p>, the function f has a derivative at $x = 3$, where $f(3) = 9$ and f is continuous at $x = 3$. Based on the two information give above what can you conclude?</p>	<p>The function $f: [0,2] \rightarrow R$ defined as</p> $f(x) = \begin{cases} 2x, & 0 \leq x < 1 \\ 1, & 1 \leq x \leq 2 \end{cases}$ <p>is not continuous at $x = 1$, f has no derivative at $x = 1$. Based on the two information given above what can you conclude?</p>
6	<p>What is the relationship between the derivative of the sum of two functions and the sum of the derivatives of each of these functions at the same point?</p>	<p>What is the similarity between the sum limit and the derivative of the sum of two functions at one point?</p>
7	<p>Assume that f is an odd function and has derivatives everywhere. Explain why the derivative of this odd function is an even function.</p>	<p>A function f has a symmetric derivative f'_s defined by</p> $f'_s(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x-h)}{2h}$ <p>exists. Explain that $f'_s(x) = f'(x)$</p>

Furthermore, a readability test was conducted by two validators in the field of mathematics education and one person in the field of mathematics. All of them have doctoral degrees and at least 20 years have teaching experience. Aspects that become validator's assessment in TMT validation include: (1) material, (2) construction, and (3) language as given [Table 9](#).

Table 9. Validator assessment results

No	Description of assessment Item	Validator		
		1	2	3
1	Is there any comprehension task content that doesn't match the derivative?	The given task related to the concept of derivatives	No, there is not	No, there is no. All content in the task of understanding the given derivative is in accordance with the material

No	Description of assessment Item	Validator		
		1	2	3
2	Can the tasks given be able to see students' understanding of derivatives?	The given task can be done if student understands the concept of derivative	Can see student understanding	Yes it can. The assignments given have accommodated to see students' understanding of the derivatives of both concepts and in solving problems
3	Does in doing this task allow to occur the communication process they are related to the relevan concept	The given assignments given enable the communication process to occur because they are related to concepts relevant	It allows for the communication process to occur. However, students are a bit difficult to understand because the material is too difficult	Yes, it does In carrying out this task it is very possible for the communication process to occur, both between participants with researcher as well as.
4	Does the sentence in the task of understanding derivatives use simple language, communication and easy to understand by students?	The sentences used are very commutative and easy to understand	Yes, there are some symbols that need to be revised	In general, the sentences in TMT are communicative and the language is simple, but there are some things that might be a little difficult for students to understand, such as in question No. 1 for the question "which of the above functions has a derivative at the point?" Shouldn't it be better to just mention the point name?
5	Are there any questions that direct students to certain answers?	The questions from the assignment are very clear and do not lead to a specific answer	No, there is not	No, there is not The questions given are very straightforward and clear which do not lead to a particular answer
	Conclusion	This is properly to be used by revising some symbols	Proper to use	Proper to use

The results obtained by the three validators concluded that the TMT instrument was suitable to be used to explore student cognition in understanding derivatives. Several suggestions were given by the validators. Suggestions from validator 2 is about symbols. These needed to be corrected. Moreover,

validator 3 suggested about the name of the point that needed to be mentioned, while validator 1 agrees (does not give advice). Based on the readability test, it shows that the three validators can understand the information given to the TMT and allow communication to occur and do not cause double interpretation. As result, the TMT 1 and TMT 2 serve as a reference for uncovering Mathematics Education Student Commognition. The result that obtained from the derivative understanding task (TMT1, TMT 2) that have been developed, we can conclude that the student commognition can be discovered. We provide the explanation as given as follow.

In the question of categories about interpreting, classifying and summarizing will bring up keywords. In interpreting, in representing the form of an image to the form of words in defining derivatives, it requires the keywords "curve", "tangents" and "secant" as the elements that make up the picture. Furthermore, in representing the form of the formula to the form of words in defining derivatives, there is the word "limit" which is an everyday word which is also used in discourse as the formal language of mathematics. In the classifying category, based on the stimulus given, it is expected to distinguish between "steps function", "absolute function" and other functions. This greatly facilitates students in classifying these functions. Furthermore, in the summarizing category, in generalizing the main characteristics of functions, the first step taken by students is to determine the nature of the function based on the given formula information. Consequently, when students start to answer the given tasks then the keywords need to be considered by them. The keywords in discourse are grouped into four phases, namely passive use, routine-driven use, phrase-driven use and objectified use (Roberts & Roux, 2018).

The visual mediator element appears in the interpreting, classifying, comparing and explaining categories. Interpreting category, students can give symbols to the dots contained in the image. In the classifying category, displays and writes the symbol of a function, for example the ladder function symbol, the absolute value function and other simple functions. In the comparing category, it requires students to think about symbolizing sentences in the problem, for example "the derivative of the sum of two functions", "the number of derivatives for each function" and "sum limit". Likewise in the explaining category, in this case using the derived function relationship with other functions. The physical objects used when communicating are called visual mediators (Zayyadi, Subanji, Hidayanto, & Sulandra, 2019).

Endorsed narrative can appear in interpreting, classifying and comparing categories during the process of understanding derivatives. Because during this process a lot has to do with mathematical definitions and theorems in mediating solving problems. Endorsed narratives that appear, for example, use the theory of left limit and right limit. Furthermore, elements of routines can appear in five categories, namely exemplifying, classifying, summarizing, inferring and explaining. In this category, you will see the processes carried out by students with the help of keywords, visual mediators and endorsed narratives. So it can be said that routine action is the core of learning. Furthermore, routine is the fundamental in which all student creativity is rooted. In addition, this is as a medium for students to

explore their expression (Lavie, Steiner, & Sfard, 2019).

CONCLUSION

This study shows that the development of the mathematics ability test (MAT) instrument can be used for the selection of research subjects. The validation results, namely content, constructs and items indicate that the developed instrument can be used in exploring commognition students. The results of the given readability test to the two students also satisfy the requirements that this instrument is properly to use. The results of the validity and reliability on small-scale trials indicate that the MAT instrument has valid and reliable.

In addition, the seven cognitive processes are used in creating task on derivative material. These are interpreting, exemplifying, classifying, summarizing, inferring, comparing and explaining. Moreover, the task given is defined as the Derivative Understanding Task (TMT). The task of understanding derivative (TMT) instrument was developed to explore aspects of university-level student commognition. This task is based on seven cognitive processes in understanding derivative which can then produce 14 auxiliary instruments in this study. The readability test process shows that the three validators can understand the information provided on the TMT and allow communication to occur and do not cause double interpretation. This indicate that the instrument is properly to use. Finally, we can conclude the instruments are properly to explore student commognition because all cognitive elements appear in each cognition category. This study contribution to literature about instrument development for explore commognition student on subject derivative.

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